Physiologic Optics

DIOPTRICS OF THE EYE, FUNCTIONS OF THE RETINA, OCULAR MOVEMENTS AND BINOCULAR VISION

BY

DR. M. TSCHERNING

ADJUNCT-DIRECTOR OF THE LABORATORY OF OPHTHALMOLOGY
AT THE SORBONNE, PARIS

AUTHORIZED TRANSLATION

From the Original French Edition, Specially Revised and Enlarged by the Author

BY

CARL WEILAND, M. D.

FORMER CHIEF OF CLINIC IN THE EYE DEPARTMENT OF THE JEFFERSON
MEDICAL COLLEGE HOSPITAL OF PHILADELPHIA

WITH 212 ILLUSTRATIONS

FOURTH EDITION

PUBLISHED BY

THE KEYSTONE PUBLISHING CO.

PHILADELPHIA, U. S. A.

1924

PUBLISHED BY THE KEYSTONE PUBLISHING CO.

TRANSLATOR'S PREFACE

Physiologic Optics is a science which, on the one side, touches the highest philosophic problems of the human mind and, on the other side, keeps in intimate contact with the practical work of the ophthalmologist, who, in his daily work of refraction, can be guided safely only by its principles.

Many are the text-books on this important subject. Some are mere compilations of older facts and some are written by men that soar so high above the field of the practical work of the ophthalmologist that their abstract scientific investigations lose almost all contact with these practical workers.

The present book is neither a mere compilation nor an abstract theoretical investigation, but a collection of all the old and new scientific facts that have any bearing on the practical work of the oculist and optician. It is written by a man who lately has probably done more original work in this line than any other since Helmholtz and Donders, and who, furthermore, has been in constant contact with practical ophthalmology. Dr. M. Tscherning, who was born in Denmark in 1854, studied ophthalmology at Copenhagen under the philosophic mind of Hansen Grut. Since 1884 he has been adjunct-director of the laboratory of ophthalmology at the Sorbonne, where, since the deplorable disability of Javal, he himself has performed the functions of the director. This laboratory, which was founded in 1876 for Javal, after he had become widely known by his translation of the "Physiologic Optics" of Helmholtz, has given a new impetus to this science in France.

Here Tscherning has made all his important original investigations, especially on ophthalmometry, the catoptric images of the eye, astigmatism, spherical aberration and accommodation. All this original work, as well as that of former investigators, is described in this book with great clearness and succinctness,

almost entirely free from tedious mathematical encumbrances. Instead of long formulæ, the experiment and simple geometrical deductions are employed to explain the observed phenomena. The translator has endeavored to reproduce the clearness and brevity of expression of the original as much as possible. How far he has succeeded in this, it is not for him to judge.

This English edition, as has been indicated on the title page, contains many additions in the text by Dr. Tscherning, who has thus brought his book thoroughly up to date. The few notes added by the translator have been included in brackets with the letter W. appended. A list of illustrations and an index have been compiled to enhance the practical value of the book.

It is true that some of the ideas expressed by the author, especially those about the use of mydriatics for ordinary purposes of refraction and the use of spectacles, are not in accord with current views about these subjects on this side of the Atlantic. But even those who cannot agree with the author on these questions will find many new facts and ideas which will make a study of the book of great interest and profit. The translator only hopes that the reader may experience the same intellectual pleasure that he felt while reading and translating this work of one of our greatest investigators in the field of physiologic optics.

CARL WEILAND, M. D., Philadelphia, U. S. A.

TABLE OF CONTENTS

BOOK I

OCULAR DIOPTRICS

CHAPTER I

OPTIC PRINCPLES

1.	Optic Properties of Bodies	1
2.	Rectilinear Propagation of Light	1
3.	Reflection and Absorption	2
4.	Regular Reflection	2
5.	Plane Mirrors. Construction of the Image	3
6.	Concave Spherical Mirrors	4
7.	Convex Mirrors	7
8.	Practical Remarks	7
9.	Refraction	9
10.	Quantity of Light Reflected. Total Reflection	10
11.	Refraction by Plates with Plane and Parallel Surfaces	11
12.	Refraction by a Prism	12
13.	Refraction by a Spherical Surface	13
14.	Infinitely Thin Lenses	17
15.	Theory of Gauss	23
	Bibliography	32

CHAPTER II

THE OPTIC SYSTEM OF THE EYE

16.	Optic Constants of the Eye	33
17.	Optic System of the Eye	38
18.	Aperture of the System	41
19.	Point of Fixation. Visual Line	44
20.	Optic Axis. Angle a	45
21.	Useful Image	46
	Bibliography	46

CHAPTER III

THE FALSE IMAGES OF THE EYE

22. General Remarks 47
23. The Images of Purkinje 48
24. Manner of Observing the Images of Purkinje 50
25. False Images of the Second Order 54
26. Manner of Observing the Sixth Image 54
Bibliography 56

CHAPTER IV

OPHTHALMOMETRY

27. Principles of Ophthalmometry 57
28. Methods of Doubling 59
29. The Ophthalmometer of Javal and Schiotz 61
30. Results of the Measurement of the Cornea 66
31. Measurement of the Angle *a* 77
32. Determination of the Position of the Internal Surfaces 80
33. Determination of the Centers of the Internal Surfaces 83
34. Direct Determination of the Radii 84
35. General Remarks 85
Bibliography 87

CHAPTER V

CIRCLES OF DIFFUSION

36. Definition 88
37. Line of Sight 89
38. Accommodation 89
39. Experiment of Czermack, Scheiner and Mile 90
40. The Optometer of Thomas Young 91
41. Effects of the Stenopaic Opening 92
Bibliography 94

CHAPTER VI

ANOMALIES OF REFRACTION

42. General Remarks 95
43. General Remarks on Ametropia 97
44. Optometers 100
45. Myopia 101
46. Choice of Spectacles 104
47. Treatment of Myopia 106
48. Hypermetropia 109
49. Aphakia 111
Bibliography 113

CHAPTER VII

SPHERICAL ABERRATION

50. Optic Principles 114
51. Phenomena Dependent on the Spherical Aberration of Lenses... 115
52. Aberration of the Human Eye. Experiments of Volkmann..... 120
53. Experiments of Thomas Young 121
Bibliography 130

CHAPTER VIII

CHROMATIC ABERRATION

54. Optic Principles 131
55. Chromatic Aberration of the Eye 133
56. Experiment of Wollaston 134
57. Results 135
58. Phenomena of Dispersion, the Pupil being Partly Covered...... 136
59. Correction of Chromatic Aberration 137
Bibliography 137

CHAPTER IX

REGULAR ASTIGMATISM

60. Optic Principles. Astigmatism Produced by the Form of the Surfaces 138
61. Defects of the Image 141
62. Astigmatic Surfaces 142
63. Astigmatism by Incidence 143
64. Astigmatism of the Human Eye. Historical 145
65. Physiologic Astigmatism 146
66. Corneal Astigmatism 147
67. Measurement of the Corneal Astigmatism 147
68. Regular Corneal Astigmatism 149
69. Relations between Ophthalmometric and Subjective Astigmatisms 150
70. Astigmatic Accommodation 155
71. Post-Operative Astigmatism 156
72. Keratoconus 157
73. Symptoms of Astigmatism 158
74. Examination of Astigmatic Patients 159
Bibliography 163

CHAPTER X

IRREGULAR ASTIGMATISM

75. General Remarks 164
76. Examination of the Eye with a Luminous Point 165
77. Different Forms of Irregular Astigmatism.................... 166
78. Rules for Analyzing the Figures of the Luminous Point....... 171
Bibliography 175

CHAPTER XI

ENTOPTIC PHENOMENA

79. Manner of Observing Entoptic Phenomena 176
80. Analysis of Entoptic Phenomena 180
81. Entoptic Observation of the Vessels of the Retina 183
82. Other Entoptic Phenomena 186
Bibliography 191

CHAPTER XII

ACCOMODATION

83. Measurement of the Amplitude of Accommodation 192
84. Mechanism of Accommodation (Historical, A) 195
85. Mechanism of Accommodation (Historical, B) 201
86. Personal Experiments 206
87. The Author's Theory of Accommodation 220
Bibliography 228

CHAPTER XII

OPHTHALMOSCOPY

88. Methods of Illuminating the Fundus of the Eye............... 229
89. Examination by the Erect Image 232
90. Examination by the Erect Image. Observations 237
91. Examination by the Inverted Image 241
92. Ophthalmoscopic Examination of the Refracting Media 245
93. Skiascopy 246
Bibliography 253

CHAPTER XIV

THE PUPIL

94. General Remarks 254
95. Action of Mydriatics and of Myotics 255
96. Movements of the Pupil 256
97. Advantage of the Position of the Pupil near the Nodal Point... 259
Bibliography 265

BOOK II

FUNCTIONS OF THE RETINA

CHAPTER XV

CHANGES WHICH THE RETINA UNDERGOES UNDER THE INFLUENCE OF LIGHT

98. Retinal Purple 266
99. Movements of the Pigment Under the Influence of Light...... 267
Bibliography 269

CHAPTER XVI

THE LIGHT SENSE

100. Psychophysical Law of Fechner 270
101. Measurement of the Light Sense 274
102. Results 278
Bibliography 281

CHAPTER XVII

THE COLOR SENSE

103. General Remarks 282
104. Phenomena of Contrast (Simultaneous) 287
105. After Images 291
106. Phenomena Dependent on the Variation of the Brightness of the Colors 293
107. Methods of Mixing the Colors 298
108. Results of the Mixtures of Colors 301
109. Abnormal Trichromasia 315
110. Color Blindness or Daltonism (Dichromasia) 317
111. Monochromasia 324
112. Clinical Examination of the Color Sense 324
113. Hypotheses on the Mechanism of Color Vision 327
Bibliography 332

CHAPTER XVIII

THE FORM SENSE

114. Central Visual Acuity 334
115. Peripheral Acuity 341
Bibliography 345

BOOK III

THE OCULAR MOVEMENTS AND BINOCULAR VISION

CHAPTER XIX

THE LAW OF LISTING

116. Centers and Axes of Rotation of the Eye 346
117. Law of Listing 348
118. Experiments of Meissner. Apparently Vertical Meridian 355
119. Historical 358
Bibliography 359

CHAPTER XX

THE OCULAR MOVEMENTS

120. Jerking Movements of the Eyes 360
121. Relative Movements of the Two Eyes 360
122. Measurement of Convergence 363
123. Relations between Accommodation and Convergence 365
Bibliography 365

CHAPTER XXI

PROJECTION OF VISUAL IMPRESSIONS

124. Projection Outwards of Uniocular Vision 366
125. Projection of the Visual Field 366
126. Projection in Binocular Vision 369
Bibliography 376

CHAPTER XXII

MONOCULAR PERCEPTION OF DEPTH

127. Influence of Accommodation 377
128. Indirect Judgment of Distance 377
129. Influence of the Parallax 380
Bibliography 381

CHAPTER XXIII

BINOCULAR PERCEPTION OF DEPTH

130. Influence of Convergence 382
131. The Stereoscope 382
132. Effect of the Stereoscope 388
133. Identical Points of the Retinæ 391
Bibliography 395

CHAPTER XXIV

STRABISMUS

134. Different forms of Strabismus 397
135. Measurement of Strabismus 400
136. Etiology of Concomitant Strabismus 400
137. Vision of Strabismic Patients 403
138. Treatment of Strabismus 404
Bibliography 406

CHAPTER XXV

OPTIC ILLUSIONS

139. Optic Illusions 407
Bibliography 412
Treatises to Consult 413

LIST OF ILLUSTRATIONS

FIG.		PAGE
1.	Luminous Source, Opaque Body, Shadow and Penumbra	1
2.	Reflection on a Plane Mirror	3
3.	Reflection on a Concave Mirror	4
4.	Reflection on a Concave Mirror	5
5.	Reflection on a Convex Mirror	7
6.	Construction of the Utilized Part of a Mirror	9
7.	Refraction	9
8.	Total Reflection	10
9.	Prism with Total Reflection	11
10.	Refraction by a Plate with Plane Parallel Surfaces	12
11.	Refraction by a Prism	12
12.	Refraction by a Spherical Surface	13
13.	Refraction by a Spherical Surface	15
14.	Refraction by a Parabolic Surface	16
15.	Construction of Image Formed by a Thin Lens	18
16.	Method of Measuring the Focal Distance of a Lens	21
17.	Principal and Nodal Points; Anterior and Posterior Focus	23
18.	Construction of the Image of an Object	24
19.	Construction to Find the Second Principal Plane	25
19.*a*.	Construction of the Cardinal Points of Two Optic Systems	27
20.	Construction to Find the Nodal Points of a Thick Lens	28
21.	Optic System of the Eye	33
22.	Optic System of the Eye of an Ox	34
23.	Images of Purkinje of the Eye of an Ox (Dead)	35
24.	Double Crystalline Images in a Case of False Lenticonus	36
25.	Diagram of the Crystalline Lens	36
26.	Position of the Cardinal Points of the Human Eye	39
27.	Pupil of Entrance and Pupil of Exit	43
28.	Reflections and Refractions by a Lens	47
29.	Manner in which a Luminous Ray is Divided in the Eye	48
30.	Position of the Seven Images in the Eye	49
31.	Corneal Images of two Lamps Observed with the Ophthalmophakometer	51
32.	The Ophthalmophakometer	53
33.	Illustration of the Principle of Doubling	58
34.	Doubling by the Two Halves of an Objective	59
35.	Plates of Helmholtz	60
36.	Doubling by an Objective, a Central Vertical Band of which has been Removed	60
37.	Prism of Wollaston	61

FIG.		PAGE
38.	Ophthalmometer of Javal and Schioetz	62
39.	Images of the Mires Seen Doubled	63
40.	Refraction by a Conical Cornea	65
41.	Radii of Curvature of the Cornea	67
42.	Diagram of Corneal Refraction	69
43.	Forms of the Image of a White Square at Different Parts of the Cornea	71
44.	Keratoscopic Images of an Astigmatic Cornea	73
45.	Keratoscopic Images of an Astigmatic Cornea	74
46.	Keratoscopic Images of a Case of Keratoconus	75
46*a*.	Keratoscopic Image of an Eye with a Large Angle *a*	76
46*b*.	Spot of Mariotte of an Eye with a Large Angle *a*	76
47.	The Ophthalmophakometer	77
48.	The Images of Purkinje Observed with the Ophthalmophakometer	78
49.	Position of the Images of Purkinje, the Lamps being Arranged Vertically	78
50.	Position of the Images of Purkinje, the Lamps being Arranged Horizontally	79
51.	Defect of Centering; Alignment of the Images Impossible	80
52.	Determining the Position of an Internal Surface of the Eye	81
53.	Determining the Position of an Internal Surface of the Eye	83
54.	Calculation of the Size of the Circle of Diffusion	88
55.	Rules of the Optometer of Young	90
56.	Magnification by Means of the Stenopaic Opening	93
57.	Retinal Image in Myopia and Hypermetropia	99
58.	Principle of Badal	101
59.	Size of Retinal Image when the Focus of the Lens Coincides with the Anterior Focus of the Eye	101
60.	Distribution of the Anomalies of Refraction	103
61.	Refraction of a Pencil of Parallel Rays by a Spherical Surface	114
62.	Spherical Aberration of a Lens	116
63.	Deformity of the Shadows of the Needles	118
64.	Experiment of Volkmann	120
65.	Distribution of the Light of the Circle of Diffusion	121
66.	The Aberroscope	122
67.	The Rules of the Optometer of Young	122
68.	The Appearance Assumed by the Line of the Optometer of Young	123
69.	Deformity of the Shadows in an Eye with Strong Spherical Aberration	126
70.	Aberration Over-Corrected Towards the Borders	127
71.	Aberration Over-Corrected Above	127
72.	Aberration Over-Corrected Everywhere	127
72*a*.	Stadfeldt's Instrument for Measuring Aberration of the Crystalline Lens (Dead)	128
73.	Achromatic Prism	132
74.	Prism *a vision directe*	132
75.	Chromatic Aberration of the Eye	135

FIG. PAGE
76. Phenomena of Dispersion 136
77. Circles of Diffusion and Focal Lines of a Regularly Astigmatic System 138
78. Focal Lines of a Regularly Astigmatic System 139
79. Construction of the Elliptical Diffusion Spot 140
80. A Torus 143
81. Focal Line of Lens Placed Obliquely 144
82. Astigmatism by Incidence; Focal Lines 144
83. Explanation of the Difference in Level (*denivellation*)* 148
84. Keratoscopic Images of a Case of Keratoconus 157
85. Forms Under which a Luminous Point is Seen by a Regular Eye 166
86. In Regular Astigmatism with Spherical Aberration 167
87. Figures of a Luminous Point Obtained by Combining a Spherical with a Cylindrical Lens 168
88. Forms which a Luminous Point Presents to the Author's Right Eye 168
89. To an Eye with Double Obliquity 169
90. Figures of the Left Eye of M. Rée 169
91. Curved Focal Line 170
92. Irregular Eye (Diplopia) 171
93. Aberroscopic Phenomena 173
94. Diagram of Variations of Refraction in the Pupil 173
95. Course of the Rays in the Author's Right Eye 174
96. Specks on the Anterior Surface of the Cornea 177
97. Striæ Produced by Winking 177
98. Prismatic Effect of the Layer of Tears 177
99. Speckled Appearance of the Entoptic Field Produced by Rubbing the Cornea 178
100. Star Figure of the Crystalline Lens 178
101. Incipient Cataract Seen Entoptically 179
101*a*. The Entoptoscope 180
102. Parallax of the Entoptic Phenomena 181
103. Determination of the Position of an Entoptic Object 182
104. Entoptic Luminous Image Surrounded by a Shadow 183
105. Entoptic Observation of the Vessels 184
106. Entoptic Observation of the Vessels 186
106*a*. Entoptic Phenomenon 190
107. Centripetal Movement of the Catoptric Image 196
108. Putting the Eye Under Water 202
109. Ciliary Muscle of Man 204
110. Ciliary Part of the Eye of a Cat 205
111. Change of Aberroscopic Phenomena During Accommodation.... 206

*[This figure 83 does not quite illustrate the actual picture that we obtain by looking at the corneal images K and L with the ophthalmometer. For with the Wallaston prism K is not seen any more, but instead of it we observe K_1 at the place indicated in the figure, and K_2 at a distance, $K K_1$ to the left of K in the direction of doubling of the prism. The same is the case with L_1, only that L_2 is displaced to the right. But to avoid complication the two images K_2 and L_2 have been omitted.] W.

FIG.		PAGE
112.	Appearance of the Luminous Point	207
113.	Appearance of the Luminous Point	208
113*a*.	Skiascopic Examination of Accommodation	210
114.	Reflection Images of the Eye	211
115.	Reflection Images of the Eye	212
116.	Reflection Images of the Eye	212
117.	Deformity of the Corneal Image of a White Square in a Case of Keratoconus	213
118.	Refraction by a Parabolic Surface	214
119.	Deformity of the Crystalline Surfaces During Accommodation	215
120.	Accommodative Phenomena of the Eye	216
121.	Accommodative Phenomena of the Eye	217
122.	Change of the Anterior Chamber During Accommodation	219
122*a*.	Reflection Images on the Anterior Surfaces of the Dead Crystalline Lens	220
122*b*.	The Dead Crystalline Lens and the Accommodated Crystalline Lens	221
123.	Crystalline Lens of the Ox	223
124.	Optic System of the Eye of the Ox	224
125.	Illumination of the Fundus by a Light for which the Eye is Accommodated	229
126.	Illumination of the Fundus by a Light for which the Eye is Not Accommodated	230
127.	Principle of the Ophthalmoscope of Helmholtz	231
128.	Magnification of the Fundus, both Patient and Observer being Emmetropic	234
129.	Line of Image of Papilla if the Fundus of Patient is Placed Free in the Air	235
130.	Magnification of Fundus if Patient is Myopic	235
131.	Construction of the Ophthalmoscopic Field	236
132.	Magnification by the Inverted Image in Emmetropia	241
133.	Influence of Refraction of the Examined Eye on the Magnification if Focus of Lens Coincides with Anterior Focus of Eye	242
134.	Influence of Refraction of the Examined Eye on the Magnification if Lens is Nearer to the Eye than in Fig. 133	243
135.	Construction of the Ophthalmoscopic Field by the Inverted Image	244
136.	Skiascopy. Plane Mirror	247
137.	Skiascopy. Concave Mirror	247
138.	Boundaries of the Skiascopic Field	249
139.	Theory of Leroy	250
140.	Theory of Leroy	250
141.	Theory of the Paracentral Shadow	251
142.	The Advantage of the Position of the Pupil Near the Nodal Point	259

FIG. PAGE
143. Experiment of Helmholtz 260
144. Hyperbolic Chessboard of Helmholtz 261
145. Artificial Eye 262
146. Image of a Window in the Artificial Eye 263
146*a*. Section of the Retina of a Frog 268
147. Experiment of Bouger 271
148. Curve Showing the Relation between the Light Sense and the Illumination 273
149. Photoptometer of Foerster 275
150. Disc of Masson 276
150*a*. Disc of Helmholtz and Disc of Benham 277
151. Spectrum of Refraction; Spectrum of Diffraction 283
152. Table of Colors after Newton 285
153. Experiment of Ragona Scina 288
154. Experiment with Colored Shadows 289
155. Disc of Masson 290
156. Curves of Parinaud to Show the Threshold for Different Rays of the Spectrum 296
157. Color Box of Maxwell 299
158. Mixture of Colors by Means of a Glass Plate 300
159. Table of Colors after Newton 303
160. Color Table of Maxwell 304
161. "Color Box" of Maxwell 305
162. Color Curves of Maxwell 306
163. Color Table of Maxwell 308
164. Color Table of Helmholtz 313
165. Color Table of Maxwell... 319
166. Color Curves of a Dichromatic 321
167. Color Table of a Dichromatic 321
168. Chromatoptometer of Chibret 326
169. Experiment of Hooke 335
170. Measurement of the Visual Acuity by a Grating 335
171. Measurement of the Visual Acuity by a Grating 335
172. Experiment of Hooke, the Optics of the Eye being Defective.... 336
173. Mariotte's Blind Spot 343
174. Phenomenon of Troxler 344
175. Determination of the Center of Rotation of the Eye 347
176. The Two Axes of Rotation Lying in the Horizontal Plane...... 347
177. Demonstration of the Law of Listing 350
178. Demonstration of the Law of Listing 351
179. Demonstration of the Law of Listing 352
180. Discs of Volkmann 356
181. Modification of the Experiment of Meissner 357

FIG. PAGE
182. Illustration of the Meter Angle 364
183. Explanation of Binocular Physiologic Diplopia 370
184. Experiment to Find the Center of Projection 373
185. Horopter of Johannes Muller 374
186. Apparent Form of the Sky 379
187. Influence of Parallax for Stereoscopic Vision 380
188. Principle of Stereoscopic Images 383
189. Stereoscope of Wheatstone 385
190. Pseudoscope of Wheatstone 386
191. Telestereoscope of Helmholtz 387
192. Binocular Ophthalmoscope 388
193. Antagonism of the Visual Fields 390
194. Suppression of one of the Images in Stereoscopic Vision 394
195 to 201. Optic Illusions.

PHYSIOLOGIC OPTICS

BOOK I
OCULAR DIOPTRICS

CHAPTER I
OPTIC PRINCIPLES

1. **Optic Properties of Bodies.**—Bodies are of three kinds; *transparent* bodies, through which we can see objects, *translucent* bodies such as ground glass, through which we perceive light, but cannot distinguish form, and *opaque* bodies.—No body is absolutely transparent. Pure water is transparent, but very little light will pass through a great thickness of water.—On the contrary very thin layers of opaque substances are more or less translucent, as all know who have examined microscopic preparations.

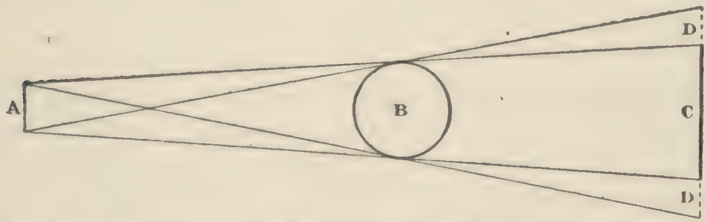


Fig. 1.—A, luminous source; B, opaque body; C, shadow; D, penumbra.

2. **Rectilinear Propagation of Light.**—In a homogeneous medium light is propagated along straight lines which are called *luminous rays.*

Shadows.—When rays emanating from a luminous point fall upon an opaque body there is produced behind the latter a shadow which is conical in shape. We can construct the form of this shadow by drawing straight lines joining the different points of

the border of the body with the luminous point. If, instead of a point, the source is a luminous surface the shadow is surrounded by a penumbra, the intensity of which diminishes more and more towards the periphery. An observer placed in the shadow C could not see any point of the luminous surface; placed in the penumbra D he would see a part of that surface, greater in proportion as he approaches the border.

Images Produced by a Small Aperture.—Rays passing through a small aperture into a dark room form on a screen an inverted image of exterior objects. By diminishing the aperture the image gains in distinctness, but loses in luminosity. Photographs may be taken in this way.

3. Reflection and Absorption.—Rays which strike the surface of an opaque object are partly *absorbed* and partly *reflected*. If the surface is not polished the rays are reflected in a diffuse manner: each point of the surface sends back light in all directions. It is through the agency of this irregularly reflected light that objects are visible, and the fact that they are visible, whatever may be the position of the observer, provided the rays are not intercepted, proves conclusively that any point whatever of the surface sends rays in all directions.

4. Regular Reflection.—The more polished the surface the less diffuse is the reflection. Thus the surface of a highly polished mirror is but slightly visible. Polished surfaces reflect rays regularly following a law which was known from remote ages, viz., that the reflected ray is in the same plane with the incident ray and the normal to the point of incidence, and that both rays form equal angles with the normal, which is expressed by saying that the *angle of incidence* and the *angle of reflection* are equal.

The effect of this reflection is to produce *images* of external objects. The image of a point is the place where the rays which emanated from that point meet again after reflection or refraction. In order that the image may be perfect, all the rays employed should meet in a point. Generally this condition is not quite fulfilled, there being more or less pronounced *aberrations*.

—A point and the image of this point we designate as *conjugate* points.—An image is *real* when the rays proceeding from a point meet again in a point; it is *virtual* when it is formed not by the reunion of the rays themselves, but of their prolongations.—A real image can be received on a screen; a virtual image cannot, but it is visible to the eye which is in the path of the rays because the optic system of the eye forms a real image of it on the retina, exactly as if the virtual image was an object.

5. Plane Mirrors. Construction of the Image.—Let fall from a point A (fig. 2) of the object a perpendicular AB on the surface, DE, of the mirror, and mark on its prolongation a point

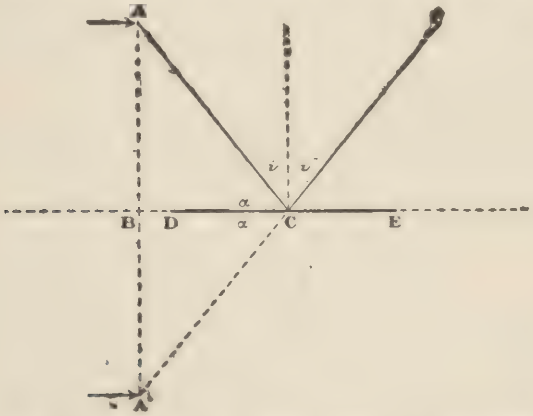


Fig. 2.—Reflection on a plane mirror. A, the object; A′, its image; DE, the mirror; AC, incident ray; CF, reflected ray.

A′ so that AB is equal to A′B. A′ is the image of A, for since AB=A′B, the two angles a are equal, and consequently also the two angles i, each of which is equal $90°— a$. The image formed by a plane mirror is *virtual, erect and equal in size to the object.*

To tell whether a mirror is true place the eye near the surface by way of observing images under as great an incidence as possible. If the mirror is not true the images of external objects are deformed. One can also notice these deformities very

distinctly by placing oneself quite a distance in front of the mirror and observing the images of distant objects.

6. Concave Spherical Mirrors.—The middle of the spherical surface is called the *apex,* a straight line passing through the center and the apex is the *axis,* and the angular measurement of the mirror is the *aperture.* In order that images may be true the aperture must be small (8 to 9 degrees). The *principal focus* of the mirror is the place where incident rays parallel to the axis meet after reflection. The *principal focal distance* is the distance of the principal focus from the mirror.

IN ALL OPTIC PHENOMENA THE COURSE OF THE RAYS IS REVERSIBLE.—If in figure 2, the ray AC is reflected along CF, an incident ray along FC is reflected along CA.—It follows that rays coming from the principal focus of a concave mirror must be parallel after reflection.

The principal focus of a plane mirror is at infinity, because incident parallel rays are still parallel after reflection.

The principal focus of a concave mirror is situated half way between the apex and center. We have, indeed $i=i$ (fig. 3), since the angles of incidence and reflection are equal, and $i=BC\Phi$ because the incident ray is parallel to the axis. It follows that $C\Phi=B\Phi$, but as the aperture is very small, we can consider $B\Phi=Q\Phi$, therefore $C\Phi=Q\Phi=\frac{R}{2}$, if we designate the radius by R.

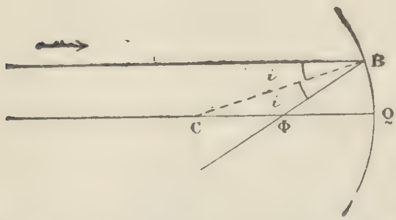


Fig. 3.—Reflection on a concave mirror. C, the center; Φ, the focus.

A ray passing through the center is perpendicular to the surface; it is consequently reflected on itself.

CONSTRUCTION OF THE IMAGE.—To find the image B_1 of a point B (fig. 4), it suffices to trace the course of two rays which have emanated from that point; the image must be at the place where they intersect after reflection. After what has been pre-

viously stated we already know the course of three rays proceeding from the point B.

1°. The ray BM, which is parallel to the axis, passes after reflection through the focus Φ;

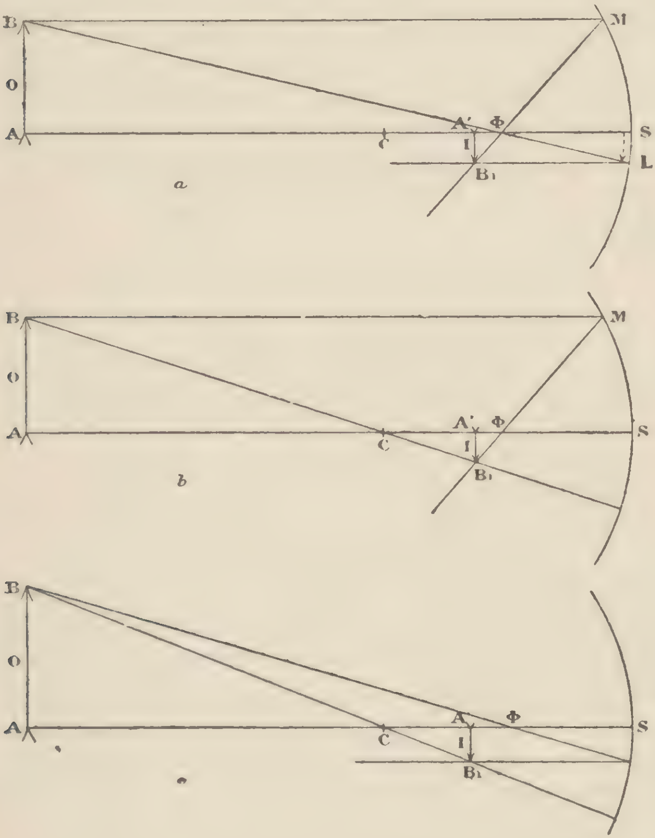


Fig. 4.—Reflection on a concave mirror. Constructions of the image, I, of an object O; C, the center; Φ, the focus. $AS=f_1$, $A'S=f_2$, $S\Phi=F$, $A\Phi=l_1$, $A'\Phi=l_2$.

2°. The ray BΦ, which passes through the focus, is reflected parallel to the axis since the course of the rays is reversible;

3°. The ray BC, passing through the center, is reflected on itself.

Two of these rays suffice for the construction. By combining them, two by two, we obtain the three different constructions shown in figure 4.

SIZE OF THE IMAGE. RELATIONS BETWEEN THE DISTANCES OF CONJUGATE POINTS.—Let us consider the line BA=O (fig. 4*a*) as the object; I is its image. And supposing SL=I and MS=O, the triangles ABΦ and SLΦ on one side, and the triangles SMΦ and A′B′Φ on the other give us the relations

$$\frac{O}{I}=\frac{l_1}{F}=\frac{F}{l_2} \text{ or } l_1\, l_2=FF \text{ (\textit{Newton})[1].}$$

The formula

$$\frac{O}{I}=\frac{l_1}{F} \text{ can also be written } \frac{O}{I}=\frac{2l_1}{R};$$

which is the formula we use later in opthalmometry.—As we have $l_1=f_1$—F and $l_2=f_2$—F, the formula of *Newton*

$$l_1 l_2=FF$$

can also be written

$$\frac{F}{f_1}+\frac{F}{f_2}=1 \text{ or } \frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F}$$

The first of these two formulæ is that of *Helmholtz;* and, as we shall see, it is altogether general. The second is identical with that of infinitely thin lenses.

By construction or formula we find that:

1°. The image of an object placed beyond the center is situated between the center and focus. It is *real, inverted* and *diminished;*

---

(1) In this formula and those which follow I designate by:
O, the object;
I, the image;
R_1, the radius of the first surface;
R_2, the radius of the second surface;
F_1, the anterior focal distance;
F_2, the posterior focal distance;
f_1, the distance of the object from the surface;
f_2, the distance of the image from the surface;
l_1, the distance of the object from the anterior focus;
l_2, the distance of the image from the posterior focus;
For mirrors and lenses surrounded with the same media on both sides we have $F_1=F_2=F$.

2°. As the course of the rays is reversible, an object placed between the center and the focus gives an image situated beyond the center, and this image is *real, inverted* and *enlarged;*

3°. An object placed between the focus and the mirror forms its image behind the mirror. This image is *virtual, erect* and *enlarged.*

7. Convex Mirrors.—As in the case of concave mirrors, the focus is placed at an equal distance between the surface and

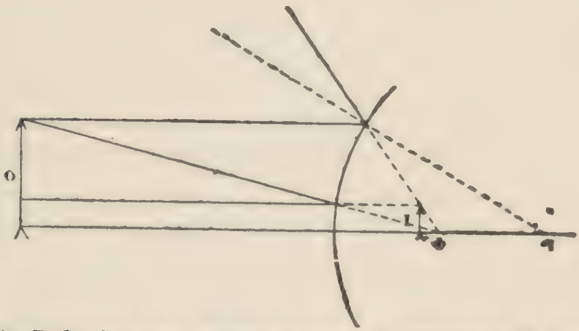


Fig. 5.—Reflection on a convex mirror. Construction of the image. C, the center; Φ, the focus.

center. The construction (fig. 5) is the same as in the preceding case, and the formulæ also, but the distances of the points situated behind the surface must be considered as negative; we have therefore

$$\frac{1}{f_1} - \frac{1}{f_2} = -\frac{1}{F}$$

The image of a real object is always *virtual, erect* and *diminished;* it is situated between the surface and the focus.

8. Practical Remarks.—One can tell whether a mirror is convex, concave or plane by placing the eye near the surface. A convex mirror forms a diminished image of the eye, a concave mirror gives a magnified image (provided the eye is between the

focus and the mirror.) The image formed by a plane mirror is the same size as the object.

To determine the focal distance of a concave mirror we can:

1. Form the image of a distant object on a screen: the distance of the mirror from the screen is equal to the focal distance;

2. Place the screen by the side of a flame and find the distance from the mirror at which the image appears distinct. The distance of the mirror from the flame is double the focal distance, for since the object and image are, in this case, at the same distance from the mirror, this distance is equal to the radius of the mirror or double its focal distance. We determine the *focal distance of a convex mirror* by finding the position of the screen at which the reflex which the mirror forms of a distant flame has a diameter equal to double the diameter of the mirror. The distance of the mirror from the screen is equal to the focal distance, as a simple geometrical construction will show.—For all small mirrors opthalmometric processes are used.

Concave mirrors, like convex lenses, make rays converge, while convex mirrors make them diverge. For this reason convex mirrors are used as opthalmoscopes when it is desirable to have a very feeble light.

A combination of a plane mirror with a convex lens acts like a concave mirror with a focal distance equal to that of the lens or half of it, according as the light traverses the lens once or twice (opthalmoscope of *Coccius*). A combination of a plane mirror with a concave lens acts like a convex mirror.

Portion of Mirrors Used.—Except in the case when an image is projected on a screen it is only a small part of the mirror that is utilized. We can find this part by constructing the image I (fig. 6) of the object O and by joining by straight lines its margin with the margin of the observer's pupil. These straight lines delimit the utilized portion of the mirror AB. We could also construct the image of the pupil and join this image to the object; the result would be the same.

9. Refraction.—When a luminous ray strikes a polished surface separating two transparent media is it divided into two, a

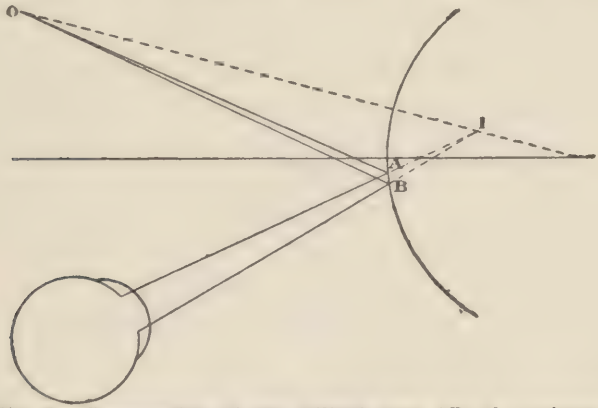


Fig. 6.—Construction of the utilized part AB of a mirror.

reflected ray which is thrown back into the first medium and a *refracted* ray which continues its course in the second (fig. 7). The three rays are in the same plane which contains also the normal to the point of incidence. The angle of reflection is, as we have seen, equal to the angle of incidence, but the angle of refraction (formed by the normal and the refracted ray) is different. Its size is determined by the law of *Descartes* (*Snellius*). *The ratio between the sine of the angle of incidence and the sine of the angle of refraction is constant, whatever may be the angle of incidence, as long as two media remain the same.*

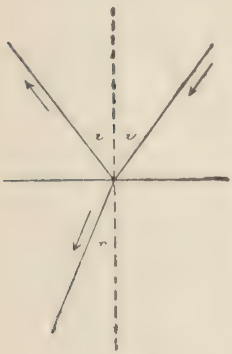


Fig. 7.

$$\frac{\sin i}{\sin r} = n$$

The symbol n denotes the *index of refraction,* and the index of air is generally adopted as the unit. The index of water in relation to air is $\frac{4}{3} = 1.333$, that of glass in relation to air is approximately $\frac{3}{2} = 1.5$. The index of glass in relation to water

is, then, $\frac{3}{2} \div \frac{4}{3} = \frac{9}{8}$, etc. In the formulæ which follow n denotes the index of the second medium as compared with that of the first.

10. Quantity of Reflected Light. Total Reflection.—The quantity of light regularly reflected increases with the *angle of incidence,* with the *difference of index between the two media,* and lastly with *the degree of poish of the surface.* In air a highly polished glass surface reflects about 4 per cent. of incident light, if the angle of incidence is negligible. Good metallic mirrors reflect about two-thirds of the incident light.

Total reflection takes place when light, propagated in a dense medium, meets at a large angle of incidence the surface which separates the dense medium from a rarer one.

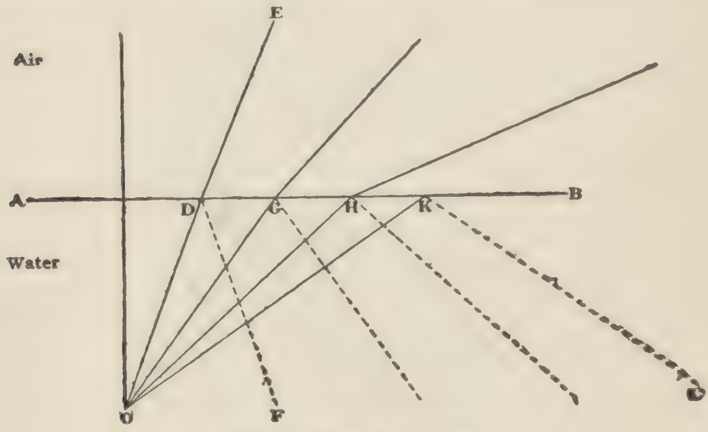


Fig. 8.—Total Reflection.

Let AB (fig. 8) be the surface separating the air from the water and O a luminous point in the water. OD is a ray which, on reaching the surface, is divided into two, DE which is refracted and DF which is reflected and much feebler; the next rays OG and OH are equally divided; the emerging ray is always more and more refracted and loses more and more in intensity, while the reflected ray gains in intensity; and when the angle of incidence reaches a certain size, the emergent ray

forms an angle of 90° with the normal, that is, it *glances* along the surface. We designate as the *critical angle* the angle of incidence which corresponds with an angle of refraction of 90°. In this case sin $r=1$; therefore,

$$\frac{\sin i}{\sin r}=n=\frac{\sin i}{1}=\sin i=n.$$

In our case $n=3/4$, sin $i=0.75$ and the critical angle is about 49°. *If the angle of incidence exceeds the critical angle all the light is reflected (total reflection)* (OK, fig. 8).

If we pour water into a glass and try to look obliquely from below upwards through the surface of the water this surface appears like an absolutely opaque metallic surface. No ray coming from above reaches the eye because all are deflected towards the bottom of the glass by refraction. If we dip a pencil in the water we see it mirrored in the surface; rays coming from the pencil reach the eye after total reflection at the surface of the water.

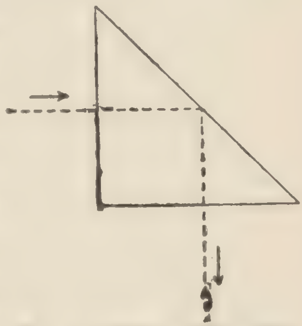


Fig. 9.—Prism with total reflection.

As this form of reflection is the most complete of all, it is frequently used in optic experiments. The most usual application of it is in the rectangular prism; looking perpendicularly at one of the faces we see an image of objects placed in front of the other face, formed by total reflection on the hypothenuse (fig. 9). Nor need the prism be rectangular; a prism of 60° gives a like result; but in every case the three faces must be polished.

11. Refraction by Plates with Plane and Parallel Surfaces.—The incident ray and the emergent ray are parallel, for we have $r=r$ (fig. 10), since the surfaces are parallel, and consequently also $i=i$. The emergent ray has suffered a displacement towards the side whence the light comes.

12. Refraction by a Prism.—Seen through a prism an object seems deflected towards the apex of the prism. The angle between the direction along which the object is seen and that in which it really is found is called the *deviation.* If i (fig. 11) is the angle of incidence, i_1 the angle formed by the emergent ray with the normal, A the angle of the prism, and d the deviation, we have

$$d=i+i_1-A$$

for

$$d=i-r+i_1-r_1$$

and

$$A=180^\circ-x=r+r_1$$

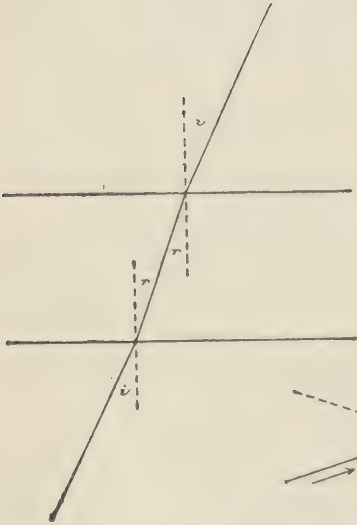


Fig. 10.—Refraction by a plate with plane and parallel surfaces.

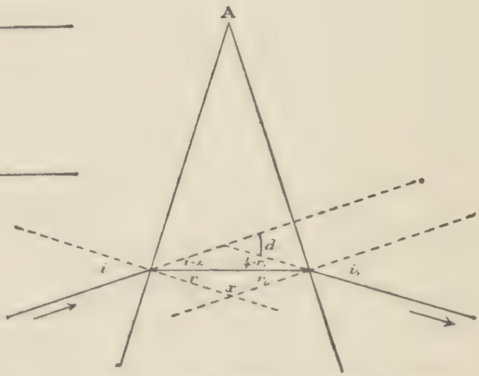


Fig. 11.—Refraction by a prism.

therefore

$$d=i+i_1-A.$$

The deviation is *least* when $i=i_1$, the course of the rays is then symmetrical, and we have:

$$A=2r \text{ and } d=2i-2r=2i-A.$$

In the formula

$$\sin i = n \sin r$$

we can replace the sines by the arcs if the latter are small; therefore and

$$i = nr$$

$$\begin{aligned} d &= 2nr - A \\ &= (n-1)A. \end{aligned} \quad (1)$$

If the prism is glass, we have $n = \frac{3}{2}$ approximately, $n - 1 = \frac{1}{2}$. *Therefore the deviation produced by a weak prism is equal to half its angle.*

13. Refraction by a Spherical Surface.—Incident rays parallel to the axis reunite at the posterior forces Φ_2 (fig. 12). The

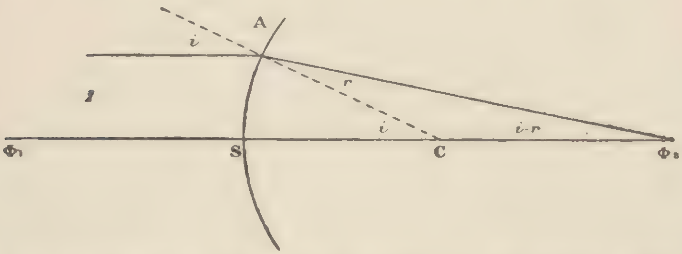


Fig. 12.—Refraction by a spherical surface. Φ_1, the anterior focus; Φ_2, the posterior focus; C, the center.

distance $S\Phi_2$ is known as the *posterior focal distance;* it is expressed by

$$F_2 = \frac{nR}{n-1}$$

for we have

$$\frac{C\Phi_2}{R} = \frac{\sin r}{\sin (i - r)}$$

or, if the angles are small,

$$\frac{C\Phi_2}{R} = \frac{r}{i-r} = \frac{r}{nr-r} = \frac{1}{n-1}.$$

Therefore

$$C\Phi_2 = \frac{R}{n-1}$$

and

$$S\Phi_2 = \frac{R}{n-1} + R = \frac{nR}{n-1}.$$

After refraction the rays coming from the anterior focus Φ_1 are parallel to the axis. Its distance $\Phi_1 S = F_1$ is called the anterior focal distance and is expressed by

$$F_1 = \frac{R}{n-1};$$

indeed, we find this value by a calculation analogous to that by which we have found the posterior focal distance.

$d = i - r + i_1 - r_4$ or for small angles
$d = nr - r + nr_1 - r_4 = (n-1)\ (r + r_4) = (n-1)$ A.]—W.

We note that

$$F_2 = F_1 + R = nF_1$$

that is to say:

1°. *The difference between the focal distances is equal to the radius;*

2°. *The ratio between the focal distances is equal to the ratio betwen the indices of the corresponding media.*

3°. In fig. 12 we have

$$\Phi_1 S = \Phi_2 C = F_1$$
$$\Phi_2 S = \Phi_1 C = F_2$$

The distance of the center from the posterior focus is equal to the anterior focal distance, and the distance of the center from the anterior focus is equal to the posterior focal distance.

---

(1) [The author here derives this formula from that for the least deviation. It may be derived in a more general way thus:

Construction of the Image.—To construct the image of a point situated outside the axis we can draw:

1°. A ray passing through the center; it is not refracted;

2°. A ray parallel to the axis: it is refracted towards the posterior focus;

3°. A ray passing through the anterior focus: after refraction is parallel to the axis.

The point of intersection of two of these straight lines is the image. There are three possible constructions, therefore, by which we may obtain the image of this point.

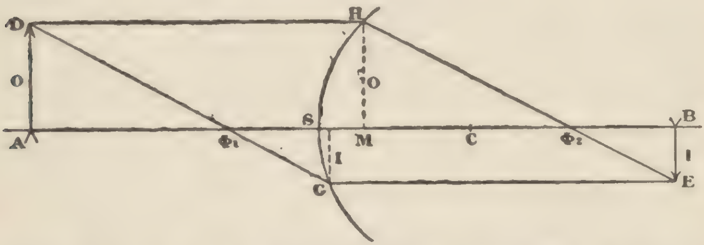


Fig. 13.—Refraction by a spherical surface. Construction of the image. C, the center; Φ_1, the anterior focus; Φ_2, the posterior focus; O, the object; I, the image. AS$=f_1$, BS$=f_2$, A$\Phi_1=l_1$ B$\Phi_2=l_2$.

Fig. 13 shows the construction by means of rays 2° and 3°. The triangles $DA\Phi_1$ and Φ_1SG and the triangles $HM\Phi_2$ and Φ_2BE being similar, we have the same relation as for the mirrors

$$\frac{O}{I} = \frac{l_1}{F_1} = \frac{F_2}{l_2}$$

whence we deduce the two general formulæ

$$l_1 \, l_2 = F_1 \, F_2 \text{ and } \frac{F_1}{f_1} + \frac{F_2}{f_2} = 1.$$

The image is *real* and *inverted* when the object is beyond the anterior focus; it is smaller than the object if the distance of the latter from the surface is greater than $2F_1$, larger if the distance is less than $2F_1$. If the object is between the focus and the surface, the image is *virtual, erect* and *enlarged* and behind the object.

If the surface is concave the radius is to be considered negative. The focal distances then become negative: $F_1 = -\frac{R}{n-1}$, $F_2 = -\frac{nR}{n-1}$, which indicates that the anterior focus is situated behind and the posterior focus in front of the surface.

If, in this latter case, the rays pass from a dense medium (with index $= n$) into a rarer medium (with index $= 1$), we must in the formulæ replace n by $\frac{1}{n}$. The focal distances then become positive again: $F_1 = \frac{nR}{n-1}$, $F_2 = \frac{R}{n-1}$. This is what happens when rays, after having passed through the first surface of a biconvex surface, meet the second.

Power of a Refracting Surface.—The refracting power of a surface is expressed in dioptries by the inverse of the anterior focal distance measured in meters: $D = \frac{1}{F_1} = \frac{n-1}{R}$. (1)

If for example the anterior focal distance is 24 millimeters (anterior surface of the cornea) the refracting power is $D = \frac{1}{0024}$ $= 42$ dioptries.

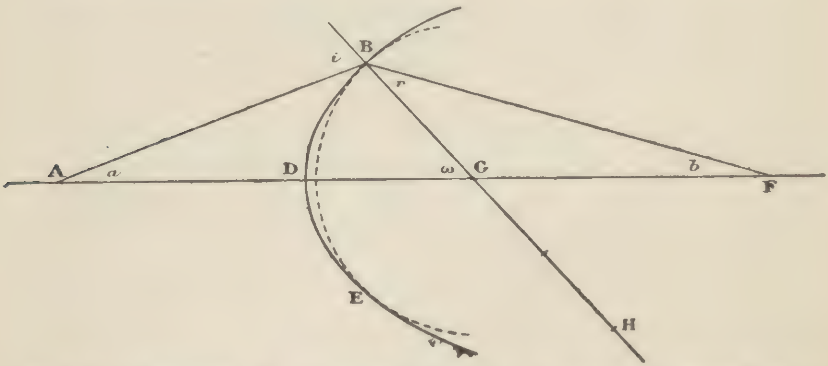


Fig. 14.—Refraction by a parabolic surface. A, luminous point; F, its image; BG, normal; BH, radius of curvature.

(1) [In other words, we define the refractive power of a convex surface at a certain point B (fig. 14) as the dioptric power of an infinitely thin plano-convex lens obtained by cutting off a piece of the refracting surface by a plane at right angles to the normal at B and very near to this point. Such detached plano-convex lens, surrounded by the first medium, has a posterior focal distance F_2 equal to the anterior focal distance F_1, equal to $\frac{R}{n-1}$ and a refracting power $= \frac{1}{F_2} = \frac{1}{F_1} = \frac{-1}{R}$. If the surface is not a sphere but a surface of revolution of the second degree, we must replace R by the normal N at the point B.]—W.

Refraction by a Surface of Revolution of the Second Degree.—If the luminous point is on the axis, refraction at a given point B (fig. 14) takes place in the same manner as if the surface was replaced by a sphere drawn around the point G where the normal BG meets the axis. If we designate as N the normal BG, the refracting power of the surface at the point B is therefore $D = \frac{n-1}{N}$.

We can indeed calculate the focal distances for a surface of revolution exactly as we have done for the sphere, and we find the same expressions by replacing R by N. It is well to note that it is the *normal* BG and not the radius of curvature BH which enters into the formulæ.—These remarks are of importance for the theory of accommodation and of keratoconus.

14. Infinitely Thin Lenses.—The theory of lenses is very simple if we can neglect the thickness. We designate as *axis* the straight line which joins the two centers of the surfaces, and as *optic center* the point where this axis crosses the lens. This point enjoys this property that a ray passing through it crosses the lens without deviation.

Focal Distance of a Biconvex Lens.—Let us designate the radii of curvature of the two surfaces as R_1 and R_2. Incident parallel rays which meet the first surface are refracted towards the posterior focus, the distance of which, as we have seen, is equal to $\frac{nR}{n-1}$. This point now acts as the object for the second surface; as it is behind the latter its distance is to be considered as negative. In the formula

$$\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$$

f_1 is therefore equal to $-\frac{nR_1}{n-1}$. F_1 has the value of $\frac{nR_2}{n-1}$ and F_2 of $\frac{R_2}{n-1}$ (§13). We have therefore

$$\frac{\frac{nR_2}{n-1}}{-\frac{nR_1}{n-1}} + \frac{\frac{R_2}{n-1}}{f_2} = 1$$

or

$$-\frac{R_2}{R_1} + \frac{R_2}{(n-1)f_2} = 1$$

$$\frac{R_2}{(n-1)f_2} = 1 + \frac{R_2}{R_1} = \frac{R_1 + R_2}{R_1}$$

$$\frac{1}{f_2} = (n-1)\frac{R_1 + R_2}{R_1 R_2} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

The posterior focus of the lens is deduced, therefore, from the expression

$$\frac{1}{F} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

The anterior focal distance is equal to the posterior focal distance, for it is clear that on rotating the lens the expression $\frac{1}{F}$ remains the same. We must replace R_1 by R_2, and *vice versa*, which does not change the expression.

Construction of the Image (fig. 15).—To construct the image A′ of a point A we can draw:

1°. The ray AC passing through the optic center: this ray suffers no deviation;

2°. The ray AD parallel to the axis: after refraction this ray passes through Φ_2;

3°. The ray $A\Phi_1$ passing through the anterior focus: after refraction this ray is parallel to the axis.

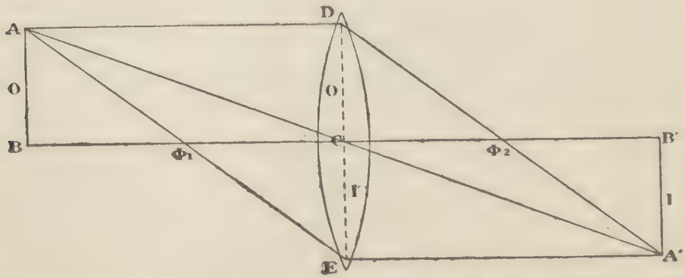


Fig. 15.—Construction of the image formed by a thin lens. $BC = f_1$, $B'C = f_2$, $C\Phi_1$ $C\Phi_2 = F$, $B\Phi_1 = l_1$, $B'\Phi_2 = l_2$.

These three rays intersect at the point A, but two suffice to find this point.

The triangles $AB\Phi_1$ and Φ_1CE on one side, and the triangles $DC\Phi_2$ and $\Phi_2B'A'$ on the other give us, as in the case of the mirrors, the relations:

$$\frac{O}{I} = \frac{l_1}{F} = \frac{F}{l_2} \text{ or } l_1\, l_2 = F^2$$

which can also be written

$$\frac{F}{f_1} + \frac{F}{f_2} = 1 \text{ or } \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}.$$

By the formula or by construction we find the following relations between object and image:

1. If the object is beyond the focus, the image is *real* and *inverted,* and on the other side of the lens. It is *enlarged* if the distance of the object from the lens is less than double the focal distance, *diminished* in the contrary case. If the distance of the object from the lens is equal to double the focal distance, the object and image are of the same size.

2. If the object is between the focus and the lens, the image is *virtual, erect* and *enlarged;* it is on the same side of the lens as the object, but farther away.

If, after having placed a strong lens on a printed sheet, we withdraw it gradually from the sheet, looking through it at some distance we see at first an erect image which is virtual and situated back of the lens and which increases in size the farther we remove the latter, until the sheet is at the focus; at that moment the image disappears (it becomes so large that a single point fills the entire field of the lens). Withdrawing the lens still farther we see an inverted image situated between the lens and the eye. It is enlarged at first, but rapidly diminishes according as the lens is removed.

Concave Lenses.—While biconvex lenses and plano-convex lenses, which act in the same manner, make incident rays converge, concave lenses make them diverge. The formula of the

focal distance remains the same, but as the surfaces are concave the radii must be considered as negative:

$$\frac{1}{F}=(n-1)\left(\frac{1}{-R_1}+\frac{1}{-R_2}\right).$$

The focal distance is therefore negative also, that is to say the focus is on the side from which the rays come. Incident parallel rays continue their course as if they come from the focus situated on the same side as the object.

The construction of the image is analogous to that which we have employed for biconvex lenses. It gives us the same relations as before with the necessary changes of the signs:

$$\frac{I}{O}=\frac{l_1}{-F}=\frac{-F}{l_2} \text{ and } \frac{1}{f_1}+\frac{1}{-f_2}=\frac{1}{-F}.$$

As long as the object is real, the image is *virtual, erect* and *smaller*. It is at the focus when the object is at infinity. According as the latter approaches the lens, the image does likewise. (1)

MENISCI.—A lens, one surface of which is convex and the other concave, is called a meniscus. According as the radius of the convex surface or that of the concave surface is smaller the meniscus is convergent or divergent (positive or negative). The positive meniscus is thicker in the middle, the negative is thicker towards the edges. These rules are valid, however, only when the thickness is negligible, which often does not happen.

METHODS OF MEASURING THE FOCAL DISTANCE OF A LENS.—The method most frequently employed by oculists consists in looking at exterior objects through the lens, subjecting the latter to slight displacements. We then notice that exterior objects are displaced in the same direction as the lens if the latter is concave, in the contrary direction if it is convex. In other words, if the eye is in front of the middle of the lens the rays reach it without any deviation; but if the eye is placed be-

(1) Generally the object and image move in the same direction in all cases of refraction, in an opposite direction in cases of reflection.

fore a peripheral part of the lens it receives rays deflected by reason of the prismatic effect of the glass, and this effect is greater in proportion as the part through which the eye looks approaches the periphery (fig. 16).—To determine the focal distance of a lens we find in the test case the glass which neutralizes it (1).

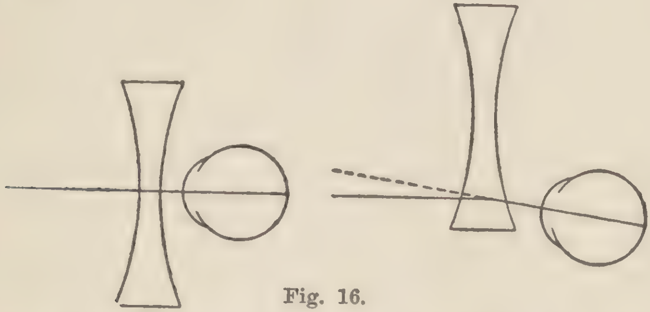


Fig. 16.

But we must remember that the numeration of the glasses in the test case is frequently not very exact.—Lenses have the same curvature on both sides; we have therefore $\frac{1}{F} = \frac{2(n-1)}{R}$; the index of the lens is approximately $n = 1.5$, which means that the focal distance and the radius are nearly the same length $\left(\frac{1}{F} = \frac{2(1.5-1)}{R} = \frac{1}{R}\right)$.—It was customary for a long time to number lenses according to their radius of curvature; as the index is generally a little more than 1.5, it would follow that the strong lenses would have a focal distance somewhat less than the number they bear, but in the case of convex glasses the error would be nearly compensated for by the influence of the thickness of the glass.

Later, numeration by dioptries (2) was introduced; and to

(1) We can also use with advantage the American spherometer, a little instrument with which we measure the radius of curvature and thus indirectly the refracting power of the glass.

(2) [In 1872 Monoyer, of France, first proposed the term *"dioptrie."* He says in the Annales d'Oculistique, Vol. 68, page 111: *"C'est le pouvoir dioptrique de la lentille d'un metre ou* 100 *centimètres de longueur focale qui doit servir d'unité. Cette unité nous l'appellerons unité métrique ou décimale de réfraction ou simplement—DIOPTRIE—si l'on veut biens nous permettre ce néologisme dérivé conformément aux usages scientifiques.* This term has been adopted all over the world and in English can have only one philologically correct translation, that is dioptry. This correct form has been employed, instead of diopter, all through this work.]—W.

obviate the necessity of changing the moulds in which glasses are ground the manufacturers simply wrote the numbers in dioptries on such of the old lenses as most nearly corresponded with such numbers. It is only recently that lenses have been manufactured strictly according to the dioptric series.

For all these reasons it may be useful for an oculist to be able to determine the focal distance directly. For convex lenses we need only form the image of a distant object on a screen. The distance of the lens from the screen is the focal distance.—For the concave lenses we place a flame at a great distance so that it forms its virtual image at the focus of the lens; we then place a screen behind the latter and find the position to give to it in order that the luminous circle formed by the lens would have a diameter equal to double that of the lens. The distance of the latter from the screen is the equal to the focal distance.

We can determine the radii of curvature by means of reflection images, by following the formulæ which we have given for the mirrors. Knowing the radii and focal distance we can calculate the index by the formula $\frac{1}{F} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$.

Refracting Power of a Lens.—The refracting power (D) of a lens is expressed in dioptries by the inverse of the focal distance measured in meters:

$$D = \frac{1}{F} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{n-1}{R_1} + \frac{n-1}{R_2}.$$

We can better realize the meaning of this expression if we recall the fact that we expressed the refracting power of a surface by the inverse of the anterior focal distance, $\frac{n-1}{R}$. The refracting power of an infinitely thin lens is, therefore, simply the sum of the refracting powers of its two surfaces.

The refracting power of an optical system composed of several infinitely thin lenses placed very near one another is equal to the sum of the powers of the lenses.

15. Theory of Gauss.—If the lenses are not so thin that their thickness can be neglected, nor placed so near one another that we can neglect their distances, we can find the position and size of the image by construction or by calculation by the rules which we have given for refraction by spherical surfaces: we construct or calculate in the first place the image formed by the first surface; this image then serves as the object for the second surface and so forth. But it is much simpler to use the theory of *Gauss*. We will briefly explain the essential points of this theory, which is applicable to every optical system composed of spherical surfaces, supposing that the system be centered, that is to say that all the centers of the surfaces are on the axis and that the aperture of the surfaces is small.

According to the theory of Gauss, every optic system has six cardinal points, namely:

Two principal points, h_1, h_2 (fig. 17);

Two nodal points, K_1, K_2;

One anterior focus, Φ_1;

One posterior focus, Φ_2.

The *anterior focal distance*, $F_1 = \Phi_1 h_1$, is the distance of the anterior focus from the first principal point; it is equal to the distance of the second nodal point from the posterior focus, $K_2\Phi_2$.

The *posterior focal distance*, $F_2 = h_2\Phi_2$, is the distance of the second principal point from the posterior focus; it is equal to the distance of the anterior focus from the first nodal point, $\Phi_1 K_1$.

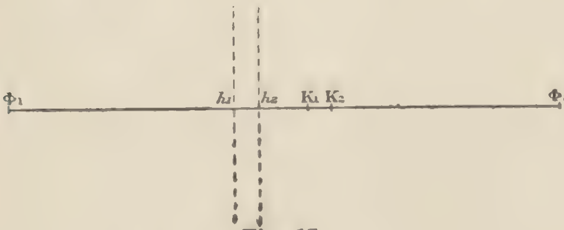


Fig. 17.

It follows that the distance of the first principal point from the first nodal point is equal to the distance of the second prin-

cipal point from the second nodal point and to the difference between the focal distances $F_2—F_1$. The distance between the two principal points is equal to the distance between the two nodal points.

The ratio between the focal distances is equal to the ratio between the indices of the first and last medium $\frac{F_2}{F_1} = n$.

We call *principal planes* two planes perpendicular to the axis and passing through the two principal points. The image of an object situated in the first principal plane is formed in the second principal plane and *vice versa*. It is the same size as the object and its direction is the same as that of the object.

A ray which, in the first medium, passes through the first nodal point, passes, after refraction, through the second nodal point, and the directions of the ray before and after refraction are parallel.

Knowing the position of the cardinal points, the image of a given point can be found by construction or calculation in a manner analogous to that which we have already employed in the case of infinitely thin lenses. To find the image of the point G (fig. 18) by construction we can choose two of the three following rays:

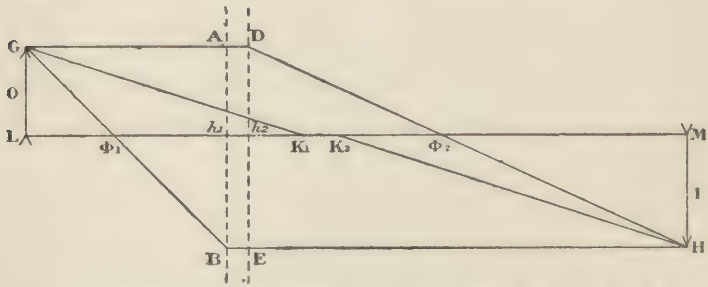


Fig. 18.—Construction of the image I of the object O. $L\Phi_1 = l_1$, $\Phi_1 h_1 = F_1$, $Lh_1 = f_1$ $M\Phi_2 = l_2$, $\Phi_2 h_2 = F_2$, $Mh_2 = f_2$.

1°. The ray GA, which is parallel to the axis, must cut the second principal plane at D, at a distance from the axis equal to Ah_1, and it must pass through Φ_2. Its direction is therefore DH.

2°. The ray GB, which passes through the anterior focus Φ_1, must, after refraction, be parallel to the axis: It will then take the direction EH.

3°. The ray GK_1, directed towards the first nodal point, takes, after refraction, the direction K_2 H, parallel to its first direction.

The triangles $GL\Phi_1$ and $Bh_1\Phi_1$ on one side and the triangles $Dh_2\Phi_2$ and $HM\Phi_2$ on the other give the relation.

$$\frac{O}{I} = \frac{l_1}{F_1} = \frac{F_2}{l_2}.$$

We have, therefore, as before $l_1 l_2 = F_1 F_2$, and we can deduce the other general formula $\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$. — Note that f_1 is reckoned as F_1 from the first principal point, f_2 on the contrary from the second principal point.

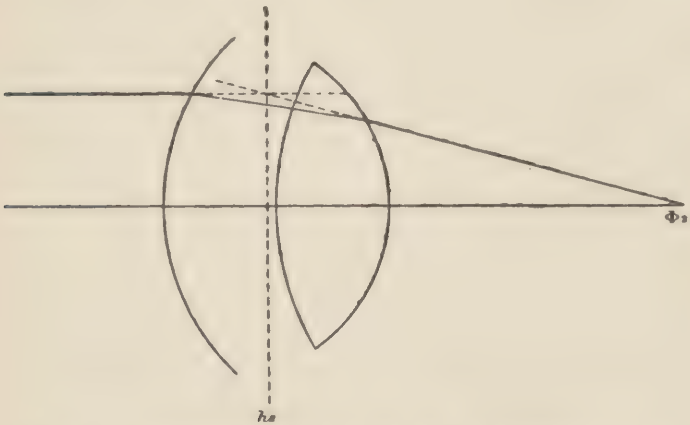


Fig. 19.—Construction to find the second principal plane.

METHODS OF FINDING THE CARDINAL POINTS OF A GIVEN SYSTEM.—*a*. CONSTRUCTION (fig. 19).—We draw an incident ray parallel to the axis and we construct its course by the *law of*

Descartes or by the formulæ which we have given for refraction by spherical surfaces. We thus find the posterior focus. We then prolong the incident and emergent rays; their point of intersection is situated in the second principal plane, and the perpendicular let fall from this point on the axis marks the second principal point h_2. Repeating the same construction with a ray parallel to the axis, coming from the other side, we find in the same manner the anterior focus and the first principal point. Knowing these four points we can deduce the positon of the nodal points, since the distance of the first nodal point from the anterior focus is equal to the distance of the second principal point from the posterior focus, etc.

b. CALCULATION.—Let us designate by A and B the two optic systems which we wish to combine, their focal distances by F'_1 and F'_2 (for the system A) and by F''_1 and F''_2 (for the system B), and the distance of the posterior focus of the system A behind the anterior focus of the system B, by *d*. We can then find the cardinal points of the combined system by means of the following formulæ in which y_1 indicates the distance of the anterior focus of the combined system behind the anterior focus of the system A, and y_2 the distance of the posterior focus of the combined system in front of the posterior focus of the system B.

$$y_1 = \frac{F'_1 F'_2}{d}. \qquad F_1 = \frac{F'_1 F''_1}{d}.$$
$$y_2 = \frac{F''_1 F''_2}{d}. \qquad F_2 = \frac{F''_2 F''_2}{d}$$

The deduction of these formulæ offers no difficulties. An incident ray, parallel to the axis, will pass after refraction by the system A, through its posterior focus, and, after refraction by system B, through the point Φ (fig. 19*a*); the posterior focus of the compound system. Its prolongation meets the prolongation of the incident ray at D so that h_2 is the second principal plane of the compound system. After the formula of *Newton* we have

$$y_2 = \frac{F''_1 F''_2}{d}.$$

On the other hand the figure gives us the relations:

$$\frac{a}{b}=\frac{F'_2}{d+F''_1}=\frac{F_2}{y_2+F'_2} \text{ or } F_2=\frac{F'_2\,(y_2+F''_2)}{d+F''_1}$$

$$=\frac{F'_2\left(\frac{F''_1F''_2}{d}+F''_2\right)}{d+F''_1}$$

$$=\frac{F'_2(F''_1F''_2+dF''_2)}{d\,(d+F''_1}$$

$$=\frac{F'_2F''_2}{d}$$

We find the value of y_1 and F_1 by supposing the light to come from the other side. Knowing thus the focal distance and the position of the foci it is easy to calculate those of the other cardinal points.

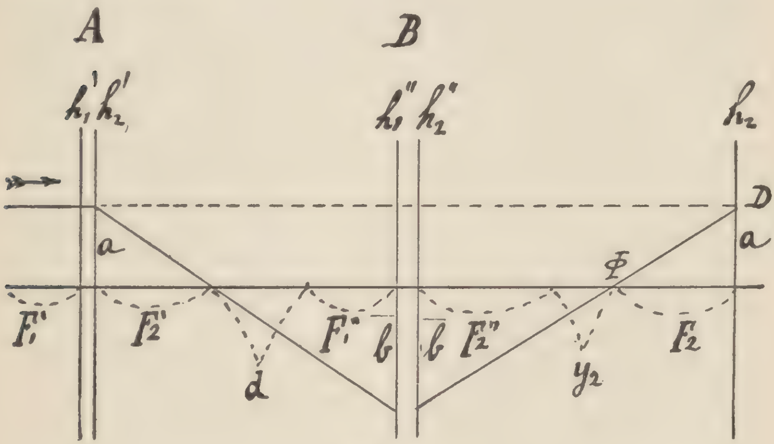


Fig. 19*a*.

In the case which the figure represents, d is negative, since the posterior focus of A is situated in front of the anterior focus of B; F_1 and F_2 are, therefore, also negative, as well as y_1 and y_2; the compound system acts as a concave lens. If $d=0$ the focal distances are infinity: incident parallel rays are again parallel after refraction. Such a system is called *telescopic;* a telescope focused on infinity by an emmetropic observer is an illustration of it. The distance d, the sign of which determines

the character of the compound system is often called the *interval;* in the cases which interest us it is nearly always positive.

Special Cases.—As the focal distances are proportional to the indices of the first and last media, they ought to be equal if the first and last media are identical, which is true for nearly all optical instruments. In this case the distance of the anterior focus from the first principal point is equal to its distance from the first nodal point, that is to say the first principal point coincides with the first nodal point and the second principal point with the second nodal point.

This is what occurs in the case of thick lenses, in which case we can find the nodal points by a simple construction. Let C_1 (fig. 20) be the center of the first surface; C_2 that of the second; C_2 A any radius whatever of the second surface, and C_1 B a radius of the first surface parallel to C_2 A. Let us draw the straight line AB, which represents the course of a ray in the interior of the lens; DB and AE indicate its direction outside the lens. It is easy to see that these two straight lines are parallel; the angles i are, in fact, equal, since the angles r are equal. Prolonging DB and AE they cut the axis at the two points K_1 and K_2, which are the two nodal points. The point O is the optic center of the lens. It is the image of K_1 in relation to the first surface, and that of K_2 in relation to the second surface.

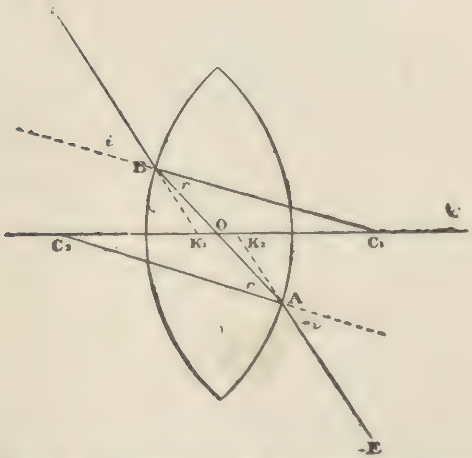


Fig. 20.—Construction to find the nodal points of a thick lens.

In an *infinitely thin lens,* the nodal points and the principal points all coincide with the optic center. If the entire system

is represented by a *simple refracting surface,* both principal points coincide with the surface, and the nodal points with the center.

The *mirrors* may be considered as dioptric systems, in which the last medium has an index equal to that of the first medium, but with the contrary sign, since the rays run in a contrary direction. The two principal points coincide with the surface, the nodal points with the center, and the focus is at an equal distance between the two (since $F_1 = -F_2$). The compound reflecting systems likewise have only one principal point and one nodal point, and the focus is situated at an equal distance between them. Such, for example, is the case in the compound systems which give rise to the images of *Purkinje* in the eye.

EXAMPLE I.—*To find the cardinal points of the crystalline lens.*

Suppose the crystalline lens has a thickness of 4 millimeters, that the radius of the anterior surface is 10 millimeters and that of the posterior surface 6 millimeters. Let us take 1.33 as the index of the aqueous humor and the vitreous body, and suppose that the index of the crystalline lens in relation to these liquids is about 1.06.

In this case each of the systems A and B is represented by a single refracting surface. The focal distances of the system A are:

$$F'_1 = \frac{R_1}{n-1} = \frac{10}{0.06} = 167^{mm}$$

$$F'_2 = \frac{R_1 n}{n-1} = \frac{10 \times 1.06}{0.06} = 177^{mm}$$

those of the system B are:

$$F''_1 = \frac{R_2}{n-1} = \frac{-6}{\frac{1}{1.06} - 1} = \frac{6 \times 1.06}{0.06} = 106^{mm}$$

$$F''_2 = \frac{n\,R_2}{n-1} = \frac{-6 \times \frac{1}{1.06}}{\frac{1}{1.06} - 1} = \frac{6}{0.06} = 100^{mm}$$

The *interval d* is the distance of the posterior focus of the system A from the anterior focus of the system B; the former

is situated as 177 millimeters behind the anterior surface, the latter at 106 millimeters in front of the posterior surface; the thickness of the crystalline lens being 4 millimeters, we will have $d=177$ millimeters $+$ 106 millimeters — 4 millimeters $=$ 279 millimeters, and

$$y_1=\frac{F'_1F'_2}{d}=\frac{167\times 177}{279}=106^{mm}$$

$$y_2=\frac{F''_1\ F''_2}{d}=\frac{106\times 100}{279}=38^{mm}$$

$$F_1=\frac{F'_1\,F''_1}{d}=\frac{167\times 106}{279}=63.4^{mm}$$

$$F_2=\frac{F'_2\ F''_2}{d}=\frac{177\times 100}{279}=63.4^{mm}$$

The anterior focus of the crystalline lens being situated at 106 millimeters behind the anterior focus of the first surface C, which is at 167 millimeters, its distance as far as that surface will be 167—106=61 millimeters, and as the focal distance is 63.4 millimeters, the first principal point of the crystalline lens will be placed at 2.4 millimeters behind the anterior surface. The second principal point will be situated at an equal distance, at 100—38—63.4=—1.4 millimeters, that is to say, 1.4 millimeters in front of the posterior surface.

Both focal distances are equal, as they must be, since the surrounding media are alike. The refracting power of the crystalline lens would be with these data $\frac{1}{63.4mm}=15.8$ D.

Example 2.—Let us consider the cornea as a simple refracting surface with a radius of 8 millimeters surrounded in front by air ($n=1$), behind by the aqueous humor ($n=1.33=\frac{4}{3}$). The distance of the anterior surface of the cornea from the anterior surface of the crystalline lens is 3.6 millimeters. *To combine the cornea with the crystalline lens the cardinal points of which we have just found.*

Here the cornea forms the system A. Its focal distances are:

$$F'_1=\frac{R}{n-1}=24^{mm}$$

$$F'_2=\frac{Rn}{n-1}=32^{mm}$$

The principal points coincide with the surface. The focal distances of the system B are those found above for the crystalline lens.

The *interval d* is the distance of the anterior focus of the crystalline lens as far as the posterior focus of the cornea: $d = 61 \text{ mm.} + 32 \text{ mm.} - 3.6 \text{ mm.} = 89.4$. With these data we find for the entire optic system of the eye:

$$y_1 = \frac{24 \times 32}{89.4} = 8.6^{mm}$$

$$v_2 = \frac{63.4 \times 63.4}{89.4} = 45^{mm}$$

$$F_1 = \frac{24 \times 63.4}{89.4} = 17.0^{mm}$$

$$F_2 = \frac{32 \times 63.4}{89.4} = 22.7^{mm}$$

The following table gives a general idea of such an optic system. By *position* of a point we mean the distance of that point behind the summit of the cornea.

Simplified Eye.

Index of aqueous humor and vitreous body	1.33
— the crystalline lens	1.41
Radius of curvature of the cornea	8^{mm}
— — — anterior surface of the crystalline lens	10^{mm}
— — — posterior surface of the crystalline lens	6^{mm}
Depth of the anterior chamber	3.6^{mm}
Thickness of the crystalline lens	4^{mm}
Anterior focal distance of the cornea	24^{mm}
Posterior focal distance of the cornea	32^{mm}
Focal distance of the crystalline lens	63.4^{mm}
Position of the anterior principal point of the crystalline lens	6^{mm}
— — posterior principal point of the crystalline lens	6.2^{mm}
Anterior focal distance of the eye	17^{mm}
Posterior focal distance of the eye	22.7^{mm}
Position of the anterior principal point of the eye	1.6^{mm}
— — posterior principal point of the eye	1.9^{mm}
— — anterior nodal point of the eye	7.3^{mm}
— — posterior nodal point of the eye	7.6^{mm}
— — anterior focus of the eye	-15.4^{mm}
— — posterior focus of the eye	24.6^{mm}

Refracting power of the cornea	....	42D.
— — — crystalline lens	....	16D.
— — — eye	....	59D.

We shall see in the following chapter that the data with which we have made these calculations are not rigorously exact; nevertheless, they give a very close approximation, generally sufficient for our purpose. Later I shall have recourse more than once to this system, which I call the *simplified eye,* to distinguish it from the complete optic system of which we shall treat in the following chapter.

Bibliography.—Complete development of the system of *Gauss* in the introduction to the physiologic optics of *Helmholtz.*

Among the numerous treatises on geometric optics, I shall cite:

Jamin and Bouty. *Cours de physique de l'Ecole Polytechnique,* 1886.—Pouillet-Müller. *Lehrbuch der Physik und Meteorologie,* 8th edition. Braunschweig, 1872. Of an easy study.—Wullner (Ad.). *Lehrmuch der Experimentalphysik.* II. Leipzig, 1877.—Lorenz (L.). *Die Lehre vom Licht.* Leipzig, 1877.

Among the more complete works, but of a more difficult study, we shall cite:

Verdet (E.). *Œuvres.* Paris 1872.—Herschel (Sir J. F. W.). *Light.* London, 1845. In French by Verhulst (P. F.) and Quetelet (A.). Paris, 1829.—Health (R. S.). *A Treatise on Geometric Optics.* Cambridge, 1877. —Gariel (G. H.). *Etudes d' optique géométrique.* Paris, 1889.

The beautiful works of *E. Abbe* resulted in considerable progress in geometric optics during the last twenty years. We will find an account of them in Czapski (S.), *Theorie der optischen Instrumente,* Breslau, 1893, and, in a more easily accessible form, in the new edition of Pouillet-Muller, by Pfaundler (L.) and Lummer (O.), Braunschweig, 1897.

CHAPTER II.

THE OPTIC SYSTEM OF THE EYE

16. Optic Constants of the Eye.—By means of the theory of *Gauss* we can calculate the cardinal points of any optic system if we know the position and curvature of the surfaces and the index of the media. To calculate the optic system of the eye

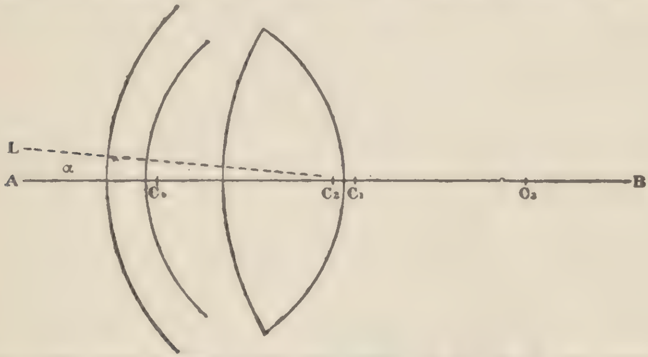


Fig. 21.—The optic system of the eye (left), C_1, C_2, C_3, C_4, the centers of the four surfaces in their natural order; AB, optic axis; L, visual line.

we must know, therefore, as exactly as possible those numbers which are frequently called the optic constants of the eye. Those which I have given in the examples in the preceding chapter are only approximate. The following table gives the constants of an eye, which I have measured as carefully as possible (fig. 21):

Optic Constants of the Eye.

Position of the anterior surface of the cornea..............	0
— — posterior surface of the cornea..............	1.15mm
— — anterior surface of the crystalline lens.......	3.54mm
— — posterior surface of the crystalline lens.......	7.60mm
Radius of the anterior suface of the cornea...............	7.98mm
— — posterior surface of the cornea..............	6.22mm
— — anterior surface of the crystalline lens........	10.20mm
— — posterior surface of the crystalline lens......	6.17mm

Index of the air....................	accepted	1
— — cornea....................	accepted	1.377
— — aqueous humor.............	accepted	1.3365
Total index of the crystalline lens........	accepted	1.42
Index of the vitreous body...............	accepted	1.3365

The positions and radii of the surfaces as stated are according to measurements which I made by methods which I shall mention later.

The only difference of any importance between them and those found up to the present arises from the thickness of the crystalline lens which, in his schematic eye *Helmholtz* put down as 3.6 millimeters, certainly too small a number to be considered an average. I have also added the numbers for the posterior surface of the cornea which I was the first to measure.

As to the indices which cannot be measured directly on the living eye I have put down 1.377 for the cornea after a measurement of *Matthiessen,* which I also have verified. Those of the aqueous humor and vitreous body are very exactly known; we can, indeed, determine them with great exactness by means of the refractometer of *Abbe,* or by other analogous methods.

Less is known of the index of the crystalline lens than of the other optic constants of the eye. It must be noted in the first place that this body is not homogeneous; its index gradually diminishes starting from the center of the nucleus towards the periphery. The curvature of its layers diminishes also towards the periphery, so that each layer takes the form of a meniscus, the concavity of which is greater than the convexity. This conclusion follows as well from anatomical researches as from optic observations which I made on the eye of an ox after death (1).

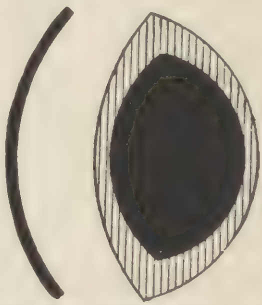


Fig. 22.—Optic system of the eye of an ox (twice enlarged.)

There is, indeed, frequently produced, in the crystalline lens, after death, a differentiation between the cortical masses and

(1) The optic constants of such an eye are as follows (fig. 22):

Radius of the cornea	15	millimeters
Position of the anterior surface of the crystalline lens	6	—
— — posterior surface of the crystalline lens	17	—
Radius of the anterior surface of the crystalline lens	14	—
— — posterior surface of the crystalline lens	8	—
— — anterior surface of the nucleus	8.5	—
— — posterior surface of the nucleus	7	—

the nucleus, probably caused by the imbition of water by the superficial parts. In consequence of this process there is produced on the surfaces of the nucleus quite a regular reflection, so that instead of two reflection images we see four (fig. 23), when the crystalline lens is exposed to the light of a flame. Now, the position of these images indicates that the curvature of the surfaces of the nucleus is considerably greater than that of the crystalline sur-

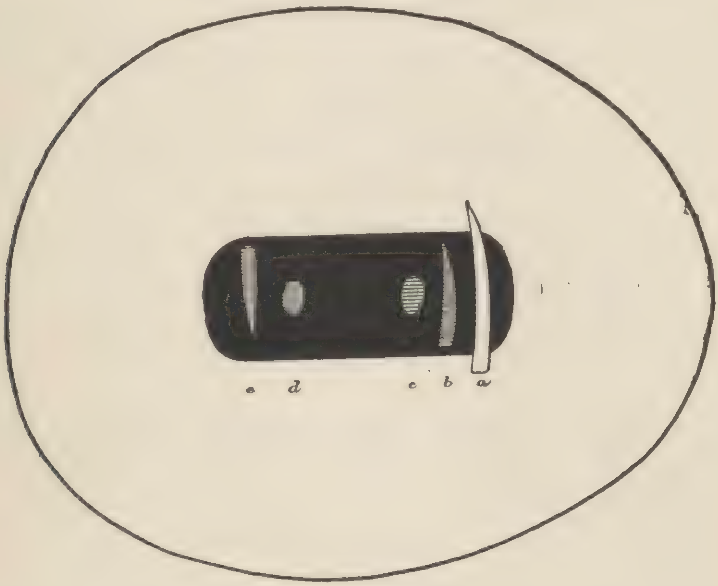


Fig. 23.—Images of Purkinje of the eye of an ox (dead). (Flame of a candle.) *a*, image of the cornea; *b*, image of the anterior surface of the crystalline lens; *c*, image of the anterior surface of the nucleus; *d*, image of the posterior surface of the nucleus; *e*, image of the posterior surface of the crystalline lens.

faces. *Dr. Demicheri* has recently described cases of alterations of the human cyrstalline lens in which we can also observe four

crystalline images; their position also indicates a greater curvature of the surfaces of the nucleus (fig. 24).

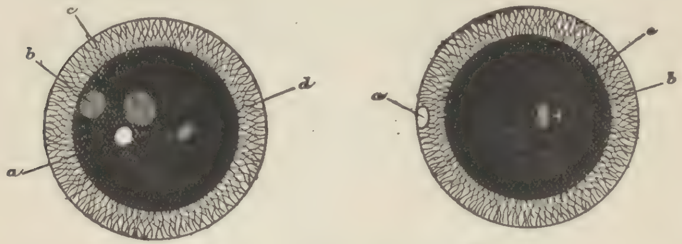


Fig. 24.—Double crystalline images in cases of "false lenticonus." After Demicheri.

A. Looking straight in front.

a, image of the cornea; *b*, image of the anterior surface of the crystalline lens; *c*, image of the anterior surface of the nucleus; *d*, image of the posterior surface of the crystalline lens, which coincides, for this direction of the look, with that of the posterior suface of nucleus.

B. Looking outwards.

a, image of the cornea; *b*, image of the posterior surface of the crystalline lens; *c*, image of the posterior surface of the nucleus.

It has long been known that, as a result of this peculiar construction of the crystalline lens, its *total* index, that is to say, the index of an imaginary lens having the same form and the same focal distance as the crystalline lens, is greater, not only than the mean index of the crystalline layers, but even than that of the nucleus.

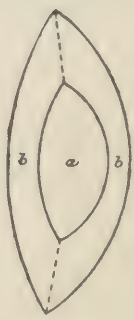


Fig. 25.

To account for this paradoxical phenomenon, we may suppose the crystalline lens divided into two parts, the nucleus and the cortical part, supposing the index uniform in each part, but greater for the nucleus. On account of its great curvature and high index, the nucleus (*a* fig. 25) would then have a very considerable refracting power, which, however, would be diminished by the influence of the cortical layers

which act as two concave lenses (*b, b*). It is clear that if the index of these layers were higher their influence would be greater, and the refracting power of the whole crystalline lens would consequently be weaker.

Thomas Young placed the index of the center of the nucleus at 1.412, and by calculation therefrom he deduced 1.436 for the total index. Later *Listing* gave 1.455 for the total index, a number adopted by *Helmholtz,* but which is decidedly too high. For his new schematic eye this latter author later adopted an index (1.4371) which was nearly identical with that of *Young.* More recently *Matthiessen* tried to determine the law after which the index of the crystalline lens varies from the center towards the periphery, and to calculate from it the total index. According to him the difference between the total index and that of the superficial layers would be double the difference between the index of the nucleus and that of these cortical layers. He has found 1.437 as the total index, and the average of his measurements of the central index approaches very close to the figures of *Young.*—Measurements which I have taken after a new method, in collaboration with Dr. Stadfeldt (1), seem, however, to show that the law of Matthiessen can be considered only as an approximation, and, on the other hand, the observations of those who have operated on cataract seem, as we shall see later, to call for a lower total index. Awaiting the result of new measurements I adopt the number 1.42.

Thanks to the special structure of this organ the refracting power of the crystalline lens is some dioptries stronger than it would have been if its index had been uniformly equal to that of the nucleus. In comparison with the total refraction of the eye the increase is not considerable; it might easily have been obtained by a slightly greater curvature of one of the surfaces. The teleologic reason for this structure is rather to be sought in

(1) According to the measurements of *Stadfeldt,* which I shall mention later on, the mean index of the crystalline lens would be 1.435, and the refracting power of the crystalline lens would be on an average 19 D. (varying between 17 D. and 24 D.).

the mechanism of accommodation. For, this mechanism would be, as I understand it, impossible without the two peculiarities which characterize the structure of the crystalline lens: the increase of density and the increase of curvature of the layers according as we approach the center.—Another advantage of this structure of the crystalline lens consists in making weaker the images of the eye which I call harmful (*nuisibles*), and which I shall mention farther on.

17. Optic System of the Eye.—Applying the theory of *Gauss* to the data which we have just stated, we find the following results:

A.—*Optic System of the Cornea.*

Position of the first principal point	— 0.13mm
— — second principal point	— 0.14mm
— — first nodal point	8.08mm
— — second nodal point	8.07mm
— — anterior focus	24.53mm
— — posterior focus	32.47mm
Anterior focal distance	24.40mm
Posterior focal distance	32.61mm
Refracting power	40.98D.

B.—*Optic System of the Crystalline Lens.*

Position of the first nodal point	5.96mm
— — second nodal point	6.14mm
Focal distance of the crystalline lens	62.46mm
Refracting power	16.01D.

Combining these two systems, we find the complete optic system of the eye.

C.—*Complete Optic System of the Eye.*

Position of the first principal point	1.54mm
— — second principal point	1.86mm
— — first nodal point	7.30mm
— — second nodal point	7.62mm
— — anterior focus	—15.59mm
— — posterior focus	24.75mm
Anterior focal distance	17.13mm
Posterior focal distance	22.89mm
Refracting power	58.38D.

Thanks to these data we may eliminate, so to speak, the entire real optic system of the eye. In the system which we have just calculated we take into consideration only the course of the rays in the air before entering the eye, and their course in the vitreous body after emergence from the crystalline lens; their course between the anterior surface of the cornea and the posterior surface of the crystalline lens remains unknown to us.

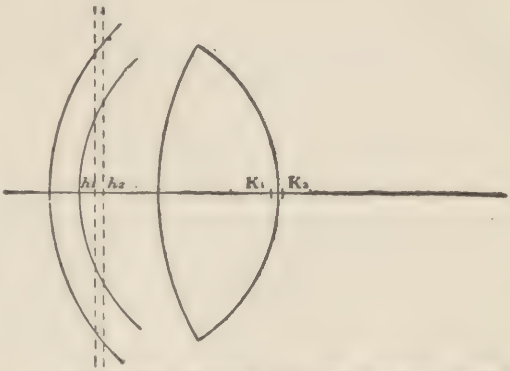


Fig. 26.—Position of the cardinal points of the human eye (magnified four times). h_1 h_2, principal planes; K_1 K_2, nodal points.

We note that the refracting power of the cornea is 2.5 times greater than that of the crystalline lens. The sum of their refracting power is not far from being equal to the refracting power of the eye, because the nodal points of the cornea are quite near those of the crystalline lens (1).

The following little table shows the refracting power of each of the surfaces:

Anterior surface of the cornea	+47.24 D.
Posterior surface of the cornea	— 4.73 D.
Anterior surface of the crystalline lens	+ 6.13 D.
Posterior surface of the crystalline lens	+ 9.53 D.
Total	+58.17 D.

(1) The refracting power of the eye would be exactly equal to the sum of the powers of its component systems, if the anterior principal point of the crystalline lens coincided with the posterior nodal point of the cornea, or if we consider the cornea as a single refracting surface, with its center. In the formula of paragraph 15 (page 27).

$$F_1 = \frac{F'_1 F''_1}{d}$$

we would have, indeed, in this case $d = F_1' + F_1''$, which gives

$$F_1 = \frac{F'_1 F''_1}{F''_1 + F'_1} \quad \text{or} \quad \frac{1}{F_1} = \frac{1}{F'_1} + \frac{1}{F''_1}$$

The posterior surface of the cornea has, up to the present, been neglected by authors; we see that it has a certain importance. Its value is negative and almost as great as that of the anterior surface of the crystalline lens. We shall see that it seems to play a part in certain forms of astigmatism.

Nevertheless, we commit only a very small error by neglecting it, that is to say, by supposing that the substance of the cornea does not exist; the anterior surface simply separating the air from the aqueous humor. By eliminating the negative influence of the posterior surface, the total refraction of the cornea should increase, but the power of the anterior surface diminishes nearly as much, since we replace the index of the cornea by the weaker index of the aqueous humor. In our case we would, by thus simplifying the matter, have found a refracting power of the cornea equal to 42.16 D. instead of 40.98 D., that is to say, we would have committed an error of 1.18 D. or about 1/50 of the total power of the eye.

The right eye, the optic system of which I have calculated (in the horizontal meridian), is the only one of which up to the present time we possess complete measurements. It is important to note that it is not to be considered as an average. The radius of the cornea is two or three-tenths of a millimeter above the average, and the length of the axis of the supposed emmetropic eye, which we have found equal to 24.75 mm., is probably also a little above the average. This eye is, therefore, to be considered relatively large, the more so as the person to whom it belongs is pretty tall in stature. A light degree of astigmatism with the rule would also act in the same way. I have measured some other eyes, but not a sufficient number to be able to establish an average.

The figures which I have just given apply only to the eye of the adult. The eye of the new-born child is much smaller (the axis measures about 17 mm. instead of 24 mm.), so that we might expect to see the curvature of all the surfaces increased

in the same proportion. This is not so: according to the concordant measurements of *Axenfeldt* and *Holth* the cornea of the new-born child differs but little from the adult cornea. This latter varies as we shall see between quite wide limits (40 to 47 dioptries) and the values which we find in the new-born child are near the higher limit.

Compensation for the diminution of the axis is made by the crystalline lens. According to the measurements of *Stadfeldt* the crystalline lens of the new-born child is as thick as that of the adult, but the diameter is 6 mm. instead of 8 or 9 mm., whence it follows that the curvature of the surfaces is very great. Following are some figures according to *Stadfeldt:*

	Radius *Ant. surface.*	*Radius* *Post. surface.*	*Thickness.*	*Diameter.*
Adult.............	11mm	6mm	3.6mm	
New-born.........	4.5mm	4mm	3.9mm	6mm

Supposing that the index is the same as in the adult, the crystalline lens of the new-born child would, therefore, be nearly twice more refracting, and the crystalline refraction in the latter would not be very far from being equal to the corneal refraction.

18. Aperture of the System.—The theory of *Gauss* supposes that the aperture of the system is very small, which is by no means the case in the eye, and many errors committed in questions of ocular refraction seem to me due to the fact that we do not sufficiently take into account the large aperture of the system. In optic instruments an aperture over ten or twelve degrees is scarcely accepted. Supposing that the pupil has a diameter of 4 millimeters, the aperture of the cornea would be 20 degrees; and a pupillary diameter of 4 millimeters is rather insufficient, for it must not be forgoten that we generally examine our patients with a very strong light. In the ordinary circumstances of life, the pupillary diameter is most frequently greater (5 or 6 millimeters), whence results a series of errors

which would be still greater but for the special precautions taken to neutralize them in part.

We must bear in mind that the pupil is seen neither in its real position nor at its true size: it appears moved forward and enlarged on account of the refraction through the cornea. It is easy to determine its apparent place and size. In our general formula, $\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$, we must put the values of the cornea of the simplified eye, $F_1=24$, $F_2=32$, and the distance of the anterior surface of the crystalline lens and of the pupil from the anterior surface of the cornea, $f_2=3.6$, and we find $f_1=-3.04$. And if the real size is 4 millimeters, we put in the formula $\frac{I}{O}=\frac{F_2}{l_2}$ the values

$$O=4^{mm},\ F_2=32^{mm},\ l_2=3.6^{mm}-32^{mm}=-28.4^{mm};$$

therefore

$$I=\frac{4\times 32}{28.4}=4.5^{mm}.$$

The pupil appears, therefore, moved forward about 0.5 mm. and enlarged by the same quantity. The iris appears at the same time swelled in front.

What we see is, therefore, a virtual image of the iris and of the pupil. We call these images *apparent iris* and *apparent pupil.* They are *aerial* images. Rays which, in the air, are directed towards a point of the apparent pupil are, after refraction by the cornea, directed towards the corresponding point of the real pupil.

If we imagine the iris and pupil seen, through the crystalline lens, by an eye located in the vitreous body, the pupil would no longer appear in its place, but the displacement would be less; it would be seen nearly 0.1 mm. farther back than it is in reality, and enlarged 0.2 mm. Rays coming from a point of the real pupil would proceed in the vitreous body as if they came from the corresponding point of the crystalline image.

If we had constructed the corneal image and the crystalline image of a point of the pupil, we would then know that a ray

directed towards the former would pass, after refraction by the cornea, through the same point, and, after refraction by the crystalline lens, through the crystalline image of the point. The apparent pupil belongs therefore to the incident rays as does the first principal point or the first nodal point, and the crystalline image of the pupil belong to the emergent rays.

The luminous cone which enters the eye is limited by the apparent pupil; in its course between the cornea and the crystalline lens, it is limited by the real pupil, and, in the vitreous body, by the crystalline image of the pupil. There are analogous phenomena in most optical instruments, wherever a diaphragm is between two lenses; Professor *Abbe* has proposed the names of *pupil of entrance* and *pupil of exit* for the images of the diaphragm.

We have seen that the principal planes are each the image of the other, and that they have this characteristic that the object and image are of the same size.

In the formula $\frac{F1}{f1} + \frac{F2}{f2} = 1$, the distances marked 1 are calculated to start from the first principal point, those marked 2 to start from the second principal point. But in this

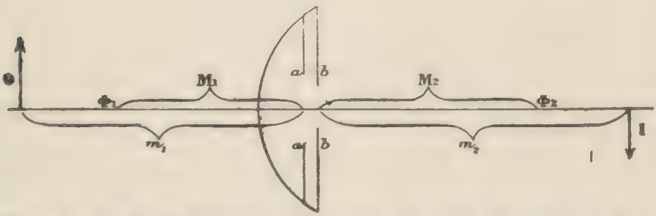


Fig. 27.—*aa*, pupil of entrance; *bb*, pupil of exit; O, object; I, image; Φ_1, anterior focus; Φ_2, posterior focus.

formula we can as well calculate the distances from any other pair of points, one of which is the image of the other. We might measure, for example, from the pupil of entrance and pupil of exit. We would thus have in figure 27 the relation $\frac{M1}{m1} + \frac{M2}{m2} = 1$ and we could find the image of an object by constructions analogous to those in which we have used the principal planes. The only difference is this: if an incident ray

meets the first principal plane at a distance from the axis equal to y, the emergent ray also cuts the second principal plane at a distance from the axis equal to y. But if the incident ray meets the pupil of entrance at a distance from the axis equal to y, the emergent ray cuts the plane of the pupil of exit at a distance from the axis which is to y in the same relation as the diameter of the pupil of exit is to that of the pupil of entrance. In our case it would be the relation of $\frac{42}{45}$. This mode of procedure is often more convenient than the classic method, more especially because it is easy by this construction to calculate the diameter of the luminous cone.

19. Point of Fixation. Visual Line.—To distinguish an object clearly it is necessary to *fix* it, that is to say, to place the eye in such a way that its image is formed on the *fovea*. The point fixed and the *fovea* are therefore conjugate foci. But we would be greatly deceived if we thought that the entire *fovea* corresponded with the point of fixation. The anatomical *fovea* has an extent of 0.2 mm. to 0.4 mm. (*Henle*) or of 0.75° to 1.50°, seen from the posterior nodal point (at 16 millimeters from the retina). Looking at the sky the fovea would cover, therefore, a part having two or three times the diameter of the moon, which corresponds to a half degee. The point of fixation is much smaller in dimension, for we can readily tell whether we fix the right border or the left border of the moon. Generally when two points closely approach each other we can still tell which one is fixed as long as we can see that there are two. It was *Javal* who specially insisted on this fact, to which he attributed great importance for the theory of binocular vision.

We designate as the *visual line* the ray which goes from the point fixed to the first nodal point, and which, consequently, after refraction, reaches the fovea as if it came from the second nodal point. If, in the aphakic eye, we neglect the posterior surface of the cornea, the visual line passes through the center of curvature of the anterior surface; it is, therefore, perpendicular to that surface. In a normal eye it is never far from being so, since the nodal points are very near the center of

curvature of the anterior surface of the cornea. The direction of the visual line does not depend on the position of the pupil. In cases of pupillary displacement it may happen that the ray which represents the visual line does not enter the eye. We shall see later (page 78) how we may determine experimentally the direction of the visual line in the eye.

20. Optic Axis. Angle. α—An exact centering would demand that the four centers of curvature, or the three, if we neglect the posterior surface of the cornea, would be on the same straight line. The centering of the eye is never exact, but the deviations that we can establish are often small. In some cases I have, however, found defects of centering relatively large in eyes, too, which functionally should be considered normal. The defect which I have most frequently met consists in this, that the center of curvature of the cornea is situated (as much as a quarter of a millimeter) below the axis of the crystalline lens. —Neglecting these deviations the optic system of the eye may be considered as centered around a straight line which is called the *optic axis* of the eye. The fovea not being placed on this line, it does not coincide with the visual line; it is directed outward and downward from the visual line and forms with it an angle of 5° to 7°, called the *angle* α (fig. 21).—We shall see later that the anterior surface of the cornea is not spherical; it is flattened towards the periphery so that it may be compared to an ellipsoid of revolution around the long axis. Certain authors designate as the angle α the angle which the line of vision forms with that axis which passes through the most curved part of the cornea (the summit). Generally the axis of the cornea very nearly coincides with the optic axis of the eye, so that both definitions amount to the same thing. But we shall see that the comparison of the form of the cornea to that of an ellipsoid is very defective. Hence it may be better to retain the old definition.

We can compare the optic system of the eye with that of an opera glass. If the optician, by a defect of workmanship, had

placed one of the lenses a little obliquely, or if he had placed the middle of this lens a litte outside the axis of the instrument, this defect would correspond with a defect in the centering of the eye.—If, on the contrary, the observer looked a little obliquely through the glass, the visual line would form with the axis of the glass an angle which would correspond with the angle a.

21. Useful Image.—The optic system of the eye forms a dioptric image, *real, inverted* and *diminished,* which is projected on the retina as the photographic image is formed on the screen of the dark chamber. The comparison between the eye and the dark chamber dates from the invention of this instrument (*Porta, Leonardo da Vinci*). But although we had from that time all the elements necessary to understand the construction of the eye, there continued, however, to prevail much confusion on this question, more especially because people could not be brought to admit that the image which serves for vision was inverted. It was *Kepler* (1604) who first explained the formation of images in general and was led to suppose the existence of an inverted image on the retina, an image which was later demonstrated by *Scheiner* on an eye from which he had removed a part of the sclera and of the choroid.—But, besides this image which I designate as the useful image, because it serves for vision, there is formed in the eye a series of other images which I have designated as false images of the eye, and which will form the subject of the following chapter:

Bibliography.—*Œuvres ophthalmoliques* of Thomas Young, edited by Tscherning, p. 134-137.—Listing (J.). *Dioptrik des Auges* in Wagner, *Handworterbuch der Physiologie.*—Tscherning (M.). *Beitrage zur Dioptrik des Auges* in *Zeitschrift fur Psychologie und Physiologie der Sinnesorgane,* III, p. 429.—Matthiessen. *Die Neuren Fortschritte in unserer Kentniss von dem optischen Baue des Auges der Wirbelthiere* in *Beitrage zur Psychologie und Physiologie der Sinnesorgane,* dedicated to Helmholtz on the occasion of his 70th anniversary. Stadfeldt (A.). *Recherches sur l'indice total du cristallin humain. Journal de Physiologie et Pathologie.* November, 1899.

CHAPTER III.

FALSE IMAGES OF THE EYE

22. General Remarks.—If we place a flame at some distance from a lens, we notice on the same side with the light two reflected images of the flame, one for each surface. Placing the eye on the other side of the lens at some distance, we see the dioptric image, which is real, and, besides, a small, indistinct

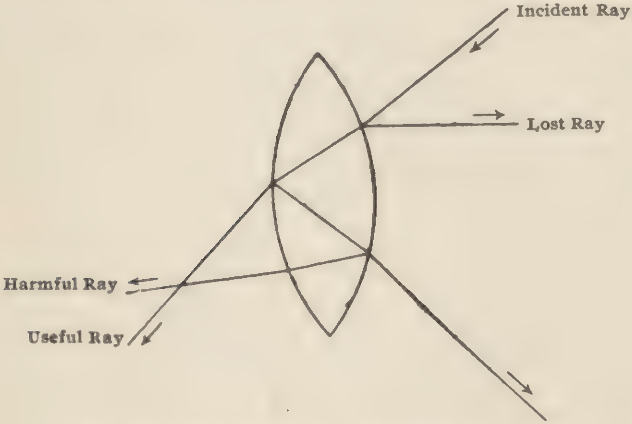


Fig. 28.—Reflections and refractions by a lens.

image due to a double reflection in the interior of the lens, a first reflection produced by the posterior surface, and a second by the anterior surface (fig. 28). The rays which form this latter image undergo, besides, a refraction by each surface of the lens. The small image is real; we can, indeed, receive it on a screen held near the lens.

The incident light is thus divided into three portions: *useful* light which forms the dioptric image of which we generally make use, the light *lost* by reflection on the surfaces, and lastly, the light reflected twice, which I call *harmful* (*nuisible*). This

harmful light may, indeed, enter the eye which is observing the useful image, where it is often a cause of annoyance, because it does not contribute to the formation of that image. A simple lens loses about 8 per cent. by reflection, and the harmful light represents only 1/500 of the incident light. In complicated instruments much more of the light is lost. In the ophthalmometer of *Javal* and *Schioetz,* the loss is about 33 per cent.

In the human eye we may also distinguish between the *useful* light which passes through the surfaces, the light *lost* by reflection, and the *harmful light,* which, having suffered two reflections, returns again towards the retina. But the eye has this peculiarity that, of all optic instruments, it is that which loses least light (about 2 per cent). The harmful light is also reduced to a minimum, but feeble as it is, it is visible nevertheless.

The useful light forms the dioptric image which serves the purpose of vision; the lost light forms four false images of the first order, called images of *Purkinje,* one for each surface; they correspond to rays I, II, III and IV, fig. 29. The harmful

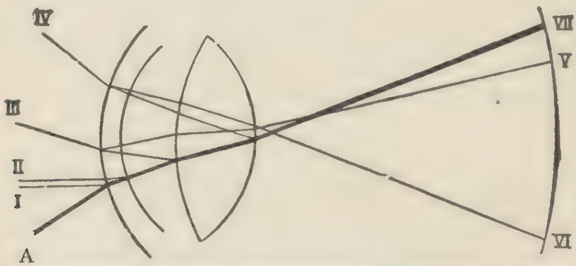


Fig. 29.—Manner in which a luminous ray is divided in the eye. A, incident ray.—I, II, III, IV, lost rays corresponding to the four images of *Purkinje;* V and VI, harmful rays corresponding to the fifth and sixth image; VII, useful ray.

light forms a series of false images of the second order, of which one only is visible (rays V and VI, fig. 29).

23. The Image of Purkinje.—These images were described at the beginning of this century by the scientist whose name they bear, but one of them, the second, was lost sight of until I

described it again some years ago. (1) The first of these images, that due to the anterior surface of the cornea, is produced by a single reflection, the others are formed by rays, which, after having suffered one or several refractions, are at first reflected, then undergo still other refractions before emerging from the eye. The optic systems which produce these images are, therefore, quite complicated, but we can always replace them by a single reflecting surface, which I call the *apparent surface*.

Suppose, for example, that we wish to study the third image of Purkinje, that produced by reflection at the anterior surface of the crystalline lens. Neglecting the weak refraction by the posterior surface of the cornea, the rays suffer, besides reflection, two refractions, one on entering and the other on emerging

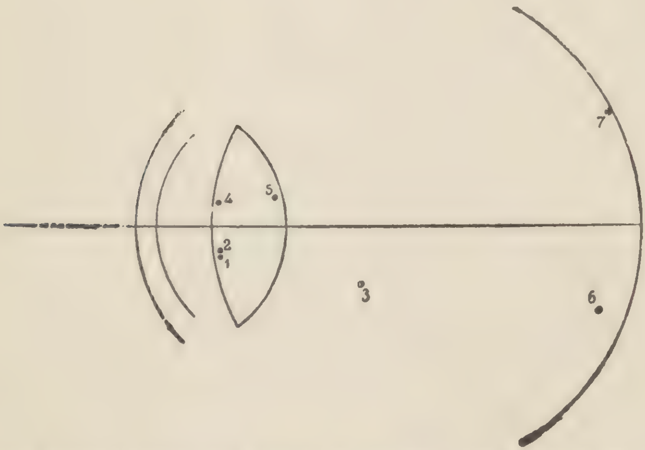


Fig. 30.—Position of the seven images in the eye. The object is supposed to be situated at 20 degrees below the visual line.

from the eye. Now, we can replace this series of refractions and reflections by a simple reflection on the apparent surface. We find the *position* of this surface by finding the position of the image of the real surface, seen through the cornea, in the same manner as we have already found the position of the ap-

(1) See *Blix,* however. Oftalmometriska Studier. Uppsala, 1880, p. 63.

parent pupil, by means of the formula $\frac{F_1}{f_1}+\frac{F_2}{f_2}=1$; with the values of the simplified eye we have $F_1=24$ mm., $F_2=32$ mm., $f_2=3.6$ mm., which gives the position of the apparent surface, $f_1=-3$ mm. We then find the position of the center of the apparent surface by finding in the same manner the image of the center of the real surface seen through the cornea ($f_1=13.5$, which gives $f_2=-17.5$). The apparent surface being at 3 mm. and its center at 17.5 mm., it must perform the function of a convex mirror of 14.5 mm. radius, placed three millimeters behind the cornea. The focus is at an equal distance between the surface and the center, that is to say at 10.2 mm. behind the cornea; it is therefore very nearly at this place that the third image of Purkinje is formed. We can also use the apparent surface to calculate the size of the image, following the formula $\frac{O}{I}=\frac{2l}{R}$ (see page 6).

To make the same calculation for the posterior surface of the crystalline lens, we must first calculate the refracting system composed of the cornea and of the anterior surface of the crystalline lens, and then the images of the posterior surface and of its center, seen through this system.—With the exception of the anterior surface of the crystalline lens, the apparent surfaces differ only slightly from the real surfaces.

The three first surfaces being convex their images are erect, while that of the fourth is inverted.—The object being generally at quite a distance, the images are formed very near the catoptric foci of the apparent surfaces. The first, second and fourth are nearly in the pupillary plane, while the third is situated at 7 or 8 mm. behind this plane (fig. 30).—Besides, the third image easily disappears behind the iris when the eye makes a slight movement, which makes this image more difficult to observe than the others.

24. Manner of Observing the Images of Purkinje.—The *first image,* that of the anterior surface of the cornea, is much the brightest; its observation offers no difficulty.

To observe the *second image* we place ourselves as when we wish to examine a patient by oblique illumination, and we examine the eye with a magnifying glass, a lens of 10 D. for example, but without concentrating the light on the eye.

Examining the corneal image of the flame, we shall see when it approaches the border of the pupil, and still better, when it shall have passed it, that it is accompanied by a small image which is situated near it. The more the images approach the edge, the more distant they are from each other; near the edge the distance may exceed a millimeter, and the small one is frequently still visible when the large one has already disappeared, giving way to the irregular reflex of the sclera.

The small image is always situated between the large image and the middle of the pupil, which indicates that the posterior surface is more curved than the anterior surface. Suppose, indeed, that we used two lamps, one on each side, and consider the distance separating the two lamps as the object (fig. 31).

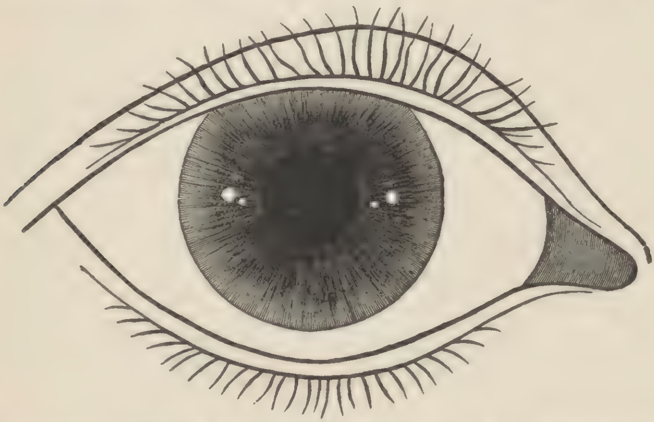


Fig. 31.—Corneal images of two lamps, observed with the ophthalmophakometer. The small images beside the large ones are due to reflection by the posterior surface of the cornea.

It is then clear that the image of the posterior surface is smaller than that of the anterior surface, which indicates that its curvature is greater. At the middle of the pupil the small image is not visible, because it coincides with the large one; they are,

indeed, situated at the same distance from the summit of the cornea.

The *third image*, the largest, always preserves, whatever we may do, a more or less diffuse appearance, due to the fact that the index varies in the superficial layers of the crystalline lens. To observe it we place ourselves as before, requesting the person whose eye is being examined to look in a direction which nearly bisects the angular distance between the eye of the observer and the flame. By moving his eye slightly from side to side the observer will quite easily see the image which presents itself as a broad glow, pale and more or less diffuse, and which changes position at the least movement of the observed eye.

After having found the image, we can concentrate the light on the eye; by this means we magnify the image, which soon fills the entire pupil. If the light is bright the pupil frequently appears white, as if the eye was affected by a ripe cataract, and we may, by examining it with the magnifying glass, thus observe anatomical details which we cannot discover in any other way. I recommend to clinicians this examination, of which I have nowhere found a description. (1) To make the experiment under the best conditions we must select a lens of large aperture, place the luminous source at quite a distance and hold the lens in such a way that its focus coincides with the catoptric focus of the surface.

The third image is, as we shall see, of great importance for the study of accommodation.

The *fourth image* does not generally offer any difficulties to the observer.—It is observed under the same conditions as the preceding one, by directing the look of the observed person a

(1) *Rings of* DEMICHERI.—Demicheri has recently (*Bulletin of the Society of Ophthalmology of Paris*) described phenomena of coloration which are observed by this method in the pupil in certain affections of the crystalline lens. The middle of the pupil appeared blackish blue; it was surrounded by a green zone, then by a yellow zone, and lastly by a red zone, near the pupillary border. The case under consideration was one of more or less mature cataract. In a case which I have examined, and in which, moreover, the crystalline lens appeared intact, the pupil was filled by this examination with an intense red, so that one would have thought it filled with blood.—These colors are probably phenomena of interference due to the reflection on the finely reeded surface of the crystalline mass, nearly like the colors which mother-of-pearl presents, but the conditions under which they are produced are still unknown.

little towards the lamp. It is small and distinct. Being inverted it moves in a direction contrary to that of the others.

For a more minute examination of these images my ophthalmophakometer may be used (fig. 32). It is composed of a small telescope, supported on a stand, and of a copper arc movable around the axis of the telescope, and bearing a scale, the zero of which coincides with this axis. The radius of the arc is 86 centimeters. The head of the observed person is fixed by a head-rest in such a manner that the eye which we are to examine

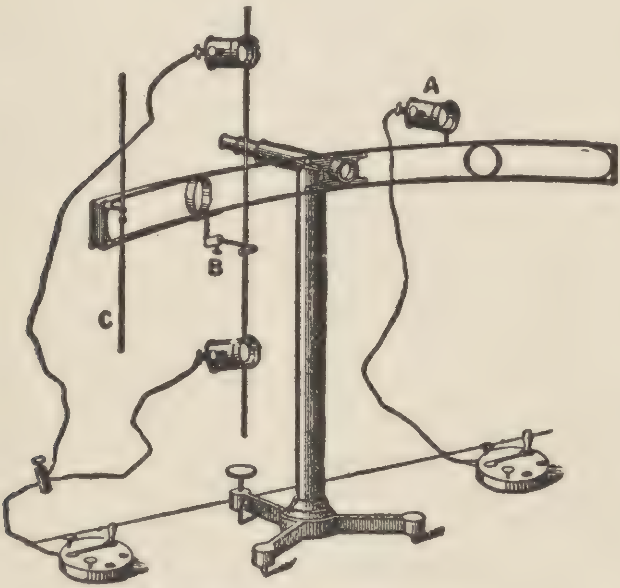


Fig. 32—The Ophthalmophakometer.

is at the center of the arc.—On the arc move several cursors, which carry electric lamps. Each lamp is enclosed in a tube closed in front by a plano-convex lens, which concentrates the light on the observed eye.—I will speak later of the manner of using the instrument for measuring the internal surfaces of the eye.

25. False Images of the Second Order.—All the reflected rays which emerge from the eye to form the images of *Purkinje,* with the exception of those of the first image, meet surfaces which again reflect a part of the light; this light is extremely feeble for most of the surfaces; it is only on meeting the anterior surface of the cornea that there is reflected sufficient light to be visible. Thus there are formed two more images, the *fifth,* produced by a first reflection on the anterior surface of the crystalline lens, and a second reflection on the anterior surface of the cornea, and the *sixth,* due to a first reflection on the posterior surface of the crystalline lens and a second reflection on the anterior surface of the cornea.—As the rays return towards the retina, these images are subjective.

The optic systems which produce these images are very complicated. They are calculated, too, by the formulæ which we have explained on page 26. The focus of the *fifth image* is near the posterior surface of the crystalline lens. It is, therefore, at this place that this image of a distant object is formed. Before reaching the retina the rays are so dispersed that they are no longer visible; I, at least, have not been able to discover the least trace of this image. Theoretically we ought to be able to make it visible by bringing the object nearer, since the image and object move in the same direction as in all the refracting systems, but the experiment did not succeed. In fact, when the flame with which we are working is moved near enough to the eye, the useful image becomes transformed into a diffusion circle, which fills the greater part of the field and prevents one's seeing anything else.

The focus of the *sixth system* is, on the contrary, very near the retina of the emmetropic eye; the image is also generally easy to observe.

26. Manner of Observing the Sixth Image.—We choose, in a half-darkened room, a point of fixation situated some distance away, and, having fixed this point, we give to the candle, held in hand, a to-and-fro horizontal motion, moving it towards and away from the visual line without, however, reaching it.

We, then, notice on the other side of the visual line a pale image of the flame. Some people see the phenomenon sufficiently distinct to be able to discern that the image appears inverted, the retinal image being erect. We discern more clearly the form of the image when we cause the candle to pass below the visual line; the image then passes above, and we see that its apex is directed downwards. Myopes see the image with greater difficulty; they often succeed better when using their correcting glasses, but they must then guard against confounding it with the images produced by repeated reflections between the cornea and the glasses.

It seems that there are persons who cannot perform the experiment successfully. If the anterior chamber is unusually deep it may, indeed, happen that the focus of the system is quite a distance from the retina, but we ought then to be able to succeed by moving the flame towards the eye or away from it.

We see, therefore, how very advisable it is that the harmful light be reduced to a minimum; in fact, if the index of the superficial crystalline layers had been higher, the sixth image would have had more brilliancy, and we would be affected with an annoying monocular diplopia. And right here we must pause to wonder at the enormous sensitiveness of the retina, for the brightness of the sixth image is really only $\frac{1}{40,000}$ of that of the useful image.

One can study the sixth image more closely, by means of the opthalmophakometer, by placing oneself in the place of the person examined, and by fixing the middle of the objective of the telescope, which corresponds to the zero of the division.

Placing the arc horizontally, and putting the lamp A which slides on the arc at some distance from the telescope, we see the image appear on the other side. We bring one of the cursors of the arc to coincide with the image, so that we may read its position on the scale. We, then, notice that the image is only approximately symmetrical with the lamp, in relation to the visual line. By causing the arc to rotate 180° in such a way as to bring the lamp into a position symmetrical with the

former, we notice that the image no longer coincides with the cursor. This is on account of the angle a. If the visual line coincided with the optic axis, the two positions of the image corresponding to two positions symmetrical with the lamp, ought to be symmetrical. We can use measurements of this kind to determine the size of the angle a.

It was while using the opthalmophakometer that I found this image, which I described as new in 1891. But *Coccius* had seen it previously, and *Otto Becker* had given the explanation of it in 1860 in a memoir which is very little known. *Heuse* described it again in 1872, but gave an erroneous explanation of it.

The images of *Purkinje* have no interest as far as the function of the eye is concerned, but they are of great importance for the physiology of vision. It is, indeed, by a study of them that we can determine the form and position of the refracting surfaces of the eye. The study of these images constitute opthalmometry, to which we will devote our attention in the following chapter.

Bibliography.—Purkinje (I. E. *Commentatio de examine physiologico organi visus et systematis cutanei.* Vratislaviæ, 1823.—Becker (O.). *Ueber Wahrnehmung eines Reflexbildes im eigenen Auge, Wiener medicinische Wochenschrift,* 1860, p. 670-672 and 684-688.—Heuse. *Ueber die Beobachtung einer neuen entoptischen Erscheinung. Graefe's Archiv.* Bd. 18, 2, p. 236.—M. Blix. *Oftalmometriska Studier.* Upsala, 1880.—Tscherning. *Recherches sur la quarieme image de Purkinje; Arch. de physiol.,* 1890.—Tscherning. *Théorie des images de Purkinje et description d'une nouvelle image. Arch. de physiol.,* 1891.—Tscherning. *Sur une nouvelle image a la fois catoptrique et dioptrique de l'oeil humain et une nouvelle méthode pour déterminer la direction de l'axe optique de l'oeil. Bulletin de la Société francaise d'ophthalmologie* 1891, p. 203.

CHAPTER IV.

OPHTHALMOMETRY

27. Principles of Ophthalmometry.—The basis of ophthalmometry is the formula $\frac{O}{I} = \frac{l}{F} = \frac{2l}{R}$ or $R = \frac{2lI}{O}$ (see page 6). To determine the radius R of the small convex mirror which forms the anterior surface of the cornea, we measure the image I of an object O, placed at a given distance l. There is never any difficulty measuring either the object or the distance; it is, therefore, to the measurement of the image that we must devote our attention.

We may say at once that we generally use as objects the distances separating two flames or two white objects (mires). The image, then, is the distance separating the images of the flames or of the mires.

The method most used by physicists for such measurements consists in placing a micrometer at the focus of the objective of the telescope with which the image is observed. The objective forms an image which coincides with the micrometer, the graduations of which permit the size of the image to be read directly by observing it through the eye piece. It has been attempted to use this method for ophthalmometry, but without success. As the observed eye cannot be kept absolutely quiet, the image is constantly changing its place in relation to the micrometer, which makes a fairly exact measurement impossible.

This is why *Helmholtz* introduced into ophthalmometry another principle which he borrowed from astronomy, where the same problem presents itself, that of doubling (*dédoublement*). It seems, however, that the method had already been used for the same purpose by *Thomas Young.*

Suppose that we desire to measure the distance I separating the two points *a* and *b* (fig. 33, 1), and that we have a process which permits us to see everything doubled at a certain distance

D. By this means instead of the two points a and b we would see four, a_1 and a_2, b_1 and b_2, and the distance a_1 a_2 would be equal to b_1 b_2 and to D, while the distance a_1 $b_1 = a_2$ $b_2 = \mathrm{I}$ (fig. 33, 2).

Suppose, now, we could make the doubling vary. By increasing it we would reach a point when a_2 and b_1 would coincide (fig. 23,3) which would take place at the moment when I would be equal to D. If we knew the amount of doubling used we would thus have measured $\mathrm{I} = a$ b, and our object would be attained. When a and b touch we say that we have obtained *contact*. If we use as objects separated flames so that a and b form two luminous points we obtain more exact measurements by giving one of them the form of two points situated on the same vertical (fig. 33,4); at the moment of contact the image of b is placed exactly between the two points a. Instead of making the doubling vary, we can make I vary, which is brought about by varying the object (displacing one of the lamps) until contact is obtained.

Fig. 33.

Generally it is useful to employ a certain degree of magnification in order to have easy measurements, and this suggests the use of a telescope placed at some distance from the eye; instruments with short focus, more or less resembling microscopes, are not practical because it is impossible to keep them in focus, the observed eye not being able to remain sufficiently quiet.

Thus, we would only have to affix our doubling apparatus to our telescope and place conveniently two flames or two white surfaces which would serve us as objects, and we would be ready to begin our measurements.

28. Methods of Doubling (Dedoublement).—*a*) A first method consists in dividing the luminous cone which meets the objective, into two halves, an upper and a lower, and displacing each half laterally, one to the right, the other to the left. We can obtain this effect:

1°. By placing before the upper half (1) of the objective a weak prism, apex to the right, and before the lower half another, apex to the left.

2°. Instead of prisms we can use plane parallel plates, placed obliquely but in a symmetrical manner in relation to the axis of the telescope. Such plates placed obliquely (see page 12) have the effect of displacing the object laterally, each on its own side; the effect is, therefore, the same as that of prisms, and the plates give better images.—This is the system employed by *Helmholtz,* who made the doubling vary by changing the inclination of the plates, and later by *Leroy* and *Dubois,* who used a constant doubling by making the object vary.

3°. We can saw the objective in two and displace the upper half a little to the left, the lower half a little to the right (fig. 34). It is easy to see that this method must produce a doubling of the image, since the optic center of the objective is, so to speak, divided into two halves, displaced laterally in relation to each other. This method gives very good images and less light is lost, since we obviate the reflection on the surfaces of the prisms or plates, but the instrument is very difficult to construct; the displacement of the two halves of the objective, in relation to each other, must be made, indeed, with an exactness that is expressed in hundredths of a millimeter.

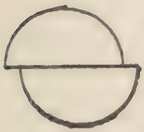


Fig. 34.

None of these methods is very practical, because all of them call for a very exact adjustment of the instrument to find the meridians of the astigmatic eye (see ch. IX).—If the eye is displaced a little during the measurement, we may find false di-

(1) I am supposing here and in what follows that it is the horizontal meridian we are measuring.

rections for these meridians. *Helmholtz* remedied this inconvenience by placing himself very far (at 1 or 2 meters) from the patient, which calls for a room prepared for this purpose and makes measurement pretty difficult.

b) A second method consists in dividing the objective into two *lateral* halves, and displacing laterally each half of the incident luminous cone. Such an arrangement can be obtained:

1°. By placing in front of the objective a double prism with apex vertical;

2°. By placing before each half of the objective a plate with plane, parallel surfaces, forming an angle with the axis of the telescope (fig. 35).

These are the plates of Helmholtz which are placed side by side instead of being placed one above the other.

Fig. 35.

3°. We can obtain the same effect by removing a vertical band from the middle of the objective and cementing together the remaining parts (fig. 36).

Systems of this order offer no difficulty in finding the meridians, but they have another inconvenience: contact depends much on the exactness of the adjustment. If, after having obtained contact the observed eye is displaced a little, so that the instrument is no longer exactly in focus, contact ceases. We may thus obtain totally false measurements of astigmatism if the observed eye is displaced between the two measurements.

Fig. 36.

This inconvenience is partly got rid of in the model of the *Javal* and *Schioetz* ophthalmometer which the optician *Kagenaar*, of Utrecht, constructed. It uses a combination of the methods *b*, 1 and *b*, 2, a combination of two very weak prisms forming an angle between them; the apex of the prisms is inwards.

c) The best method, however, is to emloy doubly-refracting crystals. *Coccius* had recourse to a place of spar; *Javal* and *Schioetz* used a *Wollaston* prism. This prism (fig. 37) is com-

posed of two rectangular quartz prisms, which are cemented together so as to form a single very thick, plane parallel plate. The two prisms are cut differently in the crystal; one has the apex parallel to the axis of the crystal, the other perpendicular to it. Each ray which passes through the prism is divided into two, and each of the two new rays is deviated a little so that they are nearly symmetrical in relation to the incident ray. (1)—By all other systems which I have mentioned the incident cone is divided into two half cones, which are a little displaced in relation to each other; the prism of *Wollaston* on the contrary produces two entire cones of half the intensity.

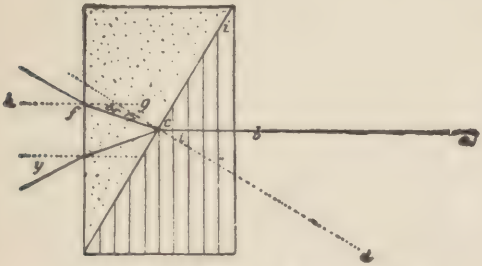


Fig. 37.—Prism of *Wollaston.*

The instrument of *Helmholtz* must be considered as an instrument for the laboratory. Investigators, like *Donders* and *Mauthner,* used it for measuring the eyes of some patients, but its use was so difficult that *Mauthner* exclaimed: "Ophthalmometry must be understood as ophthalmoscopy, only it is much more difficult." Besides it necessitates a dark room, and the complete measurement of the cornea calls for not less than 32 measurements. It is only by the labors of *Javal* and *Schioetz* that ophthalmometry has become a clinical method.

29. The Ophthalmometer of Javal and Schioetz.—The instrument (fig. 38) is composed of a telescope which carries a copper arc movable around the axis of the telescope, and with a headrest on which the head of the patient is supported; when the

(1) [A detailed theory of this prism, together with a calculation of the angles, can be found in the *Théorie de l'ophtalmométrie de la cornée* by Dr. Tscherning in Javal's *Mémoires d'ophtalmométrie,* Paris 1891.]—*W.*

telescope is adjusted to the level of the eye of the observed person, the latter is at the center of the arc.—Two white mires

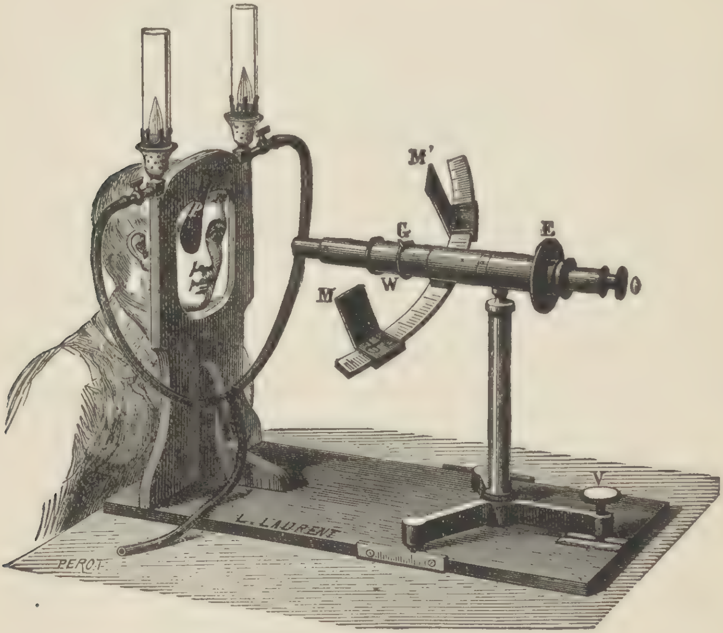


Fig. 38.—Ophthalmometer of *Javal* and *Schioetz.*

slide along the arc, and it is the distance separating them which serves as the object. By moving one of the mires on the arc, the size of the object is made to vary until it corresponds with the doubling of the prism which is constant.—The telescope has two achromatic objectives between which is the Wollaston prism, placed so as to double in a direction exactly parallel to the plane of the arc. It is, besides, provided with a *Ramsden* eye piece with a spider's thread. Each observer must begin by focusing the ocular on the thread; then the instrument is adjusted for the level of the observed eye by displacing it forwards or backwards. We then see the images of the two mires doubled (fig. 39), and by displacing the mire on the right, contact is obtained. This done we can read the distance of each mire in

degrees from the axis of the telescope on the scale of the arc, and the sum of the two figures indicates the corneal refraction. I have supposed the cornea in question spherical, otherwise we would have to begin by finding the principal meridians; but I shall reserve the description of the measurement of the astigmatic eye for the chapter on astigmatism.

Generally the patient must look into the telescope; it is only when we wish to measure the peripheral parts of the cornea also that we make him look in other directions.

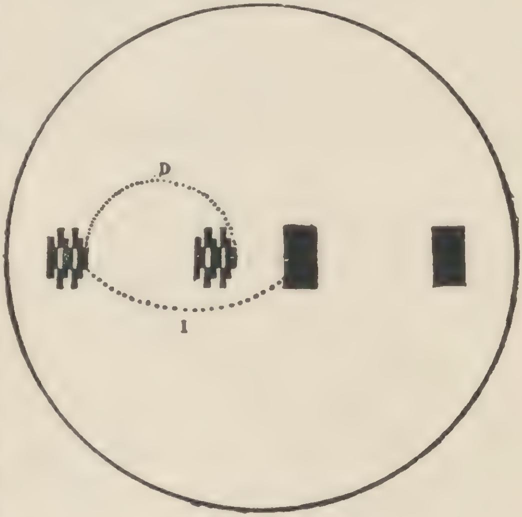


Fig. 39.

The graduation of the arc is in degrees, but the doubling is so chosen that each degree corresponds with one dioptry. This calls for an explanation.

Javal and *Schioetz* have taken as the index of the aqueous humor 1.3375 (1); the refracting power of the cornea expressed in dioptries would be, therefore (see page 16);

$$D = \frac{1}{F_1} = \frac{n-1}{R} = \frac{0.3375}{R}$$

or, expressing R in millimeters,

$$D = \frac{337.5}{R} \text{ and } R = \frac{337.5}{D}$$

(1) This value of n, very nearly correct, was selected in order that, in the following table, 45 D. would correspond exactly to 7.5 mm., which is convenient in order to regulate the instrument by a sphere type of 7.5 mm.

With this formula we calculate the following table, which gives the relation between the refracting power of the cornea, expressed in dioptries, and the radius expressed in millimeters:

Refraction.	Radius.	Refraction.	Radius.	Dioptries.	Radius.
50 D.	6.75mm	45 D.	7.5mm	40 D.	8.44mm
49 D.	6.89mm	44 D.	7.67mm	39 D.	8.65mm
48 D.	7.03mm	43 D.	7.85mm	38 D.	8.89mm
47 D.	7.18mm	42 D.	8.04mm		
46 D.	7.34mm	41 D.	8.23mm		

Placing the value which we have just found for R in the formula

$$\frac{O}{I} = \frac{2l}{R}$$

we find

$$O = \frac{2l\,DI}{337.5},$$

in which formula I designates the image which, at the moment of contact, is equal to the doubling. Let us designate by a the linear length of a degree; if this length must correspond to one dioptry, the object which corresponds with the image I must have the size Da, therefore

$$Da = \frac{2lDI}{337.5}$$

or

$$a = \frac{2lI}{337.5}$$

On the other hand as a must be one degree long, we have

$$\frac{1^\circ}{360^\circ} = \frac{a}{2\pi l}$$

therefore

$$a = \frac{2\pi l}{360} = \frac{2lI}{337.5}$$

and

$$I = \pi \frac{337.5}{360} - 2.94^{mm}.$$

In order that a degree of the arc may correspond with one dioptry, the doubling of the prism must be, therefore, 2.94 mm. This is what has been done.

The radius of the arc (l) has been selected so that the linear length of a degree may be 6 millimeters (5 millimeters in the new model).

In the last models of the instrument certain details have been changed, but the principle remains the same.—We may add, furthermore, that, in order to measure the As, one of the mires has a special form "in steps," each of which corresponds to one dioptry.—A keratoscopic disc enables us to study the general form of the cornea.

Utilized Part of the Cornea.—It is only a very small part of the cornea that is used for the measurement. Making the construction in the way indicated on page 9 we see that the images of the mires are formed by reflection on two small parts of the cornea situated about 1.2 mm. from the visual line.

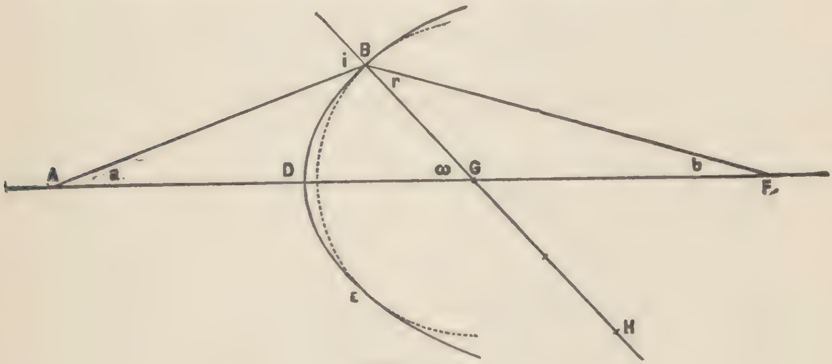


Fig. 40.

Rotating the arc these two parts move describing a concentric ring around the visual line. This ring is the only part of the cornea which sends light into the objective, and consequently also the only part on which the instrument can give information. The parts situated outside or inside this ring may have curvatures quite different from those indicated by the instrument. Suppose, for the moment, that we have to do with a conical

(hyperbolic) cornea: what we would measure would be the radius of BG of the circle BE (fig. 40), which touches the surface of the cornea at B and E (see page 16). Generally this circle coincides quite closely with the "optic" part of the cornea; but if we want to make very exact measurements we must always take into consideration this source of errors.

Exactness of the Measurements.—With a good illumination an experienced observer would not easily be led astray to the extent of a quarter of a dioptry, which corresponds to almost $\frac{1}{20}$ of a millimeter of error for the radius. Absolute reliance cannot, therefore, be placed in the second decimal of the measure of the radius. *Donders* and *Hamer* arrived at very nearly the same results using the ophthalmometer of *Helmholtz*. —Still more accurate results may be obtained by using translucent mires which are illuminated from behind by electric lamps. In these conditions an experienced observer can almost guarantee exactness to a tenth of a dioptry or thereabouts.

30. Results of the Measurement of the Cornea.—The radius of the cornea (at the summit) varies between 7 and 8.5 mm. It is extremely rare to find a cornea the radius of which is not situated between these limits, except in cases of keratoconus.

The cure (fig. 41) shows the distribution of the different curvatures in a certain number of men (emmetropes) whom I examined in collaboration with *Dr. Bourgeois*. The average was 43.1 D=7.8 mm. It is noticeable, however, that these same measurements show that the radius is greater in persons tall in stature and with a large cranial circumference. (1) Now the persons whom we examined were indeed of tall stature (*cuirassiers*). It may be, therefore, that the average length of the radius may be slightly smaller than that which I have just indicated.—It would be an error to think that one radius rather than another corresponds with emmetropia. As *Javal* says an elephant and a mouse may both be emmetropic despite the fact that their corneal radii must necessarily be very different.—It

(1) *Steiger* has since found a still more manifest relation between the radii of the corneas and the distance between the eyes.

seems that we can express the relation by saying that in the emmetropic eye there exists a constant relation between the radius of curvature of the cornea and the length of the ocular axis, so that the ocular shell of different emmetropic eyes would always be a reproduction of the same type, a little enlarged or a little diminished.—The existence of the myopia and hypermetropia of curvature (corneal) is not yet demonstrated (2) except, perhaps, for certain cases of very high hypermetropia which approach microphthalmia; but their existence is beyond doubt.

If I except cases of astigmatism, different in both eyes, it is very rare to find a difference, ever so slightly noticeable, between the corneal refraction of the two eyes of the same person, even in cases of anisometropia. Amongst the cuirassiers

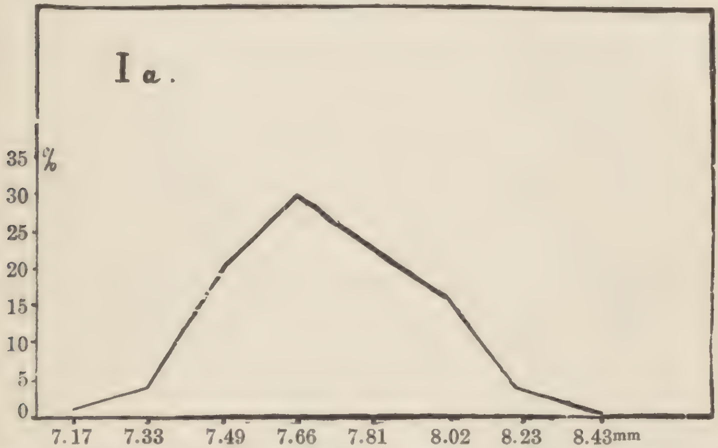


Fig. 41.—The abscissas indicate the radii of curvature of the cornea in millimeters, the ordinates the number per hundred of emmetropes in whom we meet the radius of curvature in question.

mentioned above there were not more than two per cent. who showed a difference exceeding a half dioptry between the two eyes.

Examination of the Peripheral Part of the Cornea.—

(2) See, however, the communication of *Sulzer* to the Congress of the French Society of Ophthalmology, 1896.

Up to the time when *Javal* and *Schioetz* made a clinical method of ophthalmometry there was little known of the form of the cornea. The ophthalmometer of *Helmholtz* being too complicated to make many measurements, one was limited to measuring three points of a meridian, that which corresponds to the visual line and another at some distance on either side. As the peripheral radii were found to be greater than the central radius, and as, in consequence, the cornea could not be considered as a sphere, the curvature of the second degree which approached nearest the meridian measured was calculated (see fig. 42). Thus it was that the idea was disseminated that the form of the cornea (non-astigmatic) would be that of an ellipsoid of revolution around the long axis, which axis would be directed outwards from the visual line and form an angle of about 5° (α) with this line. This idea differs widely from the reality; the cornea does not resemble an ellipsoid. *Helmholtz* insisted from the start on the fallacy of the comparison.

After the construction of modern ophthalmometers it became much easier to study this question. The second model of the *Javal* and *Schioetz* ophthalmometer is provided with a very large keratoscopic disc divided into graduations of 5° by concentric rings. After having made the usual measurements, during which time the patient looks at the center of the objective, the measurement is repeated making him look 5° to the left, 10° to the left, etc.; and, after having thus measured the right half of the horizontal meridian we measure the left half. We repeat the measurements for the vertical meridian.—Measurements of this kind have been made in Paris by *Sulzer* and *Eriksen* (fig. 42); these measurements confirmed the assertion of *Aubert* and *Matthiesen* who, using the ophthalmometer of *Helmholtz,* had said that the cornea could be divided into two parts, a central one, which is approximately spherical and which we call the *optic part,* and a peripheral one or *basilar part,* which is much flattened. *Eriksen* reckoned as belonging to the optic part that part the refraction of which does not differ more than one dioptry from the central refraction. Its extent varies a little in different

eyes. Following are the limits of the optic part compared with those of the entire cornea, after *Eriksen:*

	Optic Part.	Cornea.
Outwards	16.5°	44.7°
Inwards	14°	40.1°
Above	12.5°	38.5°
Below	13.5°	42.2°

The figures are the averages of measurements made on 24 eyes. The total width of the cornea is, therefore, not much less

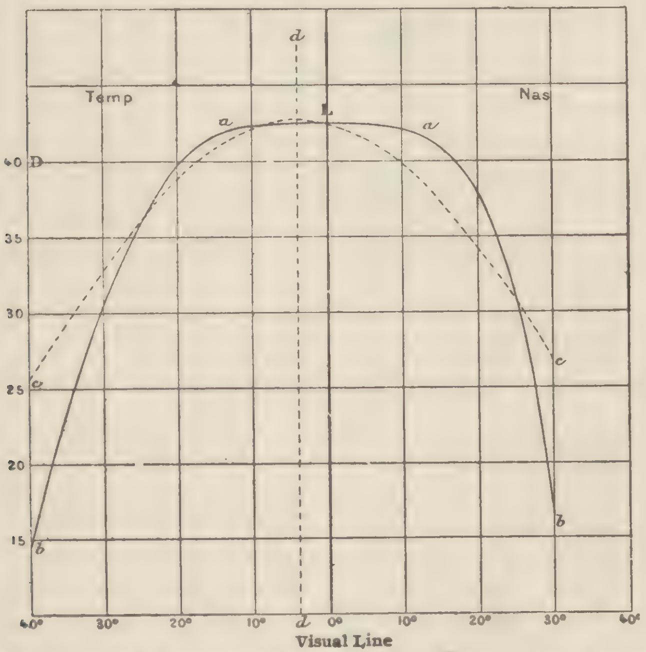


Fig. 42.—Diagram of corneal refraction after *Eriksen.*—The abscissas indicate the distance of the visual line in degrees, the ordinates, the corneal refraction in dioptries.

The full curve indicates the refraction of the horizontal meridian of a left cornea measured in graduations of five degrees. The zero corresponds to the visual line.—*aa*, optic part of the cornea; *ab*, *ab*, basilar part.—The dotted curve *cc* corresponds to the ellipsoid calculated according to the three measurements taken at 0° and 25° on the right and left of the visual line; *dd* is the axis of this ellipsoid and the distance of this line from zero corresponds to the angle which is often called the angle α.—We see that the true form of the cornea differs considerably from the ellipsoid.

than 90°, and that of the optic part is about 30°, or a third of the entire width. The horizontal diameter, as well that of the optic part as that of the entire cornea, is a little greater than the vertical diameter.

Neither *Sulzer* nor *Eriksen* have found an axis of symmetry properly so called. Nevertheless, most of the diagrams of the latter show a tendency to symmetry around an axis directed about 5° outwards and a little below the visual line. If, therefore, the comparison with an ellipsoid is persisted in, we must imagine it much more pointed than we have done up to the present, and we must suppose the summit cut-off by a section perpendicular to the axis and replaced by a spherical cap.

As far as the optics of the eye are concerned, the obliquity of the cornea plays only a slightly important role, since the optic part of the cornea is nearly spherical. This part corresponds to a linear diameter of about 4mm. When the pupil is large the basilar part may, therefore, play a certain part; according to the little table of *Eriksen* it would be especially inwards and above that its influence would be felt. But it is impossible to know anything of it without having examined each eye by itself, for the obliquity of the cornea is often compensated for by the eccentricity of the pupil. The position of the pupil varies much in different eyes. *Sulzer* found that on an average the center of the pupil is 5° outwards from the visual line, and that it is sometimes displaced upwards, sometimes downwards. This decentering of the pupil may, therefore, compensate for the obliquity of the cornea, so that it is especially outwards that we must expect to notice the effect of the peripheral flattening.

The basilar portion is less regular and much less polished than the central portion, which partly explains the slight success of optic iridectomies. The catoptric images have frequently a diffuse aspect and the ophthalmometric measurements leave much to be desired. *Eriksen* also has tried to obtain an idea of the variation of the radius of the peripheral parts by examining the form which the image of a white square assumes in the horizontal meridian, at different distances from the visual line.

We see on fig. 43 that the image becomes longer and longer until about 30° from the visual line, where it is two and a half times greater than at the center. Just at the periphery the

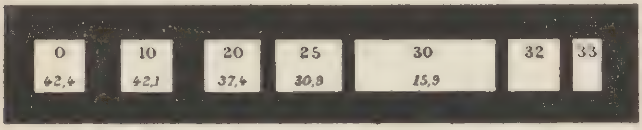


Fig. 43.—Forms of the image of a white square at different parts of the cornea (horizontal meridian, internal half), after *Eriksen*.—The figures at the top of the squares indicate the distance in degrees from the visual line; those at the bottom the refraction (in the horizontal meridian) in dioptries.

image becomes narrower, and ends as a rectangle placed upright; at this place the image is sometimes double; a second image is formed still farther away on the edge towards the sclera, and this image is inverted in the horizontal direction, but not in the opposite direction. These latter phenomena indicate that the curvature increases very considerably towards the border, and that beyond this place there is, at least in some eyes, a concavity, like a furrow which separates the cornea from the sclera. We must note that the images should increase a little in height towards the periphery at the same time that they increase in width, because the curvature diminishes also in the vertical, but much less than in the horizontal direction. This increase is not indicated on the figure.

In a general way we may, therefore, consider the portion of the cornea which plays a part in the optics of the eye as spherical, so that the angle α, understood in the sense in which we generally accept it, loses its importance.—This is why I have defined the angle α as being the angle between the visual line and the optic axis of the eye, a definition which others have also given to it.

Note, furthermore, that the normal cornea is slightly astigmatic; we reserve a special chapter for this anomaly of refraction.

The radius of the normal cornea does not fall below 7 mm., but in cases of keratoconus we may meet radii of 6 or 5 mm., or even still smaller radii, to a point where the arc of the ophthalmometer becomes too short; we cannot separate the mires sufficiently to obtain contact. The images of the mires assume in this case, as also when there are corneal opacities, irregular forms.

By the *Sulzer-Eriksen* method we determine the radius of curvature at a given part of the cornea. We obtain by this method a very good idea of the form of the cornea, but the results are not directly applicable to ocular dioptrics for the reasons given on page 16. To be able to calculate the aberration produced by a peripheral flattening of the cornea, we should know the normal (the part of the perpendicular to the cornea comprised between the latter and the visual line). To determine it *Brudzewski* made certain changes in the ophthalmometer. He replaced the arc by a larger one, reaching 170°. One of the mires is fixed at the middle of the arc so that its border when prolonged would pass through the axis of the telescope, while the other mire slides on the arc so as to be able to obtain contact. The observed person fixes the middle of the objective during all the measurements. He uses prisms of different doubling power. He begins, for example, with a prism doubling 1 mm.; and, the arc being placed horizontally, he determines the position, on the nasal side, which the movable mire must have so that he may obtain contact. He then makes the same determination on the temporal side, after having placed the arc vertically upwards and downwards. These measurements give the length of the normals to the cornea at four places, situated at 1 mm. from the visual line. He then replaces the prism by another doubling 2 mm., and so forth. Knowing the normal he can then directly calculate the aberration produced by the corresponding part of the cornea (see chapter VII).

We observe, furthermore, that the ophthalmometer lends itself very well to the examination of the curvature of the surfaces of the dead eye. *Holth* thus made a series of measurements in the laboratory of the Sorbonne. He placed the eye with the cornea

upwards under a mirror at 45° which sent the reflected image in the direction of the ophthalmometer. The mirror must not be too small, for it must allow us to measure also the peripheral parts of the surfaces by displacing the instrument. As the

Fig. 44.—Keratoscopic images of a cornea presenting a considerable astigmatism at the central part (central ring of figure C). while the remainder of the cornea is nearly exempt from it. After *Javal.*—C, direct look; H, upwards look; B, downwards; D, to the right; G, to the left.

surfaces are generally more or less misty, we are obliged to coat them with a very thin layer of oil to make them bright. It was necessary for the measurement of the cornea to make an injection into the vitreous body so as to make its tension that of the eye, but it was interesting to note how much he could change the tension of the eye without observing any perceptible alteration in the curvature of the cornea. To measure the curvature of the posterior surface of the cornea, *Holth* injected a solution of gelatine into the anterior chamber; as soon as the gelatine solidified he removed the cornea and measured the anterior surface of the cast, made bright with oil. The anterior

surface of the crystalline lens is measured directly, after the cornea and iris have been removed. To measure the posterior surface he cut the eye in two, along the equator, and, the vitreous body being removed, the eye was placed with the cornea downwards. *Holth* gave an account of the results achieved by him at the Ophthalmological Congress of Utrecht in 1899 (see also page 219).

EXAMINATION WITH THE KERATOSCOPIC DISC.—The measurement of peripheral parts of the cornea takes too much time to

Fig. 45.—Keratoscopic figures of a case analogous to that of figure 44. After *Javal*.

be of service in clinics, but we can obtain information about the peripheral parts of the cornea by means of the keratoscopic disc, a circular disc, on which are painted concentric circles of different colors. We can place it on the telescope of the ophthalmometer by taking out the double refracting prism, or simply by holding it in the hand and looking through a central aperture (*Placido*). Generally the patient looks towards the middle of

the disc; the images of the circles are then circular in a normal eye, and elongated along the meridian of least refraction in the astigmatic eye; by making the patient look towards the border of the disc it is easy to establish the peripheral flattening of the cornea.

In cases of irregular astigmatism the circles assume irregular forms; and we may often, by studying these forms, obtain important information on the anomaly in question.—Thus figs. 44 and 45 show the appearance of the disc in cases in which the central part of the cornea was affected with a pronounced astigmatism, while the middle zones were scarcely affected at all; we see, in fact, that the central ring of figure C, which corresponds to the middle of the cornea, is much lengthened, while the more peripheral rings are almost circular.—In cases of keratoconus the image of the disc is very small when it is formed at the summit of the cornea, but the least deviation of the look causes a change of form by lengthening it in the radial direction (fig. 46).

Fig. 46.—Keratoscopic figures of a case of keratoconus. After *Javal.*

We have seen (page 44) that the visual line passes through the cornea perpendicularly or nearly so. When making a keratoscopic examination the observed person looks into the telescope; the center of the concentric rings of the image indicates, therefore, the place where the visual line passes through the cornea, and if, at the same time, we illuminate the eye moderately we can account for the direction of the visual line relatively to the different parts of the eye. It may be useful to modify the appearance of the disc. Figure 46*a* shows the keratoscopic appearance of an eye affected with a high degree of astigmatism, and of which the angle α has an unusual size; the small black circle indicates the pupil, the white figure is the corneal image of a large white disc provided with a black cross, the arms of which were placed in the principal meridians; its elliptical form is due to the astigmatism. The visual line corresponds to the intersection of the two black lines. We notice that it is placed very eccentrically in the pupil so that the four quadrants of the latter are of very different size. The angle α was about 9°;

Fig. 46*a*.—Keratoscopic image of an eye with a large angle α.

Fig. 46*b*.—Spot of *Mariotte* of an eye with a large angle α, compared with that (dotted) of a normal eye. *a*, point of fixation.

the axis of the crystalline lens was directed 8.8° outwards and 3.8° downwards from the visual line.

As in every instance in which the angle α has an unusual size, the cause was to be found in the displacement of the *fovea,* a displacement which, in this case, manifests itself also by an increased distance between the point of fixation and the blind spot (fig. 46*b*). The internal border of the latter was at 15° instead of 11° or 12°.

31. Measurement of the Angle α.—For the following measurements I use the ophthalmophakometer (fig. 47, see page 53). I designate by A the cursor which carries only one lamp; by B

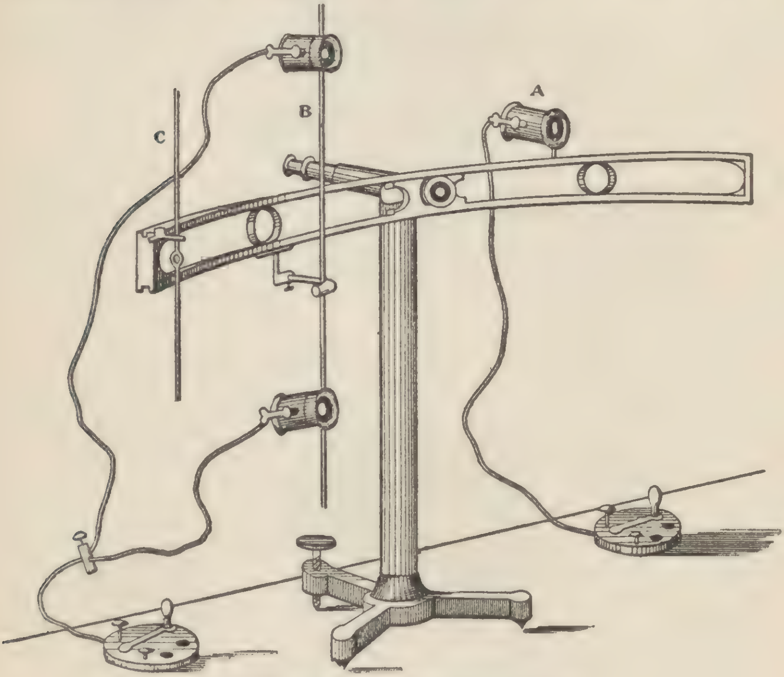


Fig. 47.—The ophthalmophakometer.

that which carries two, placed on the same vertical rod, and by C the third cursor which carries a rod on which moves a small bright ball which serves as the point of fixation.

I place the arc horizontally and the cursor B at the zero of

(1) The lamp of the cursor A is not used in this experiment.

the graduation of the arc (1) so that its two lamps are in the same vertical plane as the middle of the objective of the telescope, and I request the observed person to look towards this

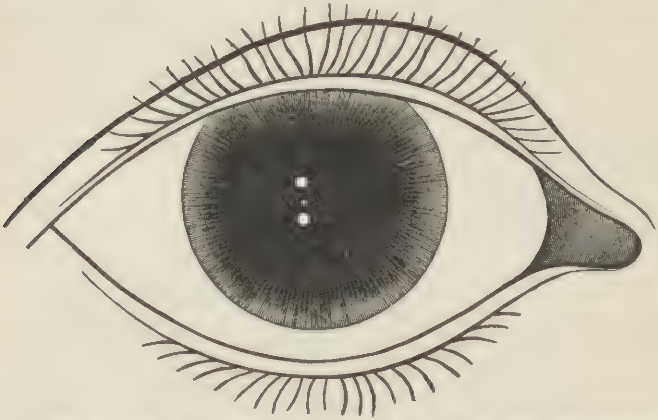


Fig. 48.—The images of Purkinje observed with the ophthalmophakometer. The two lamps B, figure 47, are in the same vertical plane as the avis of the telescope and the observed person looks at 5.7° on the nasal side, so as to align the images. The optic axis of the eye coincides in these circumstances with the axis of the telescope.

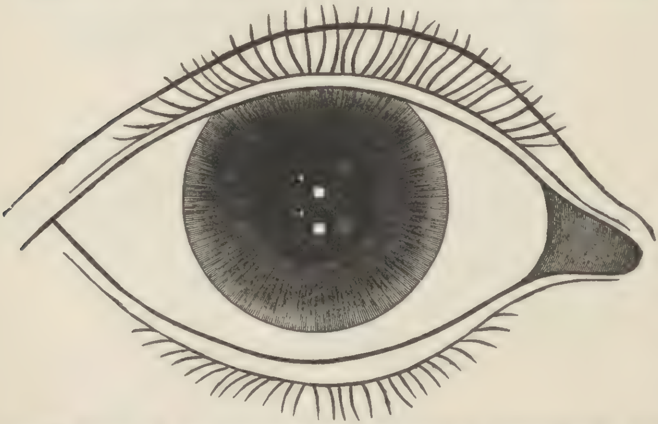


Fig. 49.—Position of the images when the observed person looks into the telescope. The position of the lamps is the same as in figure 48. At the middle, the corneal images on the right, those of the anterior surface of the crystalline lens; on the left, those of the posterior surface of the crystalline lens. The images of the posterior surface of the cornea are not visible.

latter place. It is clear that, if the surfaces of the eye were centered around the visual line, we should, in these circumstances, see the six images of reflection on the same vertical line (fig. 48) (those of the posterior surface of the cornea are not visible under these conditions). But this has never happened.

We always see, as in fig. 49, the images of the anterior surface of the crystalline lens, on the one side, those of the posterior surface of the crystalline on the other, and the corneal images in the middle. I then request the observed person to fix the

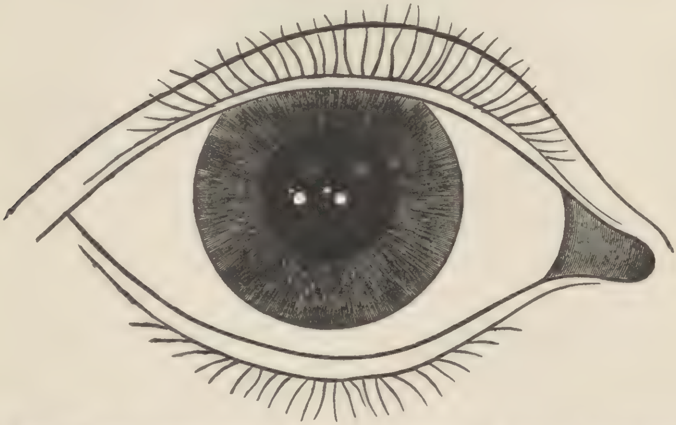


Fig. 50.—The two lamps are in the same horizontal plane as the axis of the telescope. The observed person looks into the telescope.

bright ball of the cursor C, and I displace this cursor until I see the images placed as in fig. 48. The optic axis of the eye is then in the vertical plane, passing through the axis of the telescope, and the angular distance of the cursor C from the telescope indicates how much the visual line deviates from the optic axis in the horizontal plane.—We find that it is necessary to place the cursor C on the nasal side at a distance from the telescope varying between 4° and 7° (angle $^{\alpha}$).—This angle can be determined with very great exactness.

I then place the arc vertically so that the two lamps are in a horizontal plane: generally the six images are not on a horizontal line (fig. 50); by displacing the cursor C, which the observed

person fixes, until I see all the images on a horizontal line, I determine the vertical deviation of the visual line.

The optic axis is nearly always directed outwards from the visual line, and most frequently downwards (2° to 3°); sometimes we find it, however, in the same horizontal plane, or deviated a little upwards.

Fig. 51.—Defect of centering; it is impossible to align the six images.

Defect of Centering.—We sometimes observe that it is not possible to place the six images on a straight line (fig. 51). We succeed in aligning two pairs, whichever we want, but the third remains outside. This takes place when the eye is not exactly centered; that is to say, when the axis of the crystalline lens does not pass through the center of curvature of the cornea (the posterior surface of which I neglect). We can nearly always establish slight defects of this kind, but most frequently they are negligible. When we find more considerable defects, it is generally because the axis of the crystalline lens passes a little (up to 0.25 mm.) above the center of curvature of the cornea.

32. Determination of the Position of the Internal Surfaces.—To measure the radii of the surfaces we must determine: 1° the *position* (the distance from the summit of the cornea) of the

surfaces; 2° the *position* of the centers. It is true that there exists, as we shall see, a means of determining the radii directly, but we must not forget that all the sizes which we are measuring here are apparent sizes, and that, to find the real values, we must reduce the results by a calculation following the rules which we have already given (page 49). To make this reduction it is necessary to know the *position* of the surfaces, which knowledge is likewise necessary in order that we may be able to combine the surfaces with one another so that we may proceed to calculate the entire optic system.

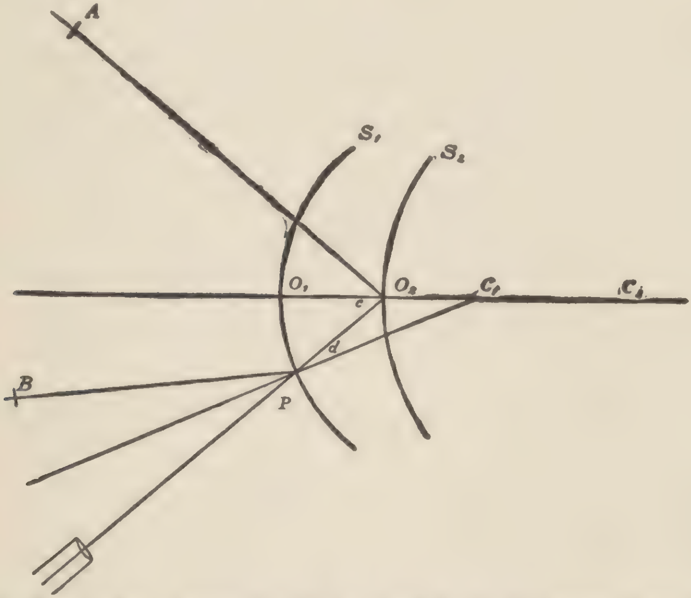


Fig. 52.—Method of determining the position of an internal surface of the eye.—S_1, anterior surface of the cornea; C_1, its center; S_2, anterior surface of the crystalline lens; C_2, its center; C_1, C_2, optic axis of the eye.

I take the anterior surface of the crystalline lens, as an example, and I suppose that we are making the measurement in the horizontal direction. It is useful to dilate the pupil.

I place the arc of the instrument horizontally, and I place also, as far away as possible from the telescope the cursor A, the lamp of which must be sufficiently brilliant that the image of the surface to be measured may be quite visible. This done, I place the cursor C, which carries the mark of fixation, at a place such that the optic axis of the eye may bisect the angular distance between the telescope and A (1). It is necessary, therefore, to have previously measured the angle α. We then displace the cursor B, the lamps of which must be very feeble so that we may see only the corneal images, until the crystalline image of A is exactly on the same vertical as the corneal images of B. Glancing at fig. 52, it is easy to see that we now possess the elements necessary to calculate the distance of the anterior surface of the crystalline lens from the summit of the cornea, for the angle c is half the angular distance of A from the telescope, and the angle d is half of the angular distance (1) of B from the telescope. Supposing that we knew the radius of the cornea R_1, which should have been measured previously, the triangle O_2 C_1 P gives us the relation $O_2 C_1 = R_1 \frac{\sin d}{\sin c}$, and we have for the distance, looked for

$$O_1O_2 = R_1 - O_2C_1 = R_1 \left(1 - \frac{\sin d}{\sin c}\right) = R_1 \frac{\sin c - \sin d}{\sin c}$$

If very great exactness is not desired, the sines can be replaced by the arcs.

Example.—Let the radius of the cornea be 7.98 mm., the distance of A from the telescope 28° nasal, that of B 16.8° nasal; we will have O_1 $O_2 = 7.98 \left(1 - \frac{\sin 8.4^\circ}{\sin 14^\circ}\right) = 3.16$ mm. The apparent depth of the anterior chamber would, therefore, be 3.16 mm., whence we find the true value 3.73 mm. by placing in the formula $\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$, the values $F_1 = 23.64$, $F_2 = 31.61$, $f_1 = -3.16$.

(1) We can imagine the two lamps of B united into one only, at the level of the lamp of A.

33. Determination of the Centers of the Internal Surfaces.—We place A above the telescope, and we move C with the mark of fixation as far as possible from the telescope, but so that the image may not disappear behind the iris; then we displace B until the corneal images of its two lamps are on the same vertical line as the crystalline image of A.

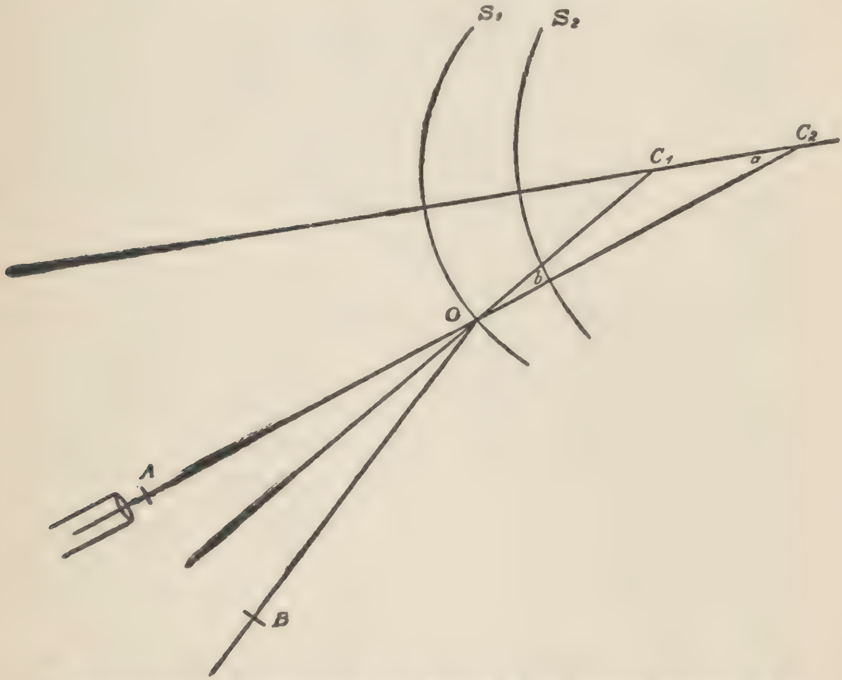


Fig. 53.—Method of determining the position of an internal surface of the eye. The letters signify the same as in figure 52.

Under these conditions, the axis of the telescope is perpendicular to the apparent anterior surface of the crystalline lens

(1) If the eye is not centered we must replace the optic axis by the line passing through the center of curvature of the cornea and the center of the surface which we desire to measure. We find this line as we found the optic axis in the preceding experiment, by aligning the corneal images witt the images of the surface to be measured.

(II) If we imagine the lamp placed at the center of the objective, the ray which reaches the observer's eye would be reflected exactly on itself, which can take place only if it meets perpendicularly the apparent surface.

(II). We find the angle a (fig. 53) by adding (subtracting) the angle x to the angular distance of C from the telescope. The angle b is half of the distance of B from the telescope; we have $C_2C_1 = R_1 \frac{\sin b}{\sin a}$ and the distance sought equal to

$$R_1\left(1 + \frac{\sin b}{\sin a}\right) = R_1 \left(\frac{\sin a + \sin b}{\sin a}\right).$$

EXAMPLE—In the same eye as before let $a = 5.1°$, the distance of B from the telescope 12.4° temporal and that of C from the telescope 9.9° nasal. We would then have for the distance sought $7.98 \left(1 + \frac{\sin 6.2°}{\sin 4.8°}\right) = 18.28$ mm. and the apparent radius would be 18.28mm.—3.16 mm.=15.12 mm. The position of the real center would be 13.78 mm. (1) and the radius of the real surface 13.78 mm.—3.73=10.05 mm.

34. Direct Determination of the Radii.—In fig. 49, as well as in figs. 50 and 51, the ratio between the distances separating the two images of the same kind is equal to the ratio between the apparent radii. We may, indeed, consider the distance separating the two lamps as an object, three images of which are formed on the pupil; these images are proportional to the radii following the formula $\frac{O}{I} = \frac{2l}{R}$, since O and l are the same in the three cases.

We can make sufficiently accurate measurements of the radii if we make use of two cursors similar to A and two others similar to B. We place the lamps A in such a position as to be able to observe clearly the images produced by the anterior surface of the crystalline lens. Then we displace the cursors B, the lamps of which must be feeble, until the corneal images of the lamps of each are on the same straight line as one of the crystalline images of A. We consider the distance which separates the cursors A as object for the anterior surface of the crystalline lens, and that separating the cursors B as object for

(1) [Considering that we have again obtained this apparent position with reference to the refraction of the cornea, we must therefore in the formula $\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$ put $F_1 = 23.64$; $F_2 = 31.61$ and $f_1 = -18.28$, this gives $f_2 = 13.78$.]—W.

the cornea. As the images are alike, the radii must be inversely proportional to the objects. Knowing the radius of the cornea, we can, therefore, calculate the apparent radius of the anterior surface of the crystalline.

To determine the astigmatism of the surface we must repeat all the measurements in the vertical meridian.

The posterior surface of the crystalline lens is measured exactly like the anterior surface. As to the posterior surface of the cornea, its image is not visible at the middle of the pupil. We must, therefore, limit ourselves to measuring the peripheral parts. The direct determination of the position of the surface, following the method indicated in paragraph 32, is not applicable for the same reason, but the position of the center can be determined after paragraph 33, and the length of the radius as we have just explained, which gives indirectly the thickness of the cornea. It is necessary to have previously measured the radius of the anterior surface of the cornea at the place where we are making the measurement, for generally this place is so peripheral that the flattening of the cornea makes itself felt. Besides, the posterior surface undergoes, towards the periphery, a flattening analogous to that of the anterior surface, so that the relation between the radii of the two surfaces seems almost the same everywhere.

35. General Remarks.—We can, therefore, thus measure on the living subject all the optic constants except the indices. But we must not deceive ourselves as to the exactness of these measurements; excepting those of the anterior surface of the cornea, they are not very exact. In fact, the crystalline images are feeble, and those of the anterior surface of the crystalline lens very diffuse, which causes the measurement to become less certain; there are also other sources of errors, such as that made by comparing the surfaces to spherical surfaces. It may happen also that the observed eye does not fix exactly at the moment of observation. When we wish to determine, for ex-

ample, the radius of the anterior surface of the crystalline lens, we have to depend on three measurements, that of the radius of the anterior surface of the cornea, that of the position of the anterior surface of the crystalline, and that of the position of its center. The errors of these measurements are added in the final result. I do not think, therefore, that we can guarantee an exactness of more than half a millimeter in the final result. As far as the optics of the eye are concerned, this want of exactness does not present any considerable importance. Indeed, it must not be forgotten that the difference of index of the media which separate the internal surfaces is very slight, making it unnecessary to know the radii very exactly; an error of half a millimeter in the measurement of the anterior surface of the cornea corresponds to about 3 D., whilst the same error in the measurement of the anterior surface of the crystalline lens corresponds only to a third of a dioptry.—But, as to the thickness of the crystalline lens, which is only 4 mm., an error of half a millimeter presents a vast importance. The much disputed question of knowing whether the crystalline lens changes its thickness during accommodation can with difficulty be decided by the observation of the crystalline images, for the alleged change (an increase of 0.4 mm.) does not exceed the limit of error.

Ophthalmometry of the cornea has passed the doors of the laboratories, and has been introduced into clinics where it is daily rendering great service. It might be asked, therefore, whether the measurements of the internal surfaces could not also find clinical application. Indeed there often exist between the astigmatism indicated by the ophthalmometer and subjective astigmatism, differences the cause of which it is very natural to look for in the internal surfaces, and which we might hope to disclose by these methods. I have made some measurements of this character, but I do not think they have a great future. They are always very complicated; it would be necessary, in fact, to measure the radius of each surface at least in two meridians, and as each radius calls for two measurements (of the surface and the center) this would involve already 12 measure-

ments; it would then be necessary for us to calculate the real values in order to deduct the astigmatism of each surface and lastly to combine these astigmatisms with that of the anterior surface of the cornea. This is already sufficiently complicated, but it becomes more so if, as is probable, the main meridians of the internal surfaces coincide neither with one another nor with those of the anterior surface of the cornea.—It is true that it would be possible to simplify the methods for practical application, and to replace the calculations by approximations, but I do not think the result is worth the trouble, more especially as it is probable that we frequently would not find what we look for, the explanation of the differences between ophthalmometry and subjective astigmatism, for these differences are probably frequently due to the fact that the peripheral parts of the cornea have an astigmatism different from that of the central parts, which we measure with the ophthalmometer.

Bibliography.—Aubert (H.). *Pfluger's Archiv.* Bd. 35, p. 597, 1885. —Javal (E.). *Mémoires d'ophthalmométrie.* Paris, 1890.—Sulzer (D.). *La forme de la cornée humaine ėt son influence sur la vision. Arch. d'opht.,* 1891.—Eriksen. *Hornhindemaalinger* (Danish). Arahus, 1893.—Tscherning (M.). *Beitrage zur Dioptrik des Auges* (*Zeitschrift fur Psychologie und Physiologie der Sinnesorgane,* III, p. 429).—Brudzewski (K.). *Beitrag zur Dioptrik des Auges. Arch. fur Augenheilkunde.* XL, 3.

CHAPTER V

CIRCLES OF DIFFUSION ON THE RETINA

36. Definition.—Receiving on a screen the image of a distant luminous point, and moving the screen forwards and backwards, we see that there is only one position in which there is formed a distinct image of the point. In every other position we see on the screen a luminous spot of the same form as the aperture of the lens, which spot is the larger the farther it is removed from the distinct image. This luminous spot is called *circle of diffusion.*

The same thing happens in the eye, with this difference that, not being able to move the retina backwards or forwards, we move the luminous point which amounts to the same. The round form of the image of diffusion is due to the round form of the pupil; if we look, for example, through an aperture which is triangular and smaller than the pupil, the image of diffusion is triangular and is somewhat improperly called *circle of diffusion.*

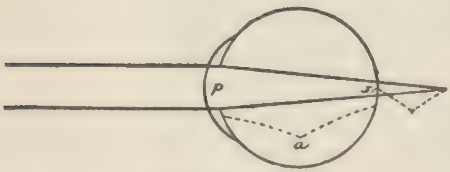


Fig. 54.

It is easy to calculate the size of the circle of diffusion (fig. 54). If the diameter of the pupil (of exit) be designated by p, its distance from the retina by a and the distance of the distinct image from the retina by d, we have for the diameter of the circle of diffusion the expression

$$x = p \frac{d}{d + a}.$$

If, instead of a luminous point, we observe an object the image of which is formed in front of or behind the retina, each point of the object produces on this membrane a circle of diffusion which is overlapped by the next circle, except near the

borders of the diffuse image. There is also formed around the shape of the object a border, the width of which is equal to half of the diameter of a circle of diffusion, and the intensity of which diminishes towards the periphery. The object is, therefore, seen a little enlarged and with ill-defined borders.

37. Line of Sight.—When we perform the act of sighting we try to make two points, situated at different distances, coincide; as we can only see one point distinctly at once, it is generally supposed that we make the image of one of the points coincide with the center of the circle of diffusion of the other. Now the center of the circle of diffusion corresponds with the middle of the pupil; it would be necessary, therefore, to place the second point on the line which joins the point which is fixed to the center of the apparent pupil, a line which is called the *line of sight*. This reasoning is subject to caution. Indeed, in order to be able to sight, it is necessary to see the second point pretty distinctly, which requires that it be not too far removed, optically, from the point fixed. The circle of diffusion of the point of sight is, therefore, so small that we commit only a very small error when we consider it as a point. We must also note that the rule according to which the circle of diffusion should everywhere have the form of the pupil, is not strictly correct. By reason of astigmatism and other irregularities of the eye, there nearly always exists, as we shall see in chapter X, a part in front of or behind the focus, where the circle of diffusion is far from having the form of the pupil; it assumes more or less irregular forms, and the light is no longer distributed in a regular manner. In sighting, then, we make the image of the point fixed coincide with the brightest part of the circle of diffusion, which has nothing to do with the center of the pupil. In order not to complicate the terminology, it would, therefore, be preferable to dispense with the expression *line of sight*.

38. Accommodation.—We know that the eye can change its focus, adapting itself for shorter distances than that for which it is adapted in a state of repose. Holding a book at 50 centi-

meters and placing a veil between the book and the eyes, at 20 centimeters, we can see distinctly, sometimes the threads of the veil, and sometimes the letters.—If we illuminate the fundus of an emmetropic eye with the aid of a plane mirror, by using a flame placed at a great distance, we see a distinct image of the flame projected on the fundus of the eye, if the observed person looks in the distance. If, on the contrary, he fixes an object located nearer, the image forms a circle of diffusion which, most frequently, fills the entire pupil. The contrary takes place when the flame is placed at a short distance.

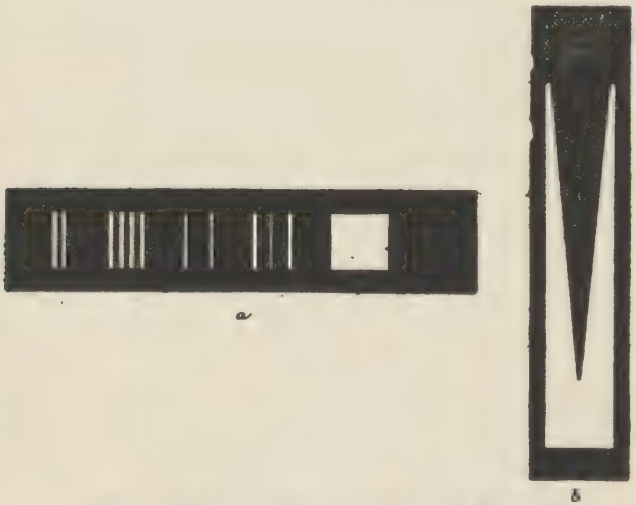


Fig. 55.—Rules of the optometer of *Young*.

39. Experiments of Czermak, Scheiner and Mile.—Looking towards an illuminated surface (the sky, for example) through a pin-hole made in a dark screen, we see the opening under the form of a circle of diffusion. If we move a second screen, held nearer the eye, in front of the opening, it seems to move in a direction contrary to that in which it really does move. If, on the other hand, we move the second screen in front of the first, it seems to move in the direction of its real displacement (*Czermak*).

Looking towards an illuminated surface through two openings,

the distance of which is smaller than the diameter of the pupil, we see two circles of diffusion which partly overlap. A needle is then placed so that we see it in the part common to the circles of diffusion, and another farther away in the same direction. That one of the two needles which we fix is seen single, the other double. If it is the nearer needle that is seen double, the image on the left disappears, if we cover the opening on the right. (1) If it is the other needle that is seen double, the contrary takes place (*Scheiner*).—It is easy to repeat this experiment with a lens, and it is also a very good way of determining the focal distance of the latter (by replacing the needle by a luminous point).

If we look at the more distant of the two needles in the experiment of *Scheiner* through a single small opening, we shall see that a slight movement of the screen causes the nearest needle to move in the contrary direction. On fixing the nearer of the two needles the other seems to move in the same direction as the screen (*Mile*).

It is easy to account for these phenomena when we sketch the course of the rays, not forgetting that the eye inverts the phenomena when projecting them outwards.

40. Optometer of Thomas Young. (2)—The experiment of *Scheiner* forms the basis of the optometer of *Thomas Young*,

(1) To render the experiment more striking to my pupils, I had a plate of red gelatine glued in front of the opening on the right. But, after having explained the theory of the experiment, I met with very vigorous protestations; all declared that it was the needle on the right which appeared red. It is thus, in fact, when we look towards the sky, but we must not conclude from this that it is the needle on the right which belongs to the opening on the right. The phenomenon is analogous to that of colored shadows, of which I will speak in chapter XVII. If one places oneself in such a way that the needle is eliminated, it is the image on the left which appears red. One of my pupils, M. *Johnsson*, has studied the chromatic phenomena which are observable under the same circumstances, by looking at the needle towards the sky, but without the interposition of the colored plate. One sees them specially well by dilating the pupil and using the slits of the optometer of *Young*. When the needle is situated on the near side of the point which is fixed, one of the images is seen green, the other purple; each image is bordered with red on the side which looks towards the other image, with blue on the opposite side. These phenomena, which depend on the chromatic aberration of the eye, are not yet well explained.

(2) Not being able to procure any part of this instrument, I had it constructed again by M. *Werlein*, modernizing it a little.

which appears to me to be one of the most important instruments for the study of physiologic optics. It has the form of a little rule. On one of the faces is drawn a fine white line on a black ground. We look along this line, through a lens of + 10 D. In front of the lens moves a small horizontal rule, in which are different groups of slits (fig. 55*a*). Placing the two slits, which are at the middle of the horizontal rule, in front of the lens, they act like the openings in the experiments of Scheiner. Each point of the line appears double, except that which is seen distinctly; an emmetrope, not using his accommodation, must, therefore, see two lines which intersect at the *punctum remotum,* or artificial far point, at 10 cm. from the eye. To determine the refraction of any person we make him look in the instrument, and put a small cursor at the place where he sees the lines intersect. A dioptric scale, placed along the line, then permits the refraction to be read off directly.—We then determine the near point (*punctum proximum*) in the same manner.—The other groups of slits permit the determination of the refraction of the different parts of the pupillary space. We can also use the little vertical rule (fig. 51*b*) which has the form of a very pointed triangle; by lowering it more or less, we eliminate a smaller or greater part of the middle of the pupil.

The instrument does not lend itself very well to the examination of patients, for it is quite difficult for an inexperienced observer to use it without using his accommodation. For one who can control his accommodation, the instrument permits the measurement simultaneously of the refraction and the amplitude of the accommodation; the refraction can be determined in different meridians by making the instrument rotate around its longitudinal axis. It was thus that *Young* discovered the astigmatism of his own eye.

The observations made with this optometer are, moreover, of the greatest importance for the study of the nature of accommodation (see chapter XII).

41. Effects of the Stenopaic Opening.—Looking through an opening smaller than the pupil, we diminish the circles of dif-

fusion so that objects which we first see dimly become more distinct. This is why myopes see better at a distance by looking through a small opening. We can also make use of it as a magnifying glass; we can, indeed, move very close to the eye the object which we desire to examine, and in this way obtain a very large retinal image. The more the diameter of the opening is diminished, the more distinct the image becomes, but it loses at the same time in brightness. We cannot exceed a certain minimum limit without blurring by diffraction the distinctness of the image. (1)

As the stenopaic opening effaces, so to speak, the effect of the anomalies of refraction, it is harmful in all cases in which we desire to determine refraction. This is why we place patients with their backs towards the window when we examine their vision. We must also avoid the small apertures in the ophthalmoscopes which are used to determine refraction; a too strong illumination is equally hurtful.

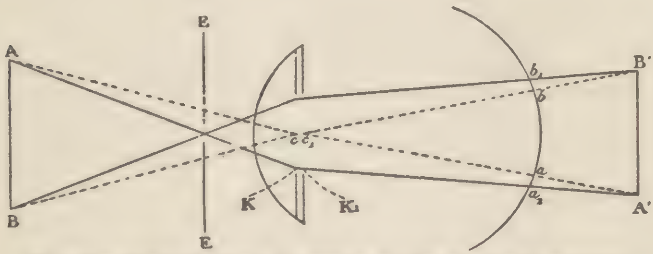


Fig. 56.—Magnification by means of the stenopaic opening.

Examining an object placed very near the eye through a stenopaic opening, we shall see that the object seems to enlarge as we gradually move the screen away from the eye. Following is the explanation of this fact.

Let AB (fig. 56) be an object, and A_1B_1 its image formed by the optic system of the eye. As the object is near the eye, the

---

(1) Looking at a luminous point which we see distinctly, through a very fine opening, we observe that it becomes enlarged into a small luminous surface surrounded with brilliant rings. This effect of diffraction begins to make itself slightly felt starting from an aperture of the pupil or of the opening of about 2 millimeters.

image is formed quite a distance behind the retina. To determine the position of the indistinct image on the retina, we draw the ray Ac passing through the middle of the pupil of entrance; after refraction it continues its course as if it came from c_1, the center of the pupil of exit. Its direction is $A'c_1$, since it must pass through A′, the image of A. The point a is, therefore, the middle of the circle of diffusion which A forms on the retina, and ab is the diameter of the image of diffusion.—Let us now interpose the screen EE with its stenopaic opening. The only ray which passes from A through this opening takes the direction AK and, after refraction, the direction K_1A'; it meets the retina at a_1 and a_1 b_1 is the size of the retinal image. We see that this image is larger than ab and that it would become larger still if we moved the screen farther away.—Myopes looking at distant objects through a stenopaic opening see them diminish if the opening be moved away a little.

Bibliography.—The study of the influence of circles of diffusion on vision has been very much neglected by modern authors. The best work done on this question is the following, which dates from the last century:

Jurin (J.). *Essai sur la vision distincte et indistincte,* in Robert Smith, *Cours complet d'optique,* translated by Pezenas, Paris, 1767.—Scheiner (C.). *Oculus.* Innspruck, 1619.—Mile (J.). *Pogg. Ann.,* XLII, 40.—*Œuvres de Young,* edited by Tscherning, page 112.—Tscherning (M.). *L'optometre de Young et son emploi.* *Arch. de physiol.* October, 1894.

CHAPTER VI

ANOMALIES OF REFRACTION

42. General Remarks.—We have thus far treated the optic system of the eye as if it were perfect, but it has really many defects. *Helmholtz* said that if an optician had delivered to him an optic instrument as imperfectly made as the eye, he would have considered himself within his right in refusing it; expressing himself in quite forceful language. The remark of *M. Mascart* appears to me nearer the truth. He said that the eye has all possible defects, but only to such an extent that they are not harmful. We have already seen that this is the case with diffraction, which begins to make itself felt, starting from a pupillary diameter of 2 millimeters, almost the lowest limit of this diameter. It is the same with chromatic aberration, spherical aberration, etc. An optician need not be so careful with an objective, the images of which are intended to be magnified five times, as with another the images of which are to be magnified twenty or thirty times. In the same way eyes frequently have all the visual activity we can expect considering the retinal structure, and a greater degree of optic perfection would be superfluous. It is true that many eyes which are considered normal, have optic defects which diminish their *visual* acuity, which should be nearly double that called *normal* acuity; but for most occupations, the acuity known as *normal* amply suffices.

We can divide anomalies of refraction into three groups:

1°. Anomalies "of the Screen."

a. Axial myopia.—Screen is too far away from the optic system.

b. Axial hypermetropia.—Screen is too near the optic system.

c. Oblique position of the screen.—This last anomaly is not generally recognized. It seems to play a part in diminishing the visual acuity in certain forms of very high myopia, in which the

summit of the staphyloma does not correspond exactly with the *fovea*. It is evident that, if the optic system of the eye were perfect, all the rays emanating from a point would meet exactly in a point on the screen, and the obliquity of the latter would play no part, for the extent of distinct vision is so small that the difference of distance of the different parts of the image from the optic system cannot have much influence. But if the rays do not meet exactly in a point, as is nearly always the case, it is clear that the circle of diffusion on the retina must be larger when the retina is placed obliquely, and that this must diminish visual acuity.

2°. ANOMALIES OF THE REFRACTING SURFACES.

$\left.\begin{array}{l}\textit{Myopia}\\ \textit{Hypermetropia}\end{array}\right\}$ *of curvature.*

Regular astigmatism.

Spherical aberration.

Chromatic aberration.

Keratoconus.

Lenticonus.

Aphakia.

Luxation of the crystalline lens.

All the forms which are classified under the name of *irregular astigmatism.*

3°. ANOMALIES OF THE INDICES.

False lenticonus.

The anomally which *Demicheri* has recently described under the name of false lenticonus is the only anomaly of the indices which has been established up to the present. In these cases we see with the ophthalmoscope the same play of shadows that is characteristic in keratoconus; it is due to a great difference of refraction between the middle of the pupil which is very myopic (as high as 10 D. and more), and the periphery which is hypermetropic (3 to 4 D) The explanation is probably to be found in a diminution of the index of the peripheral layers of the crystalline lens, a change which must diminish the refraction of the

peripheral parts of the pupil and greatly increase the central refraction, following the explanation which we have given on page 36. We find in these cases the images of Purkinje doubled (see page 35), the surfaces of the nucleus giving rise to a quite regular reflection; these cases are analogous to that which I have found in the case of the eye of a dead ox, probably also due to the imbition of water by the superficial parts.

43. General Remarks on Ametropia.—We designate as the *far point* (*punctum remotum*) the place for which the eye is focused when in a state of repose. It is, therefore, the conjugate focus of the *fovea*. By making an effort of accommodation, the eye can focus itself for shorter distances. The nearest point for which the eye can adapt itself is called the *near point* (*punctum proximum*). We generally express the distance of the *near point* and that of the *far point* in dioptries; the difference between the two numbers is called the *amplitude of the accommodation*. The determination of the *far point* is quite easy, and forms an important part of the work of the oculist; that of the *near point* is not very certain, since its position depends on an effort of the patient, the strength of which may vary from day to day; for that reason the determination of the near point is frequently neglected in clinics.

We consider as normal the *emmetropic* eye, that is to say, an eye such that, in a state of repose, the image of distant objects is formed on the retina. In the *myopic* eye this image is formed in front of, in the *hypermetropic* eye behind, the retina. We designate these two anomalies under the common name of *ametropia*. The emmetropic eye has its *far point* situated at infinity, that of the myopic eye is at a finite distance. As to the hypermetropic eye, its remote point is virtual. It is necessary that the rays converge before entering the eye in order that they may reunite in a point on the retina. This point towards which the rays must converge, before entering the eye, and which is consequently situated behind the latter, is the *far point;* its distance is to be put down as negative. The degree of

ametropia is indicated by expressing in dioptries the distance of the eye from the remote point. (1)

In the great majority of cases, myopia and hypermetropia are due to an anomally in the length of the eye: the myopic eye is too long, the hypermetropic eye too short. An increase or a diminution of 1 millimeter in the axis of the eye corresponds to an ametropia of two dioptries and a half. Let us place in the formula of Newton, $l_1 l_2 = F_1 F_2$, the values of the simplified eye $F_1 = 17$ millimeters, $F_2 = 22.7$, and we will have $l_1 l_2 = 386$, in which formula l_1 is the distance of the far point from the anterior focus and l_2 the distance of the retina from the posterior focus of the eye. If $l_2 = 1$ millimeter, $l_1 = 386$ millimeters, which corresponds to about 2.5 D.; if $l_2 = 2$ millimeters, $l_1 = 193$ millimeters or about 5 D., and so on.

Myopia is corrected by placing in front of the eye a concave glass so that the image which it forms of distant objects may be situated at the *far point* of the eye. On account of the distance of the glass from the eye its focal distance is a little shorter than the distance of the eye from its far point. The subjective examination always results, therefore, in our finding a somewhat higher myopia than really exists. The difference is insignificant for low degrees of myopia, considerable for high degrees. If we move the glass away from the eye, its effect diminishes.—When selecting the correcting glass, we must take great care to select the weakest concave glass which corrects, because young myopes see as well with stronger glasses, the excess of correction being neutralized by accommodation. After having found the correcting glass, we may try the effect of moving it gradually away from the eye. If the patient continues to see well the glass is too strong.

Hypermetropia is corrected by means of a convex glass, which brings the image of the distant object to the *far point* situated

(1) From which part of the eye one should start to calculate ametropia is a disputed question; it seems to me that the simplest way is to calculate it, starting from the summit of the cornea. Some have preferred to calculate it from one or other of the cardinal points of the optic system, but as these points have not the same position in all eyes, nor in all the meridians of the same eye, nor even for all parts of the same meridian, confusion would result.

behind the eye. The focal distance of the glass being a little greater than the distance of the eye from the *far point,* the correcting glass is a little weaker than the hypermetropia. The hypermetrope can increase the strength of his glasses by moving them a little away from the eye.—The correcting glass is the strongest convex glass which the patient tolerates without loss of visual acuity, but he can also see as well with weaker glasses by using his accommodation.

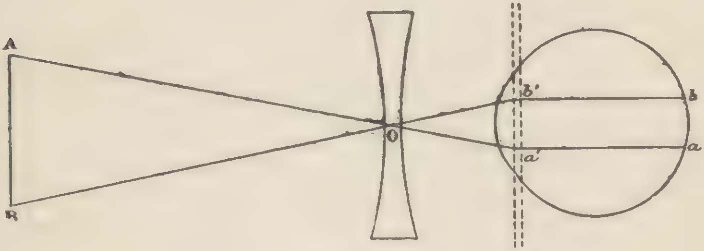


Fig. 57.

The retinal image of an object seen under a given angle is larger in the myopic eye and smaller in the hypermetropic eye than in an emmetropic eye, because the distance of the posterior nodal point from the retina is greater in the myopic eye, less in the hypermetropic eye.—But, this effect disappears when we correct the ametropic eye, by placing the correcting glass so that its optic center coincides with the anterior focus of the eye. Then the image is always the same size, whatever the ametropia may be. For, the rays AO and BO (fig. 57) pass through the lens without deviation and are parallel, after refraction by the same optic system of the eye, so that the size of the image is always the same, whatever may be the distance of the retina.—If we place the correcting glass in front of the anterior focus, the retinal image of the myopic eye is smaller, that of the hypermetropic eye larger, than the image of the emmetropic eye, which is easy to see by a construction analogous to that of fig. 57. We first construct the image formed by the glass, and draw the rays passing through the extremities of this image and through the anterior focus.

Patients often say that the concave glasses diminish objects. This may be attributed to the fact that the glass is placed in front of the anterior focus, or simply to the fact that exterior objects, seen distinctly, appear smaller, because of the disappearance of the circles of diffusion. But the cause may also be that the glass is too strong; for if the patient uses his accommodation the anterior focus approaches the eye and the image becomes smaller for this reason.

44. Optometers.—The use of the test case lenses and of the visual acuity chart, placed at a distance, is always the best of the subjective methods. A very great number of optometers have been constructed, but none of them has succeeded in superseding the test case; they have this defect in common that they superinduce an effort of accommodation which makes the myopia appear too strong. The best are those which are operated at a great distance, like the optometer of *Javal,* but even these seem sometimes to give too strong degrees of myopia. The optometer of *Javal* is composed of two discs, nearly like the discs of the ophthalmoscope for refraction, but much larger: one of the discs has spherical lenses, the other cylindrical lenses; a special mechanism permits the axis of all the cylindrical lenses to be adjusted in the direction we desire.—Other optometers are founded on the use of a single convex lens; by displacing the object in relation to this lens, we can form the image of it at any distance whatever, and thus find the place where it appears distinct. Optometers of this kind have been constructed by *Coccius, Donders, Sous,* and many others. The optometer of *Graefe* was a Galilean telescope; we know that myopes are obliged to shorten their opera glasses to see distinctly. By providing the opera glass with a scale it may, therefore, be used as an optometer.—So also may the telescope, the use of which was proposed by *Hirschberg.*

Among all these optometers I shall mention specially only that of *Badal,* on account of its admirable principle. It is composed of a single convex lens, the focus of which coincides with the anterior nodal point of the eye. The position of the latter

is made secure by an eye-rest. A diminished copy of the chart of *Snellen* is placed on the other side of the lens, movable forwards and backwards. By displacing the object we can make

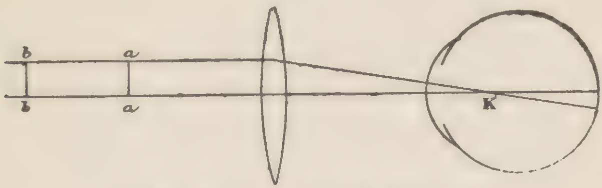


Fig. 58.—Principle of *Badal.*

the image appear anywhere, and it is easy to see (fig. 58) that the retinal image remains always the same size, no matter whether the object is at *bb* or at *aa*, etc. We can therefore

Fig. 59.

measure the visual acuity with this optometer. The same result is obtained by making the focus of the lens coincide with the anterior focus of the eye (fig. 59).

45. Myopia.—There exist two forms of axial myopia, one which depends on near work, and one which does not. (1)—*Myopia* from near work appears usually at an age ranging from 6 to 15 years; it often stops at the age of 25 years. It attains medium degrees and does not seem to exceed the limit of 9 D. Complications, except staphyloma, are rare.

---

(1) Even eliminating these two forms of myopia, it is probable that there would still remain a certain number, due to a congenial disagreement between the optic system and the length of the axis of the eye, for it is not probable that all normal eyes are constructed so as to be exactly emmetropic.—But myopia between 2 D. and 9 D. is so rare among uneducated persons, that this third form must comprise only light degrees.

Dangerous myopia is sometimes congenial and stationary; as a rule it develops in early infancy, and continues to increase during the whole life. At the age of 20 years it generally exceeds 9 D. This form of myopia is to be considered as a malignant choroiditis, and it is to it that dangerous complications of myopia belong; like most choroidal affections it seems to be a little more prevalent among women.

In 1882 and 1883 I examined about 7,000 young Danish conscripts, by determining their refraction by means of the upright image. The influence of near work is seen in the following list:

		Myopes.
I	Students	32 per cent.
	Persons employed in offices and in trade	16 —
	Artists, etc.	13 —
	Tailor, shoemakers, etc.	12 —
II	Workmen (hard labor)	5 per cent.
	Agriculturists (peasants)	2 —

We see that the very great frequency of myopia in the educated classes comprises only the lowest degrees. The very high degrees are rather more frequent in the illiterate (fig. 60).—Among the peasants I have even met more cases of myopia greater than 9 D. than of myopia between 2 D. and 9 D.

It is, therefore, a great exaggeration to regard myopia from near work as a public calamity, as is done especially in Germany. One exaggeration leads to another. It was thought formerly that myopic eyes were stronger than others because they did not become presbyopic. After the discovery of the ophthalmoscope very grave complications in cases of strong myopia were continually met with, and thus originated the idea expressed in the celebrated phrase of Donders, "I do not hesitate to declare that every myopic eye is a diseased eye," a phrase which *Cohn* adopted as his motto in the first of the great compilations of statistics of school children ever made. Later, many others were made, but without important results. They show conclusively that myopia is more frequent and more pronounced in the higher classes of the schools; but as the pupils of these classes are older, and as the myopia is a condition that develops with age,

these statistics do not establish definitely the influence of near work.

The distribution of the two forms of myopia in the two groups was the following:

	In all.	Myopes < 9 D.	Myopes > 9 D.
I	2.336	407 (17 per cent.)	13 (0.56 per cent.)
II	5,187	169 (3 —)	38 (0.73 —)

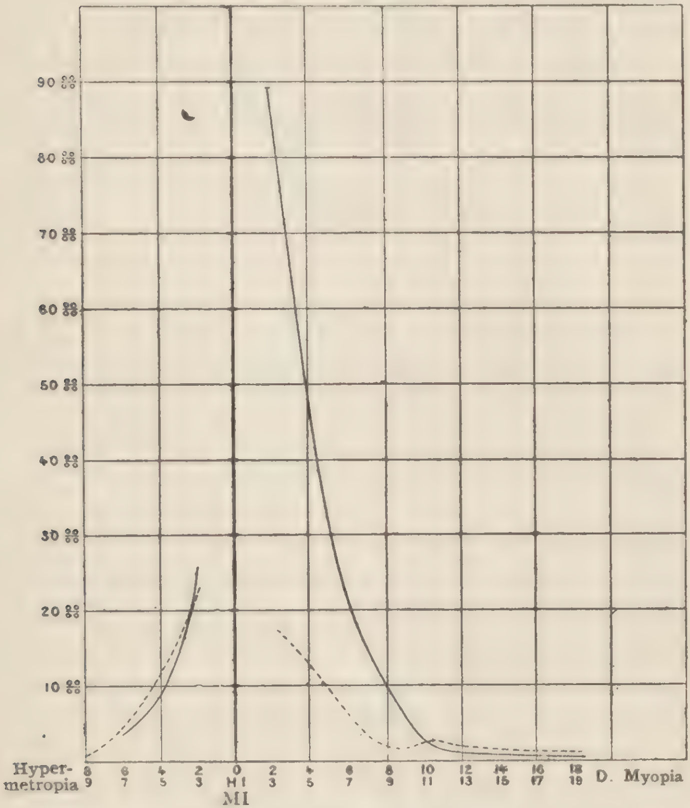


Fig. 60.—Distribution of the anomalies of refraction among the young population of Copenhagen.

——— Educated. ...... Uneducated.

A satisfactory explanation of the mechanism by which near work produces myopia has not yet been given. *Donders* named

three factors: first, the inclined position of the head which produces hyperemia of the globe with a tendency to distention; second, the fatigue of the eyes, which would be the result of prolonged reading, and which would also produce hyperemia; third, the compression which the external muscles would exercise on the eye, during convergence for a near point.—*Arlt*, who, by his autopsies, proved for the first time in 1854 that myopia is due to a lengthening of the globe, laid special stress on the action of the superior oblique while reading. The eye being directed downwards, this muscle may, indeed, compress one of the veins and thus produce the development of hyperemia. *Stilling* tried to further develop this theory by finding the predisposition to myopia in a special form of the orbit (very low—*Hypoconchia*) which would give to the muscle a direction more likely to compress the eye.

In spite of the slight degree of accommodation which myopes need (1), the theory of the accommodative origin of myopia has, however, many believers, and I think they are right; but as the mechanism of accommodation was scarcely known until recent times; it is not wonderful that the solution of the problem of myopia from near work was sought in vain.

46. Selection of Spectacles.—Although myopia from near work is not to be considered as a true diseased condition of the eye, it always causes a disagreeable feeling which it is our duty to prevent as much as possible. As it is near work which produces myopia, young myopes must be made to work at as great a distance as possible; and, on account of the probable influence of

---

(1) It is possible that myopes often accommodate more than we think. In low degrees they frequently work within their *far point*, because by bringing the work near they can see more detail. As to high degrees, other circumstances may bring about a quite remarkable accommodation. This is why *Javal* said that a myopic eye may be focused at once for the extremities and the middle of a line of a book. If the myopia is 10 D., the length of the line is 10 cm., and if the ends of the line are seen distinctly without accommodation, the patient is obliged to accommodate about two dioptries when reading the middle, unless he keeps the book or his head in continuous motion, or contents himself with seeing diffusely a part of the line.

accommodation, we must suppress the latter as much as possible, or annul it.—We are very frequently consulted on the question of glasses by parents who are worried at seeing their children become myopes.—If the myopia is low, under three dioptries, we give correcting glasses for distant vision, and nothing for near vision, (1) recommending the patient to be careful as to the distance of the book while reading. We place the normal distance for work at 33 centimeters.—If the myopia exceeds three dioptries we give for near vision correcting glasses diminished by 3 D. For example, if the myopia is 6 D. we give 3 D. for near vision. For distant vision we may give correcting glasses or a supplementary glass to superimpose on the spectacles.—But, in giving concave glasses for near vision we must forcibly impress upon myopes the necessity of observing the minimum distance of 33 centimeters when working; otherwise the glasses would be rather harmful by superinducing an effort of accommodation which might cause the myopia to increase.

When the myopia exceeds 9 D., it becomes necessary to regard it as dangerous, and great care in the use of the eyes must be recommended. Generally it is preferable not to completely correct myopia, but only sufficiently so that the patient may not be too much annoyed in moving around. As the acuity is frequently diminished we can no longer insist on as great a distance for near work; thus we may give correcting glasses diminished by 4 to 5 D. for near work, which places the far point at 25 or 20 centimeters respectively.—The patient must be advised never to work with his head lowered; in the latter case where the distance of the work is 20 cm. a desk must be used.—Patients frequently ask us for advice as to illumination. No artificial light, except an arc lamp, is hurtful to the eyes; the stronger it is the better, because artificial illumination never attains the degree of illumination of a bright day; but it may be useful to protect the eyes with a shade.

(1) [In the United States we prefer to let these myopic patients wear their glasses constantly, especially as these eyes are usually more or less astigmatic. The sucess of this method is proved by the careful investigations of Dr. S. D. Risley. See his article on School Hygiene in the System of Diseases of the Eye by Norris and Oliver, Philadelphia, 1897.]—W.

When the myopia is very high, spectacles are frequently of no service, as the patients do not accept them. It is then necessary to restrict near work as much as possible. For distant vision a small telescope sometimes gives good service. In order to obviate the necessity of accommodation, patients should be advised to lengthen it as much as possible.

47. Treatment of Myopia.—Each of the two theories by which myopia from near work has been explained has given rise to a treatment of this defect. The theory of convergence led to the attempt to stay the progress of myopia by performing a tenotomy of the external rectus as soon as there was a slightly pronounced latent divergent strabismus (which was called insufficiency of the internal recti—*exophoria*). Certain surgeons performed thousands of operations of this character: the result was very doubtful, and we may consider this operation as abandoned. The theory of accommodation led to treatment by atropine; but, before speaking on this subject, I shall say a few words on the use of atropine for the determination of refraction, a method which is still very much in vogue in some countries.

De Wecker held decided views on the abuse of atropine in ophthalmic practice, and, as far as its use for the determination of refraction is concerned, I am in perfect agreement with him.—We know that young hypermetropes are accustomed to correct part of their hypermetropia by using their accommodation, and that they cannot relax this accommodation without becoming trained to it by means of convex glasses, at least as long as they fix a specified object. To make all the hypermetropia manifest we must instill atropine in order to paralyze the accommodation. It is this perfectly correct observation which gave rise to the idea that generally a better determination of refraction would be obtained by using atropine, and which resulted in the ciliary muscle being held responsible every time a difference of refraction before and after the instillation of atropine was found. By putting atropine in the emmetropic eye we often find a light degree of hypermetropia, which *Donders* was wont to explain by assuming a "tonus of the ciliary muscle." Frequently also we

see myopia diminish slightly under the influence of atropine, and this diminution has been attributed to the existence of a "spasm of accommodation," which would disappear as soon as the accommodative muscle would be paralyzed.

These errors originated in the belief that refraction must necessarily be the same in the whole pupillary space. It is nothing of the kind: there nearly always exist differences which are frequently very considerable. Thus there is in my eye a relatively great difference, nearly 4 D., between the upper border and the lower border of the pupil (see page 173).—When we instil atropine, the pupil is dilated and the basilar position of the cornea, which is much flattened, comes into play. As the flattening of these parts is often considerable enough to overcorrect the spherical aberration, we find that the refraction of these peripheral parts is generally less than that of the central parts. A quite slight dilation of the pupil suffices in order that the area of these parts, which, in ordinary conditions, are excluded, may be greater than that of the ordinary pupil; it is this fact which makes us judge specially by them in the determination of refraction. If the peripheral flattening of the cornea is less, or if the extent of the optic part exceeds the ordinary limits, which sometimes happens, we may, thanks to the spherical aberration, obtain an increase of refraction by instilling atropine. Such cases have been observed among others by *Javal;* they were very difficult to explain with the ideas which have been held on the subject up to the present, since it could not be supposed that the use of atropine could cause a spasm of the accommodation. We observe like phenomena with photographic objectives the aberration of which is not well corrected; the focus changes on changing the aperture of the diaphragm.—Except in cases of latent hypermetropia, we obtain, therefore, generally a better idea of ocular refraction by the ordinary examination without atropine.

Atropine treatment has been used in cases of progressive myopia; the ciliary muscle would be kept paralyzed for 15 days or a month, in order to arrest the progress of the myopia, the

special purpose being to counteract the spasm of accommodation, which was supposed to be the cause of the progress of the myopia. This treatment does not seem to have been effective.— In cases where the eyes are exposed to great danger, for example in detachment of the retina, it may, however, be useful to procure for them complete rest by instilling atropine and forbidding work altogether for some time.

Some years ago, on the advice of *Fukala*, the profession began to treat high degrees of myopia by removing the crystalline lens, generally by a discission followed by extraction. This treatment, which *Donders* pronounced criminal at a time when surgical operations were more dangerous than now, often seems to give very good results, not only because those operated on become emmetropic or nearly so after the operation, but also because they gain considerably in visual acuity for distance. We have already seen that the size of the retinal image of the myopic eye, corrected by a glass placed at the anterior focus, is equal to the image of the emmetropic eye. Now, in the emmetropic eye the retina is situated about 16 millimeters behind the posterior nodal point; in a myopic eye, which has become emmetropic by the extraction of the crystalline lens, the retina is situated at the posterior focus of the cornea or about 24 millimeters from the nodal point. As the size of the image depends only on this distance, we see that the linear enlargement of the image by the operation is about a half. Often it gains still more because the correcting glass is placed not at the anterior focus but a little in front, which has the effect of diminishing the image. The loss of accommodation, which is, indeed, of very little use to myopes of a high degree, cannot counterbalance these advantages; nevertheless there is reason for prudence in recommending this operation, for it is not without danger. When making the discission (followed by paracentesis) we may fear glaucomatous complications or iridocylitis as a consequence of a too great swelling of the crystalline lens. If extraction is performed an accidental loss of the vitreous body may sooner or later produce a detachment of the retina.

48. Hypermetropia.—The hypermetropic eye is too short. The retina being too near the optic system, the hypermetrope cannot, without an effort of accommodation, reunite on the retina parallel or diverging rays. When the hypermetropia is high, the amplitude of accommodation diminishing with age, there comes a time when the patient can no longer correct his hypermetropia by accommodation (*absolute hypermetropia*).—The degree ot hypermetropia is expressed by the strongest convex glasses with which the patient can distinguish distant objects distinctly. To disclose all the hypermetropia, it is often necessary to paralyze the ciliary muscle by means of atropine, because the patient has formed the habit of accommodating as soon as he fixes an object, and he cannot suddenly rid himself of this habit even when we put before his eye a convex glass which should eliminate any necessity of accommodation.—That part of hypermetropia which we cannot make manifest by the ordinary examination is called *latent hypermetropia (Donders)*; it diminishes with age, and it need not be regarded as a very definite quantity. We can often, by working a little with the patient, make him accept stronger and stronger glasses. In the dark room where the patient does not fix, hypermetropia frequently becomes manifest in its entirety which permits it to be determined with the refraction ophthalmoscope or by skiascopy.

Accommodative Asthenopia.—The hypermetrope, being obliged to use part of his accommodation to neutralize his defect of refraction, generally becomes fatigued more quickly than the emmetrope by near work. The essential symptom of this *accommodative asthenopia* is that, while reading, the letters become blurred. When this symptom appears, the patient reads with ease for some time; then the letters begin to become indistinct, so that he is forced to rest a while. If he begins again he gets along well for a shorter time than before, after which the same phenomenon is reproduced. If the patient still continues there supervene fatigue, orbital pains, etc.; but these phenomena are secondary, and we must not, from their appearance, decide on hypermetropia as the cause in the absence of the essential

symptom, viz., the indistinctness of the letters after reading for some time. We need no longer attribtue the complaints of patients to a low degree of hypermetropia. Low degrees of hypermetropia manifest themselves, as a rule, only by the premature appearance of presbyopia. We may easily correct a low degree of hypermetropia, even in young people, but we must not expect to obtain great results. The complaints of the patients have generally other causes.

Boehm, Stellwag and others recommended the use of convex glasses in cases of accommodative asthenopia, but to *Donders* belongs the credit of having brought them into general use. His labors, indeed, contributed greatly to dispel the fear which earlier oculists had of strong convex glasses. They considered asthenopia as the forerunner of amblyopia, and believed that the giving of convex glasses was conducive to the development of the latter.

Hypermetropes generally prefer a great distance tor work in order not to fatigue their accommodation. But, when the hypermetropia is very high, which demands an effort of accommodation much too fatiguing, we see patients choose a very short distance, moving the book to within a few centimeters from the eyes. They see better, thanks to the considerable enlargement of the retinal images. It is true that they are blurred; but, on bringing the object nearer, the circles of diffusion increase less quickly than the images, and moreover, the patients can diminish them by winking their eyelids.

The rule of *Donders* for the selection of spectacles was to correct the manifest hypermetropia plus one-fourth of the latent, that is to say, to give, for young people, convex glasses a little stronger than those which they accept for distant vision. I consider this rule a wise one; others correct all the hypermetropia. Generally the patients are dissatisfied at the beginning, before becoming accustomed to the spectacles; the glasses annoy them, and it is advisable to forewarn them that they will do so for some time. This annoyance is greater the stronger

the glasses, which is one reason for not correcting all the hypermetropia. Another reason is that patients are much more annoyed when, for one reason or another, they cannot wear the glasses, since they have lost the habit of overcoming their hypermetropia by accommodation.

If the hypermetropia is low or medium (1 to 3 D.) there is no reason for giving glasses for distant vision, at least to young people who easily correct their hypermetropia by accommodating; we may leave them free in this regard. If the hypermetropia is high or if there is a tendency to strabismus, the glasses must be worn constantly. (1)

49. Aphakia.—It is very rare to find true hypermetropia which exceeds 7 D. (see fig. 60). The higher degrees are met with only in aphakia (absence of the crystalline lens).

The degree of hypermetropia of the aphakic eye can be calculated by means of the formula $\frac{F_1}{f_1} + \frac{F_2}{f_2} = 1$. With the values of the simplified eye we have $F_1 = 24$, $F_2 = 32$, $f_2 = 24.7$, which gives $f_1 = 81.2$. The *far point* is therefore situated at 81.2 mm. behind the cornea; the eye will be corrected by a convex glass of 96 millimeters = 10.4 D., placed at 15 millimeters in front of the cornea. We find, in fact, that nearly all the emmetropes operated on for cataract are corrected with a glass of from 10 to 11 dioptries.

But it would be an error to apply this number to the ametropias, and to think that we could always find the post-operation refraction by diminishing the ante-operation refraction by 11 D. To find the correcting glass for ametropias we must calculate

---

(1) [In this country our reasoning upon this point is quite different. As people with hypermetropia, higher than 3 D., accommodate with great difficulty, they do not keep it up very long at a time or sometimes avoid to correct accommodation by reading very near with diffuse but enlarged images as has been so well explained by the author. They thus frequently rest their eyes more than the persons with lower degrees of H. who use their accommodation more constantly and on that account show more asthenopia. At any rate the constant correction of the lower degrees of hypermetropia has relieved many cases of obstinate asthenopia.]—W.

it in the same way as for emmetropes. It is thus that Dr. Stadfeldt has calculated the following little table:

Before operation	H. 7	H. 5	H. 3	H. 1	E	M. 1	M. 3	M. 5	M. 7
After operation	H. 15	H.13.8	H.12.5	H.11.3	H.10.6	H.10.1	H.8.9.	H. 7.8	H. 6.6
Before operation	M. 9	M. 11	M. 13	M. 15	M. 17	M. 19	M. 21	M. 23	M. 25
After operation	H.5.5	H.4.4	H 3.4	H. 2.3	H. 1.3	H. 0.2	M. 0.8	M. 1.8	M. 2.7

Comparing this table with the following table which has been made up from a series of results from operations published by *Pflueger,* we see that the agreement is sufficiently satisfactory.

Before oper.	M 10	M 11	M 12	M 13	M 14	M 15	M 16	M 18	M 22
After —	H 5	H5.5	H3.5	H3.5	H3.5	H 1	H2.5	M 2	M 2

Dimmer has directed attention to a slight source of error in the ordinary examination of aphakics. The lenses of our test cases are biconvex, while those which the optician makes for patients are generally sphero-cylindrical, the cylindrical surface being turned towards the eye. Now, the optic center of biconvex lenses is situated at the middle of the lens, while that of plano-convex glasses is situated at the apex of the convex surface. It follows that the spherical effect of the sphero-cylindrical glass is a little greater than that of the biconvex glass, having the same focal distance, the posterior focus being situated a little nearer the glass in the former case. The error may reach a half dioptry. For some time test cases have been manufactured in Austria in which the strong convex glasses are plain on one side.

Ostwalt has laid stress on the influence which the distance of the glass from the eye exerts on the power of sphero-cylindrical glasses. Supposing, for example, that an eye is corrected by +11 D. with +3 D. cyl., placed at 15 millimeters in front of the eye. Such a glass has, in one of the principal meridians, a

focal distance of 91 millimeters, in the other of 71 millimeters. The *far point* of the eye is thus found in one of the meridians at 91 mm.—15 mm.=76 mm. (13.1 D.), in the other at 71 mm. —15 mm.=56 mm. (17.9 D.). Its astigmatism is, therefore, really 4.8 D. and not 3 D. As far as the subjective examination is concerned this difference plays no part, since the glasses with which we examine our patients are at the same distance from the eye as those which the patient will wear, but it is not so with the ophthalmometer, which tells the true astigmatism of the eye; we must recollect, therefore, that in this case the number furnished by the ophthalmometer is higher than that which suits the patient.—In the case of simple cylindrical glasses the same influence makes itself felt, but to a much less extent; a convex cylinder of 6 D. thus corresponds with a true astigmatism of 6.5 D., a concave cylinder of 6 D. with 5.5 D.

Bibliography.—Donders (F. C.). *On the Anomalies of Accommodation and Refraction of the Eye.* London, 1864.—Mauthner (L.). *Vorlesungen uber die optischen Fehler des Auges.* Wien, 1876.—Landolt (E.). .*La réfraction et l'accommodation de l'oeil* in Wecker and Landolt. *Traité complet d'ophthalmologie.* Paris, 1883.—Boehm (L.). *Das Schielen.* Berlin, 1845.—Arlt (F.). *Die Krànkheiten des Auges,* I-III. Prag., 1851. —Stellwag v. Carion. *Die Ophthalmologie vom naturwissenschaftlichen Standpunkte aus.* I-II. Erlangen, 1853.—Tscherning (M.). *Studien uber die Aetiologie der Myopie. Arch. f. Opth.,* XXIX, I, 1883.—Dimmer (F.). *Zur Glaesercorrection bei Aphakie.* Kl. M. f. A. 1891.—Ostwalt (F.). *Einige Worte uber Glasercorrection bei Aphakie.* Kl. M. f. A. 1891.—Demicheri (L.). *Faux lenticone.* Ann. d'oc. 1895.

CHAPTER VII

SPHERICAL ABERRATION

50. Optic Principles.—When the aperture of a spherical lens is not very small, the rays proceeding from a point of the object do not, after refraction, reunite exactly at a point, as would be essential to form a good image; the borders of the lens are more refracting than the center. Thus the test case lens, the center of which has a refraction of 20 D., refracts 25 D. towards the borders. Generally speaking, the same is true of all refracting and reflecting systems (fig. 61). It is possible, nevertheless, to

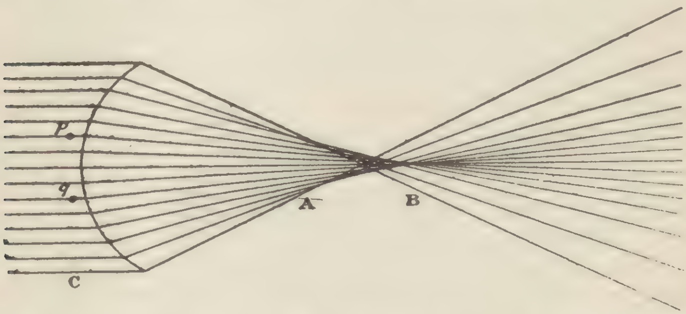


Fig. 61.—Refraction of a pencil of parallel rays by a spherical surface. Spherical aberration. At A, the rays are condensed towards the border; at B, towards the axis of the pencil; *p*, *q*, two needles.

construct systems of large aperture, which present only very little aberration (aplanatic lenses), and others in which the aberration is over-corrected, the borders being less refracting than the center (*lentilles suraplanétisées*).

The degree of aberration increases as the square of the aperture of the lens and as the cube of its refracting power. It depends, besides, on the distance of the object and the form of the lens. A plano-convex lens presents less aberration than a biconvex lens, if the spherical side is turned towards the incident rays supposed to be parallel; it presents more in the contrary direction. It is for this reason that the objectives of opera

glasses are bulged in front. The best form of simple lens is that which the English call *crossed lens* (periscopic), in which the radius of the posterior surface is about six times greater than that of the anterior surface. We give here the refracting power, at 15 millimeters from the axis, of different lenses, all having at the middle a refraction of 20 D. The incident rays are supposed to be parallel.

Crossed lens.	Plano-convex with the convex surface in front.	Bi-convex.	Plano-convex with the plane surface in front.
21.1 D.	22.3 D.	23.6 D.	23.8 D.

It is evident that, the weaker the aberration of the lens, the more aperture can be given to it without the aberration interfering with the distinctness of the image. The crossed lens is little used, because the plano-convex lens is nearly as good. Besides, for the correction of chromatic aberration, compound lenses are usually employed (a *flint* lens and a *crown* lens cemented together). Both glasses can then be cut in such a way as to neutralize the spherical aberration also, until the total aberration becomes almost nothing for a given distance of the object.

51. Phenomena Dependent on the Spherical Aberration of Lenses. —I am going to explain some experiments by which the spherical aberration of lenses may be studied. In order to have very marked phenomena we must use a strong lens, 20 D. (convex) of the test-case, for example, or, better still, a strong plano-convex lens (the objective of an opera glass), the plane side of which is turned towards the luminous source, placed at a great distance.

a. Application of the Principle of Scheiner.—We place on the lens an opaque screen in which we have previously made, not two apertures as in the experiment of *Scheiner*, but four, which are equidistant, placed on the horizontal diameter of the lens, two central ones, 2 and 3, and two peripheral, 1 and 4 (fig. 62). The object being a distant luminous source, we receive the images on a white screen placed behind the lens.

First, placing the latter beyond the focus, we see (fig. 62 A) four luminous spots which correspond to the apertures of the screen, but which are placed in reverse order. The distance be-

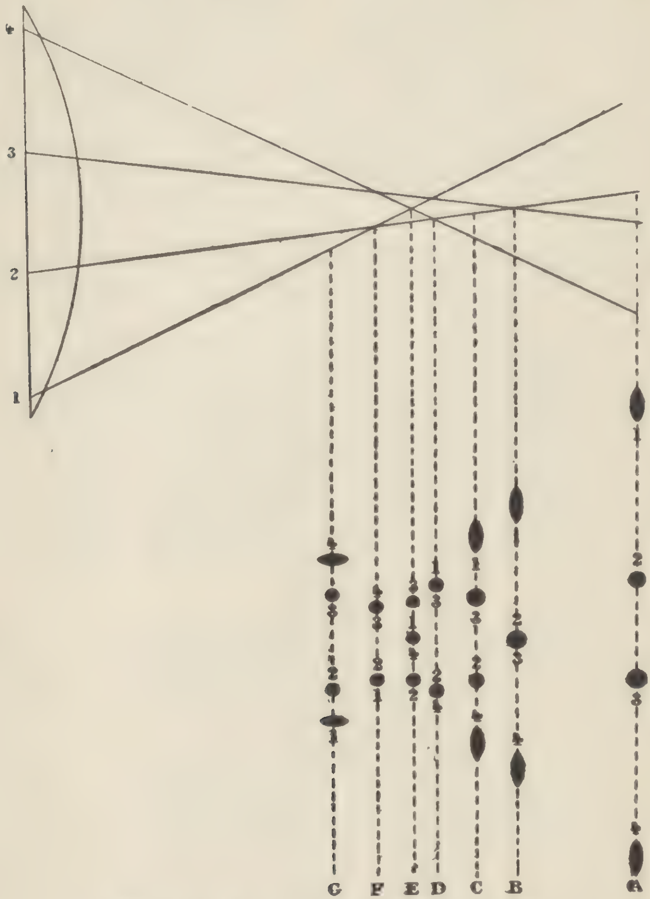


Fig. 62.—Spherical aberration of a lens.

tween the central spots is less than that which separates each of the peripheral spots from the neighboring spot. The two central spots reproduce the form of the source enlarged, while the two peripheral spots are elongated in the horizontal direction, especially if the aberration is strong. The pencils passing

through the peripheral openings are, indeed, *astigmatic by incidence* (see ch. IX). By moving the screen nearer, the two central spots are blended into one (fig. 62 B). At this moment the screen is at the focus of the central part of the lens, while it is still beyond the focus of the peripheral parts. Advancing the screen still more, the spots 1 and 4 approach and are blended (fig. 62 E, focus of the peripheral part), while spots 2 and 3 are again separated. Finally we have four spots, as at the beginning of the experiment; but they are now arranged in the same order as the apertures; the distances separating the two spots on each side are less than the distance between the central spots. We observe also that the peripheral spots are now elongated in the vertical direction.—If the lens is very large we can observe all the different phases shown on fig. 62.

To determine the degree of aberration, we have only to measure the distances of the positions E (focus of the peripheral parts) and B (focus of the central part) from the screen. The difference between these two distances, expressed in dioptries, tells the degree of aberration. To have more accurate measurements it is advisable to cover, each time, the two apertures we are not using; for the determination of E, we cover the central aperture, for that of B the peripheral apertures.—We can also cover the two apertures situated on the same side and determine the focal distance on the other side (the position F, fig. 62), but it is not necessary in order to determine the course of the rays: we can, indeed, construct figure 62 by knowing the positions B and E only.

b. Examination of the Circles of Diffusion.—Examining the circle of diffusion, without putting the screen with the openings on the lens, we see that as long as the white screen is situated beyond the focus, the light is concentrated at the middle of the circle; the brightness diminishes rapidly towards the borders. When it is situated within the focus, we see, on the contrary, a luminous disc surrounded by a more brilliant circle. This phenomenon is easy to understand: we see, in fact, in figure 62, that the rays are condensed towards the border,

between the lens and the focus, while they are concentrated around the axis beyond the focus.

c. Deformity of the Shadows.—Put the white screen beyond the focus, and place a knitting needle against the lens. We then see the shadow of the needle in the circle of diffusion and observe that this shadow is straight only if the needle coincides

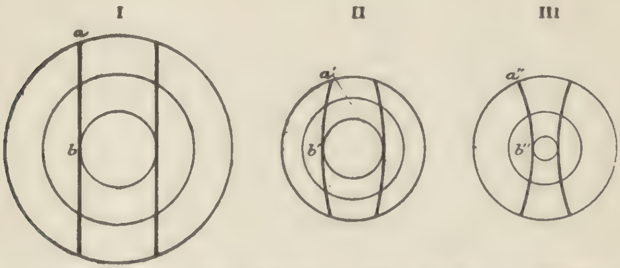


Fig. 63.—Deformation of the shadows of the needles. Successive sections of the pencil of figure 61. Section I is supposed to be made at C (fig. 61), section II at A, section III at B, the two latter enlarged; *ab*, a needle; *a′ b′* and *a″ b″*, its shadows.

with a diameter of the lens; otherwise it is curved, with its convexity towards the center. If the screen is between the focus and the lens, the shadow is concave towards the middle, but the curvature is much less pronounced.

To understand these deformities let us suppose the lens divided into concentric zones of the same width. A glance at figure 62 shows that after refraction the corresponding zones of the circle of diffusion diminish in width towards the periphery, when the screen is situated between the focus and the lens, while they increase in width towards the periphery beyond the focus. In figure 63, I shows the lens seen from the front and divided into concentric circles; the two straight lines represent two needles. In figure 63, II represents a circle of diffusion between the lens and the focus. We see that the zones become narrower towards the edge, and we understand that the point *a′* is relatively nearer the center than the point *b′*, which gives the shadow its curved form. Knowing the position of the concentric circles of the diffusion spots, it is easy to construct the form of the shadow,

since the shadow of a point of the needle must be at the same angular distance from the horizontal diameter as the point itself. In figure 63, III represents a circle of diffusion beyond the focus.

An over-corrected lens gives all the phenomena here mentioned, but in the reverse order, while a corrected lens (*aplanatic*) gives none of them. The circles of diffusion of an aplanatic lens have the same brightness in their whole extent, and the shadow of the needle remains straight everywhere. To give a good image a lens must be approximately aplanatic. The preceding experiments can be used as a verification of the aplanatism of a lens.

d. Application of the Principle of Foucault.—We obtain very pretty phenomena by using the method by which Foucault studied his telescopes. We place a luminous point a little beyond the focus of the lens which we wish to study, so that its image is quite distant (2 to 3 meters). The observer takes his place beyond this image, so that his eye is in the luminous pencil on the axis of the lens, which he approaches gradually. Under these circumstances the eye sees luminous the parts of the lens which send rays to it. If the lens were aplanatic, all the rays would meet at the focus, and, reaching this point, the observer ought to see the entire lens luminous. At some distance from the focus, he would see, on the contrary, only a small central part luminous, the other rays not entering his eye. If the lens is affected with spherical aberration, we observe the following phenomena: placed very far off we see only a quite small central spot, which increases in diameter accordingly as we approach the focus where it attains its maximum; but even here it is far from filling the entire lens. Approaching still nearer we see a luminous ring become detached and separated from the central part by a dark zone. This ring dilates more and more accordingly as we approach the lens, while the dark zone becomes enlarged. On reaching a certain point, the ring extends to the borders of the lens and disappears. The phenomena are still clearer if we look through a narrow diaphragm.—It is easy to account for the nature of these phenomena by glancing at figures

61 and 62. Thus, if we suppose the pupil of the observer reduced to a point and placed at the intersection E, fig. 62, it would receive rays 1 and 4, and the borders of the lens would appear luminous, while the parts 2 and 3 would be black, the corresponding rays passing to one side of the pupil. There will always be a small, luminous spot at the middle, since the axial ray always enters the eye. The distance, in dioptries, between the place where the ring appears and that where it disappears, tells the amount of the aberration.—If the aberration is overcorrected we have the same phenomena in the reverse order: placed at the focus, we must move away in order to see the ring; the further away we move the more it increases, until finally it disappears.

52. Aberration of the Human Eye. Experiments of Volkmann.—This scientist examined the aberration of the eye by repeating the experiment of *Scheiner* with four openings located as indicated in figure 64, C. Looking at a pin placed beyond the *far point* through these openings, it is seen quadrupled (fig. 64, A, *a*); and by moving closer to it he observed the different phases illustrated in figure 64, A, in the order in which they are shown in the figure, and which corresponds to the spherical aberration. It is easy to account for this phenomenon by comparing figure 64 with figure 62. In the position *b,* the pin is at the *far point* of the central parts of the pupil, since the two central images are reunited; it is still beyond the focus of the peripheral parts since the peripheral images are not yet blended. Most of the time, the persons examined observe the same phe-

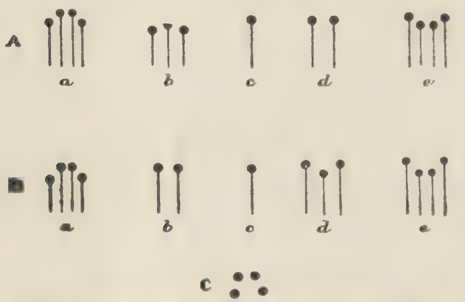
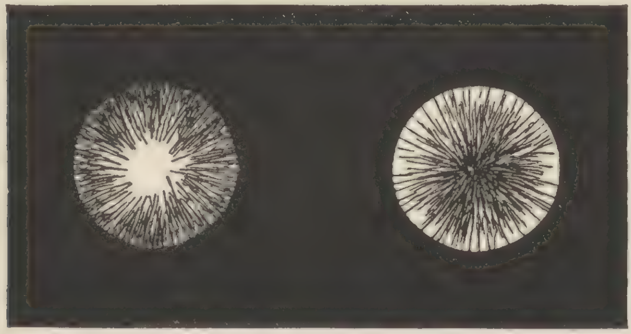


Fig. 64.—Experiment of *Volkmann.*—*a,* corresponds to the most distant position; *e,* to the nearest position of the needle. A, phenomena observed by an eye with strong spherical aberration; B, by an eye with over-corrected aberration.

nomena in the same order, but some see them in the reverse order (fig. 64, B), which indicates over-corrected aberration. In the position *d* (fig. 64, B) the pin is at the *far point* of the central parts and within the far point of the peripheral parts.—It is probable that these latter persons used their accommodation, for it is quite rare to find over-corrected aberration in an eye in a state of repose; I have, however, met instances, especially among persons having a large pupil. On the contrary, during accommodation, it is the rule that the aberration is over-corrected, as we shall see later on.

53. Experiments of Thomas Young.—Long before *Volkmann's* time, *Young* had already performed a series of experiments much more conclusive, but which had been forgotten.

a. A myopic eye sees a distant luminous point as a circle of diffusion, the brightness of which is concentrated at the middle, if the eye has spherical aberration (fig. 65, I). If the aberration is over-corrected, or if the luminous point is inside the far



I II

Fig. 65.—Distribution of the light of the circle of diffusion in an eye with strong aberration (Antonelli). In I the luminous point is beyond; in II within the focus.

point, it is the borders that are the more luminous; an aplanatic eye, or one nearly so, sees the circle of a uniform brightness. To repeat the experiment, when one is not myopic, one places in

front of the eye a convex lens of 3 to 4 dioptries. Many eyes, the optic system of which is irregular, perceive eccentric concentrations of the light; I shall return to this immediately. (1)

b. Bringing a needle in front of the eye, made myopic, while the experiment *a* is being performed, we see the shadow of the needle in the circle of diffusion. If the shadow remains straight everywhere, there is no perceptible aberration; if it is curved, its concavity towards the periphery indicates ordinary aberration; its concavity towards the center indicates

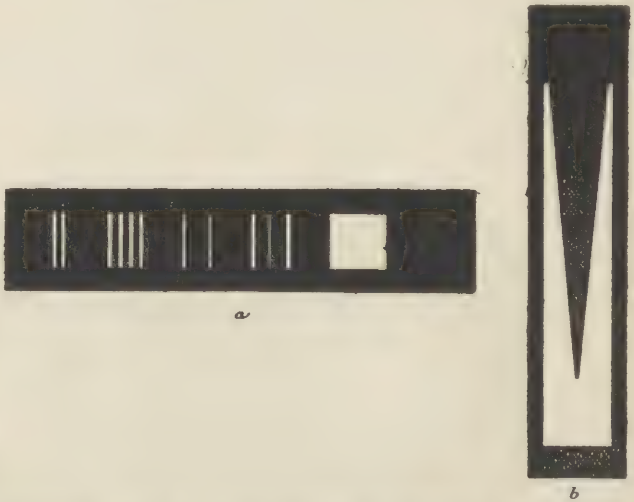


Fig. 66.—The aberroscope. Fig. 67.—The rules of the optometer of Young.

over-corrected aberration. We can perform the experiment in the different meridians and thus prove that the aberration is not always the same in the different directions.

I have constructed a little instrument, the *aberroscope* (fig. 66), consisting of a plano-convex lens which, on its plain side,

---

(1) *Young* does not mention the experiment under this form, but it is a sequence of other experiments which he describes. For the experiment *b*, he used the bars separating the four slits of his optometer.

carries a micrometer in the form of little squares. We look at a distant luminous point through the lens, moving it 10 or 20 centimeters from the eye in order to observe whether the lines then appear curved or not.

Fig. 68.—I and II. The appearance assumed by the line of the optometer of *Young*, seen through four slits by one eye with strong spherical aberration. O, position of the eye; *a* (*a'*) *far point* of the peripheral parts; *b* (*b'*) *far point* of the central parts.

III. The appearance of the line, seen in the same circumstances by one eye (left) with marked obliquity. The external part of the pupillary space is more refracting than the internal part.

c. The Optometer of Young enables us to measure spherical aberration directly. In the horizontal rule (fig. 67), on the left, are two slits, very narrow and very close. We look at the line

through these slits and determine the central refraction by observing the intersection of the two apparent lines, as I have explained in chapter V. Care must be taken to place the slits so that both the lines appear of the same distinctness, which takes place when the slits are almost at the middle of the pupil. This done, we bring the quadrangular aperture in front of the lens, and gradually lower the vertical rule which has the triangular plate, so as to exclude a continually increasing part of the middle of pupil. We then see two intersecting lines which separate more and more, until one of them disappears at the moment when the width of the plate is equal to the diameter of the pupil. We then raise the rule a little, so as to again see two lines, and measure the refraction. The difference between this measurement and that made with the two slits placed at the center indicates the degree of aberration.

Young made two measurements at once by using four slits of the horizontal rule. The experiment thus performed is much more elegant and sure, but it is often difficult to succeed, especially if the pupil is not dilated. It is easier to succeed if the slits are brought together in pairs, leaving a central interval a little greater than that between the pairs.—With the four slits we see four lines (fig. 68, I); if there is spherical aberration the two central lines intersect farther away (at *b*) than the peripheral lines (*a*). Very frequently the lines partly blend, so as to give the appearance shown in figure 68, II. Figure 68, III, shows the appearance which the line assumes to an unsymmetrical eye (left), the external part of the pupil being more refracting than the internal.

We can also measure with the two slits the refraction at the middle of the pupil, as we did just before, and then displace the slits successively towards either border until one of the lines begins to disappear. We thus determine the refraction near the two borders. This experiment, by which we determine the position of the point *c*, figure 68, I, is analogous to that described

on page 118, in which we covered the two apertures situated on the same side of the lens to measure the refraction on the other side. The measurements made with the slits placed peripherally generally differ more from those obtained with the central slits than do the measurements made with the triangular plate, which is so also in the case of the lens.

Skiascopic Examination.—While the methods which we have just mentioned are quite delicate, skiascopy furnishes us with a convenient means of examining the aberration of the human eye. For this purpose it is necessary to use *skiascopy with a luminous point,* a method which has been with good cause recommended by *Jackson,* and which is nothing more than an application of the principle of *Foucault.* We observe the pupil, while we form a distinct image of a luminous point on the retina. We surround a flame with an opaque tube pierced with an opening of one centimeter diameter; it is the image of this opening that we project on the retina with an ophthalmoscope, and care must be taken in selecting the mirror so that this image may be distinct; in other words, so that the image of the opening formed by the mirror is near the place for which the observed eye is focused. If the observed person is emmetropic, we place the light at 50 centimeters or one meter behind him, and examine with a plane mirror. If he is myopic, we use, on the contrary, a concave mirror which projects the image of the luminous point near his far point. In all cases it is advisable that the opening of the mirror be quite small, about 2 mm. The pupil of the observed person must be dilated.

To examine the aberration, we make the observed person emmetropic, and, placing ourselves at 50 centimeters distance, we project a light on the eye. Generally we will see at once the phenomena of aberration: the borders of the pupil are luminous, separated from the central light by a dark zone. We approach until the ring disappears; if this takes place at 25 centimeters from the observed person, the aberration is positive and 4 D. If we do not perceive the ring, we move back as far as one meter;

if it does not yet appear, we try whether the aberration is over-corrected: we make the observed person myopic 3 D.; if the ring appears, we increase the myopia until it disappears. If it disappears with myopia of 4 D., the aberration is—2 D., since we must take off 2 D., the observer being at 50 centimeters. *Brudzewski,* who determined the aberration of a certain number of persons in this way, said that it is rare not to meet with positive aberration in some part of the pupil. It happens, indeed, quite often that the ring is incomplete, or even that there remains only a very small section of it. Negative aberration is met with most frequently inwards or upwards in the pupil where the corneal flattening begins soonest.

RESULTS.—Examined with the aberroscope most people indicate a certain degree of aberration, which corresponds closely to the nearly spherical (toric) form of the optic part of the

I II

Fig. 69.—Deformity of the shadows in an eye with strong spherical aberration (Antonelli). I, in a state of repose; II, during accommodation. In the latter case the aberration is nearly corrected.

cornea (fig. 69).—Since the peripheral parts of a spherical surface are too refracting, we can correct the defect by flattening it towards the periphery. We also sometimes find people whose aberration is corrected, or even over-corrected, towards the borders, where the basilar part of the cornea comes into play (fig. 70). And, if the pupil is placed a little eccentrically, we may thus find aberration in one direction and over-corrected

aberration in another (fig. 71). Thus the middle of my pupil is slightly myopic and the upper part slightly hypermetropic, while the lower marginal part measures a myopia of three dioptries, which may even reach four dioptries when the pupil is dilated. I have, therefore, spherical aberration below (and on both sides), over-corrected aberration above.—One of my friends, who is an astronomer, has aberration in the vertical meridian, while the horizontal meridian is corrected.

Some are met with who have slightly over-corrected aberration in the entire pupillary space (fig. 72). These are probably

Fig. 70.—Aberration over-corrected towards the borders.

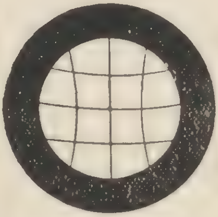


Fig. 71.—Aberraton over-corrected above

Fig. 72.—Aberra- over-corrected everywhere.

persons in whom the spherical part of the cornea is of little extent.—The ophthalmometric measurements of *Brudzewski*, which I have mentioned, page 72, enable us to calculate directly the degree of the aberration of the cornea. They show that there exist, in this regard, considerable variations. Corneal aberration is, as a rule, positive, negative aberration being rather an exception. Positive aberration is especially pronounced in cases of corneas of great curvature which is not surprising, since the aberration increases in very close proportion to the central refraction. Negative aberration is met with most frequently on the inner side, sometimes above or below, very rarely outside.

The greatest degree of aberration which *Brudzewski* found was + 4.5 (temporal side), the least —2.2 D. (nasal side). Generally it varied between +3 and —1.5. The numbers are calculated for a distance of 4 mm., starting from the axis; they correspond, therefore, to a maximum dilation of the pupil; the values diminish as we approach the axis.

Stadfeldt measured the aberration of the dead crystalline lens by the method of *Foucault*. When the crystalline lens was taken from the eye, in its capsule and with the zonula, he fixed it in a cork ring which he then placed in a small tube filled with serum and closed in front and behind by plane parallel plates of glass. He placed this tube on the support A (fig. 72*a*), which moved along the graduated rule E D. The lens C concentrated the light of a flame on a very fine opening pierced in the screen B D.

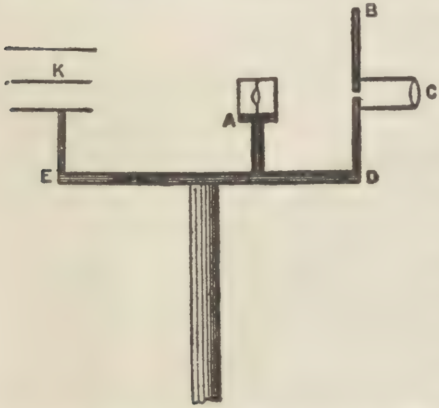


Fig. 72*a*.—*Stadfeldt's* instrument for measuring the aberration of the crystalline lens (dead).

The crystalline lens was observed with a telescope, placed at some distance in the direction K; an ocular micrometer permitted the measurement of the diameter of the aberration ring, corresponding to a given distance between A and the plate B D.—The determination of the focal distance of the central part is less exact by this method. To have a more exact measurement, *Stadfeldt* removed the plate B D, and placed a microscope of slight magnifying power in the tube K. He then sighted towards an object placed at a great distance. By displacing the cursor A, leaving the microscope motionless, he put the latter in focus, first for the image of the distant object formed by the crystalline lens, and then for the posterior surface of the crystalline lens itself. The difference between the two positions of the

cursor A enabled him to calculate the focal distance of the crystalline lens.

By these methods *Stadfeldt* proved that a central part of the crystalline lens (up to a distance of 2 mm. from the axis) may be considered as aplanatic. This part is surrounded with a zone (up to 3.5 mm. from the axis); the aberration of which is over-corrected (about 2 D.). Very close to the borders the aberration changes sign and becomes positive. The over-correction is due to the diminution of the index towards the periphery, but very close to the borders the increase of curvature of the surface is so great that the diminution of the index is not sufficient to correct the aberration.

Although aberration may sometimes be very pronounced, it does not seem to hurt the visual acuity much as long as it continues entirely regular, a remark which *Graefe* made on the occasion of his celebrated case of aniridia. The reason is that patients do not use the part of the cone of which the diameter is smallest, but another part near B, figure 61. Placing a screen at this place, the image of a point is presented as a point surrounded with a slightly luminous halo; if the brightness of the object is feeble, as is most frequently the case in the ordinary circumstances of life, this halo is too slight to be perceived, and the image becomes quite good.—We see (fig. 61) that a section of the caustic (the most luminous part of the cone) has the form of the head of an arrow. The point of the arrow is directed backwards in eyes with ordinary aberration and forwards in those with over-corrected aberration; it corresponds to the focus of the central rays, and it is this point which serves for vision; but, as it is very pointed, it follows that the determination of the refraction cannot be of very great exactness. The spherical aberration acts, in this regard, as a narrow diaphragm. If a lens is diaphragmed much it becomes very difficult to determine its focus exactly.—Thanks to this form of the caustic, very regular eyes can have a very beautiful visual acuity despite a strong aberration; but, in most eyes, the refraction is irregular, so that patients have not this advantage (see chapter X). I

think, however, that they generally select the place where the section of the caustic is smallest, and not that where the cone has the least diameter.

Bibliography.—*Œuvres de Th. Young*, p. 153.—Volkmann (A. W.) in *Wagner. Handworterbuch der Physiologie*, Art. *Sehen*, p. 292.—Meyer (H.). *Ueber die spharischen Abweichungen des menschlichen Auges.* Poggendorfs Ann. LXXXIX, p. 540.—Tscherning (M.). *Die monochromatischen Abweichungen des menschlichen Auges. Zeitschr. f. Physiol. der Sinnesorgane*, VI, p. 456.—Stadfeldt (A.) and Tscherning (M.). *Une nouvelle méthode pour étudier la refraction cristallinienne, Arch. de physiol.*, July, 1896.—Jackson. *Skiascopy.* Philadelphia, 1896.—Stadfeldt (A.). *Recherches sur l'indice total du cristallin humain. Journal de Physiologie*, November, 1899.—Brudzewski (K.). *Beitrag zur Dioptrik des Auges. Archiv. ur Augenheilkunde*, XL, 3.

CHAPTER VIII

CHROMATIC ABERRATION

54. Optic Principles.—By receiving on a screen a pencil of white rays which, after having passed through a slit, has traversed a prism, we obtain what is called a spectrum, a luminous band containing the entire gamut of the colors of the rainbow, arranged in the following order: red, orange, yellow, green, blue, violet. Each white ray is divided into colored rays which are refracted differently, the red the least, the violet the most, which we express by saying that the index of refraction of the glass is greater for the violet. If we speak of the index of a medium, without more particular specification, it is generally the index of the yellow rays (the sodium line) that is meant.—The difference between the index of the violet and that of the red is called the *dispersion* of the medium. Instead of receiving the spectrum on a screen, we can observe it directly by looking at the slit through the prism. For this observation the prism is frequently combined with an astronomical telescope (spectroscope).

In order that the spectrum may be really pure we must: 1° make use of a very narrow slit; 2° interpose a lens so that the rays of each color may be reunited on the screen in a distinct image of the slit. The spectrum is, therefore, in reality composed of a whole series of images of the slit; if these images are not distinct they are partly overlapped and the colors are not pure. —To obtain a very great purity of colors, special precautions must be used: we project the spectrum on a screen pierced by a slit at the place where the color we desire to examine is formed. Through this slit an eye situated behind the screen receives the light of this color, mixed with a little white light, due to diffusion in the substance of the prism and lens. To eliminate this white light, we observe the slit through a second

prism. It forms a spectrum which is very weak everywhere, except at the location of the color we desire to examine (*Helmholtz*).—The length of the spectrum depends on the size of the angle of the prism and on the degree of dispersion of the glass: a prism of *flint* glass produces a spectrum much longer than a prism of *crown* glass.—Beyond the red there are *ultra-red* rays, which are invisible, but which have a greater caloric effect than the visible rays. Beyond the violet rays there are likewise *ultra-violet* rays, which, in ordinary circumstances, are invisible, but which act on photographic plates. They can be made visible by overlaying the screen with a "fluorescent" liquid (sulphate of quinine, fluorescence, etc.). Struck by the ultra-violet rays, these substances send back visible rays, generally bluish or greenish.—With certain precautions we can see directly a part of the ultra-violet rays, perhaps because the retina itself is fluorescent. Thus *Mascart* mentions a physicist who could distinguish the lines of *Fraunhofer* in the ultra-violet part of the spectrum as far as the photographic plate could reproduce them. We cannot make the ultra-red rays visible because they do not pass through the media of the eye (*Bruecke*).

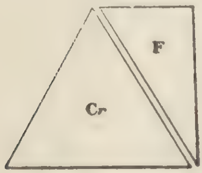


Fig. 73.—Achromatic prism.

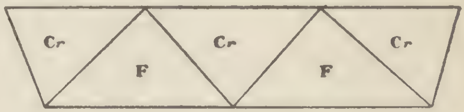


Fig. 74.—Prism *a vision directe*.

Generally, the media which have a greater index have also a greater dispersion, (1) but the index and dispersion are not

(1) This assertion is true for the glasses which we generally use, but not for the new glasses manufactured by *Abbe & Schott* at Jena since 1886. They succeeded in making one part of crown glass (with baryta basis) which has scarcely any more dispersion than the ordinary crown glass, but the average index of which is equal to that of very dense flint, and another part, of crown glass, with low index and relatively high dispersion. The new glasses are imported for the manufacture of microscopic objectives (apochromatic systems, see the following page) and also for photographic objectives. Under the name of *isometropic glasses,* they have been used for spectacle-making purposes, but, in this respect, they present no advantage.

proportional. Thus *flint* glass, for example, gives a dispersion nearly double that of *crown* glass, while its index is 1.7 and that of crown 1.5.—If we combine a prism of crown glass with another of flint glass in an inverse manner, the angle of which is nearly a half less, the dispersion may be neutralized, while there remains a quite considerable part of the refraction of the crown glass. Such a combination constitutes an *achromatic prism* (fig. 73).

We can also construct combinations of prisms which give no deviation to the emerging ray, but which have a quite considerable dispersion: we call these combinations prisms *a vision directe* (fig. 74); they are much used for the construction of spectroscopes.

By passing through a lens the colored rays are also separated. As the index is stronger for the blue rays (violet), the blue focus is nearer the lens than the red focus. This is the reason why the circle of diffusion of a convex lens is bordered with red inside the focus and with blue beyond.—Lenses may be made achromatic by the same system as prisms: a convex lens of *crown* glass is combined with a concave lens, half as strong, of *flint*. The circles of diffusion of such a lens no longer present red and blue borders, but there still remains traces of other colors (green and purple). *Zeiss* at Jena caused these latter to disappear also by combining several glasses of different kinds, specially manufactured for this purpose (apochromatic systems).

55. Chromatic Aberration of the Eye.—The eye is not achromatic as was for a long time believed. The question has played quite a curious part in the history of optics. *Newton* thought that the dispersion of a medium was proportional to its index and that, consequently, the construction of an achromatic objective was a chimera; this is why, forsaking astronomical telescopes, he adopted catoptric telescopes. But *Euler* concluded that, the eye being achromatic, it must be possible to construct achromatic lenses, and this remark led *Dolland,* the optician,

to construct objectives thus corrected. Later *Wollaston* demonstrated that the eye is not achromatic. This is not the only time that useful results have been arrived at by starting from a false hypothesis.

56. Experiment of Wollaston.—A luminous point seen through a prism gives a linear spectrum. But, making the experiment, we observe that we cannot see distinctly at once the entire extent of the spectrum. If the luminous point is at a great distance, the emmetropic eye sees the red extremity of the spectrum as a distinct line, while the blue extremity is enlarged and frequently divided into two ("like the tail of a swallow"). If we go nearer, taking care not to use our accommodation, we find a distance at which we are focused for the blue extremity, while the red extremity is, in turn, diffuse. The observer can, therefore, determine his *far point* for each extremity of the spectrum; the difference gives the degree of chromatic aberration.

Wollaston has likewise directed attention to another phenomenon of chromatic aberration: the colored borders which are seen along the lines of the optometer of *Young*.

EXPERIMENTS WITH THE COBALT GLASS.—Placing a luminous point, such as an opening in an opaque screen, inside the *near point,* we see a circle of diffusion bordered with red exactly as when we made the analogous experiment with the lens; it is more difficult to see the blue border which surrounds the point, when it is situated beyond the *far point.* The experiment is much more striking when the point is observed through a cobalt glass. These glasses allow only the blue and red rays to pass; looking at a luminous point situated inside the *near point,* through such a glass, we see it blue and surrounded by a red halo. If the luminous point is situated beyond the *far point,* we see, on the contrary, a red point surrounded with blue.

EXPERIMENTS OF FRAUNHOFER.—This scientist determined the distance at which he could see distinctly a spider thread placed sometimes in the red light, sometimes in the blue light of the spectrum. We thus obtain very exact results.

57. Results.—*Young* estimated the chromatic aberration of the eye at 1.3 D., *Fraunhofer* found 1.5 to 3 D., *Helmholtz* gives 1.8 D. The number is difficult to determine exactly, since the lowest limit of the visible spectrum is not well defined.—The dispersion of the eye is a little greater than it would be if the eye were filled with water.

The eye, therefore, is not achromatic, and, as we have seen, it is easy to convince oneself of it when the object is situated beyond the *far point* or within the *near point*. But when the object is at such a distance that it can be seen distinctly, we do not see colored borders. The explanation which is given of this

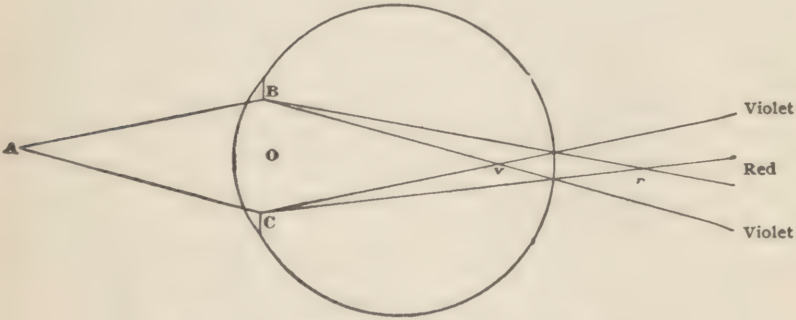


Fig. 75.—Chromatic aberration of the eye.

fact is the following: Let A (fig. 75) be a luminous point which sends the cone ABC into the eye. After refraction the white rays are divided into colored rays; the red rays form the cone B*r*C, the violet rays, which are more refracted, the cone B*v*C, and the eye accommodates itself in such a way that the retina is between the two foci, placed so that the red diffusion circle covers the blue one (see fig. 75). The intermediary rays of the spectrum, the yellow and the green, which are the most luminous, are then concentrated at the middle of the diffusion circle, where they coincide with a part of the red and a part of the violet, while the peripheral parts of the red and violet form a purple border all around; but this border is very narrow, and, as it is formed by the extreme rays of the

spectrum, which are very slightly luminous, it is too weak to be perceived.—When observing a luminous point with an astronomical telescope, the objective of which is not very well achromatized, the same phenomena are seen: if the telescope is focused for a nearer point, the circle appears surrounded with blue; in the contrary case it is bordered with red, and, when the point is seen distinctly, it is surrounded by a very narrow purple border. —The same thing occurs if the point A be replaced by a white object: in the latter case we do not see colored borders.

58. Phenomena of Dispersion, the Pupil Being Partly Covered. —It is different if a part of the pupil be covered by a screen. Let us fix, for example, the sash bar of a window through which we see the sky. Covering the right half of the pupil with a screen, we see the border *aa* (fig. 76) become colored blue, the border *bb* yellow. In order to explain this fact let us examine the point *a,* the last luminous point of the window on the right, and suppose that the point A in figure 75 is this point: by covering the half (BO, fig. 75) of the pupil, instead of a circle of diffusion uniformly illuminated by violet and red, we have a circle the right half of which is violet and the left half red. This latter half is covered by the circle of diffusion of the following point of the window on the right, and is not visible; there remains, therefore, a blue border (violet) along the sash bar. Of the point *b* it is, on the contrary, the red half (yellow) of the circle of diffusion which is not covered.—We frequently observe very striking phenomena due to the chromatic aberration of the eye, by fixing black objects on a white ground, placed at a distance for which the eye cannot accommodate itself. Looked at towards the sky, the slits of the optometer of Young present thus very vivid colorings.—The chromatic aberration increases with the diameter of the pupil. To study it, it is useful, therefore, to make use of mydriatics.

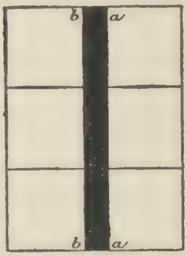


Fig. 76.

59. Correction of the Chromatic Aberration.—We could correct the chromatic aberration of the eye with a concave lens of *flint,* exactly as we can correct the chromatic aberration of a convex lens of crown glass. The dispersion of flint glass is about three times that of the eye. As the refracting system of the eye is about sixty dioptries, a concave flint lens of about twenty dioptries would be necessary to correct this aberration. A myope of twenty dioptries, who would correct his ametropia with a flint lens, would have, therefore, at the same time corrected his chromatic aberration. An emmetrope would be obliged to add to this lens a convex achromatic lens of twenty dioptries to remain emmetropic. The attempts which have been made in this direction (*Helmholtz, Javal*) have not given a very marked improvement of the visual acuity.

Bibliography.—*Œuvres de Young,* p. 154.—Wollaston, *Phil. trans.,* 1801, p. 50.—Fraunhofer (J.) *Gilberts Ann.,* LVI, p. 304.—v. Bezold (W.). *Graefes Arch. f. Ophth.,* XIV 2, p. 1.

CHAPTER IX

REGULAR ASTIGMATISM

60. Optic Principles. Astigmatism Produced by the Form of the Surfaces.—To account for the form of the astigmatic pencil, the following experient may be made. We combine a convex cylinder, with its axis horizontal, with a convex spherical lens; the combination of +3 cyl. with +6 sph. answers very well.

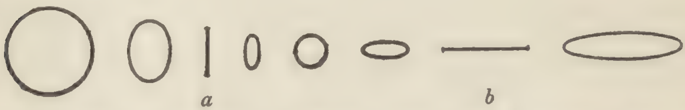


Fig. 77.—Circles of diffusion and focal lines of a regularly astigmatic system. After *Fuchs.* (In order that the figure may agree with the text, we must suppose the first focal line *a* horizontal, the second *b* vertical.)

The pencil, which emanates from a distant luminous point and is refracted by the sphero-cylindrical combination, is received on a screen which is gradually moved away from the lens. Then, instead of a circle of diffusion, the diameter of which diminishes according as the screen is removed in order to become a point when the screen is at focus, and to again become circular beyond, we obtain the forms illustrated on figure 77.

The two straight lines are called *focal lines;* the distance which separates them is called *interfocal distance,* and the meridians of the optic system to which they correspond are the *principal meridians.* Together the rays no longer form a cone in which all the rays pass through a point, but a more complicated system, characterized by this peculiarity, that all the rays pass through two short straight lines perpendicular to each other (the focal lines). The system is known as the conoid of *Sturm.*

The first focal line is at the focus of the meridian of greatest refraction (in our case, the vertical meridian); it is parallel to

the meridian of least refraction; the second focal line is at the focus of the meridian of least refraction and parallel to the meridian of greatest refraction. The diffusion spots are everywhere elliptical, except at one point of the interfocal distance where the luminous spot is circular.

In the principal meridians, refraction takes place as if the lenses were spherical; an incident ray parallel to the axis cuts the latter at the focus of the meridian. The rays which are not situated in the principal meridians do not meet the axis; their course will be indicated later on.

The length of the focal lines is proportional to the distance of these lines from the lens. Let F′ (fig. 78) (1) be the distance

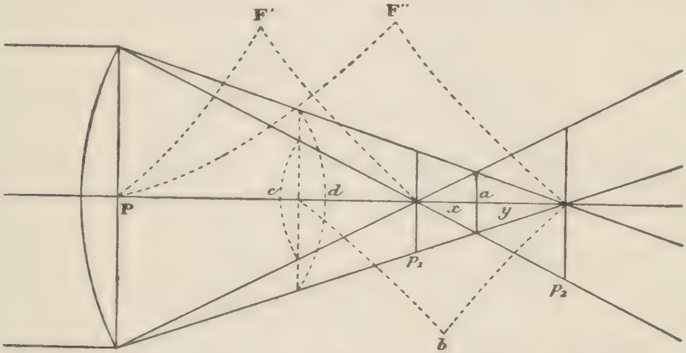


Fig. 78.—p_1, horizontal focal line; p_2, vertical focal line.

of the first focal line, F″ that of the second, P the diameter of the lens, p_1 and p_2 the lengths of the two focal lines. Then we have

$$\frac{p_1}{P} = \frac{F'' - F'}{F''} \text{ and } \frac{p_2}{P} = \frac{F'' - F'}{F'}; \text{ consequently by dividing}$$

$$\frac{p_1}{p_2} = \frac{F'}{F''}$$

The circle of circular diffusion is at *a,* where the diameters are equal. *It divides the interfocal distance into two parts,*

(1) We must suppose that the vertical meridian has been made to rotate 90° around the axis, so as to be able to draw the two focal lines in the same plane.

which are proportional to the focal distances. For, designating the diameter at this place by a, and the two parts of the interfocal distance by x and y we have:

$$\frac{a}{p_1} = \frac{y}{x + y} \text{ and } \frac{a}{p_2} = \frac{x}{x + y}, \text{ therefore, by dividing,}$$

$$\frac{y}{x} = \frac{p_2}{p_1} = \frac{F''}{F'}$$

All the other diffusion spots are ellipses, of which it is easy to calculate the axes. Placing a screen at a distance b from the second focal line, we see (fig. 78) that the axes c and d of the ellipse are found by the equations $\frac{c}{P} = \frac{b - (F'' - F')}{F'}$ and $\frac{d}{P} = \frac{b}{F''}$, equations which give as the relation between the axes:

$$\frac{c}{d} = \frac{b - (F'' - F')}{F'} \times \frac{F''}{b}$$

Knowing the axes we can find the ellipse by construction (fig. 79). We make a circle with half the long axis d (fig. 78) as

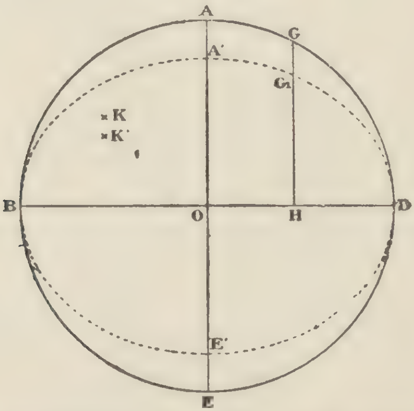


Fig. 79.—Construction of the elliptical diffusion spot.

radius, and draw therein two diameters, a horizontal BD and a vertical AE, and mark the points A′ and E′ so that OA′=OE′= $\frac{c}{2}$. BD and A′E′ are then the two axes of the ellipse, and we

can find any point whatever G_1, of the ellipse, by letting fall the perpendicular GH on the long axis, and marking the G_1 so that $\frac{G_1 H}{GH} = \frac{c}{d}$.

We can use this construction to find the course of the rays which are not situated in the principal planes. Suppose, indeed, that one of these rays passes through a given point of the lens. If the optic system were spherical and of the power of the meridian of least refraction, we would have a circle of diffusion of diameter BD, in which it would be easy to find the point K through which the ray would pass, since the circle would be only a diminished image of the lens. Having determined the position of the point K, we find the point K′ through which the ray really passes, by diminishing the distance of K from the long axis in the proportion $\frac{c}{d}$.

Application of the Principle of Foucault.—Let us place the luminous point a little beyond the focus of our sphero-cylindrical combination. The focal lines are then formed at quite a great distance. We receive the *horizontal* focal line on a screen which is then removed and the eye put in its place; we will then see a *vertical* luminous band which passes through the lens, while the parts on the right and left are dark. As we have already seen (page 119) the eye sees luminous the parts of the lens which send light to it, and it is easy to see that it receives under these circumstances all the luminous rays from the vertical meridian, while it does not receive rays coming from the lateral parts which intersect in other points of the horizontal focal line, to the right and left of the eye. Placing the eye in the vertical focal line we see a horizontal band.

61. Defects of the Image.—As the image of a point is never exactly a point, the image of an object can never be really distinct. Outside the focal lines, the outlines are all more or less dull. If the screen is at p_1, the horizontal lines only are distinct, if it is at p_2, it is the vertical lines that are distinct. The image is better at p_1 than at p_2, since the first focal line is the shorter.

With a cylinder which is strong compared with the spherical glass, the image becomes so poor that it is unrecognizable; with +6 spherical combined with +3 cylindrical of our test case, it is impossible to form an image on a screen. If, on the contrary, we place this combination sufficiently far from the eye that the image may be seen inverted, this image is pretty good, because the pupil of the observer forms a diaphragm; but it is deformed, all the dimensions parallel to the meridian of greatest refraction being greatly diminished.

62. Astigmatic Surfaces.—We have so far obtained astigmatic refraction by a combination of spherical and cylindrical surfaces, but we can obtain the same result by refraction through a single refracting surface.—If the aperture is very small, this result is obtained with any surface whatever. (1) For, a small part of any surface always presents two principal meridians, perpendicular to each other, one of maximum and the other of minimum curvature. The incident rays, situated in these planes, remain there after refraction and go to meet the axis after refraction; the rays which are not situated in these meridians do not meet the axis, but pass through two focal lines, perpendicular to the axis and situated in the principal meridians.—Among the surfaces for which this is true, even for quite a large aperture, at least approximately, there are two specially noteworthy: the ellipsoid with three axes and the tore.

By rotating an ellipse around its long axis, we obtain an ellipsoid of revolution. And if we suppose that it undergoes a flattening in a direction perpendicular to the long axis, we obtain an *ellipsoid with three axes.* The luminous point must be on the long axis.—The two principal meridians are elliptical (as is every other section of this surface).

The *tore* is the surface which is obtained by making a circle rotate around an axis situated in its plane (*ab*, fig. 80). By cutting a part near A, we would have an astigmatic surface the

(1) We must except the plane, sphere, the part near the axes of the surfaces of revolution, and that near the points called umbilical of other surfaces, supposing the incidence normal. Otherwise, the refraction is always astigmatic.

principal meridians of which would be circular; one would have the same radius as the circle (R_1); the radius of the other (R_2) would be equal to the distance of the axis from the apex of the circle. The luminous point must be on the prolongation of AO.

Even with these surfaces a pure astigmatic action is not obtained, when the aperture is a little large. It is clear that on account of the spherical aberration the peripheral parts of the principal meridians of the tore must have a greater refraction than the central parts; also the astigmatism of a peripheral zone becomes greater than that of the central part, since the refraction increases more rapidly towards the periphery in the most curved principal meridian.—On account of the flattening towards the periphery, the aberration is less for the ellipsoid; one of the meridians may even be aplanatic for a distant object, but then the other meridian is either over-corrected or under-corrected, so that the astigmatic effect is never pure.

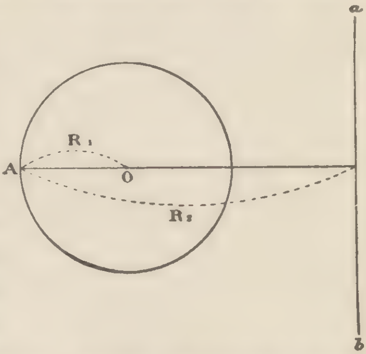


Fig. 80.—By the revolution around the straight line *ab*, the circle produces a torus.

63. Astigmatism by Incidence.—Let us place a spherical lens at some distance from a luminous point and form the image of this point on a screen; then make the lens rotate around a vertical axis. The screen immediately ceases to be at the point; we must move it nearer the lens, and we find at the same time that the refracted pencil is astigmatic. The horizontal focal line is farther from the lens than the vertical focal line. *The refraction has, therefore, increased in both meridians, but more in that which contains the axis of the lens and the luminous point.*

The focal lines are far from being distinct, especially if we do not use a small diaphragm. They are rather diffusion spots greatly lengthened in one or other direction.—But the pencil has *one* true focal line which, in our case, is horizontal; we find

it by making the screen rotate around a vertical axis, but in a direction the reverse of that of the lens (fig. 81).

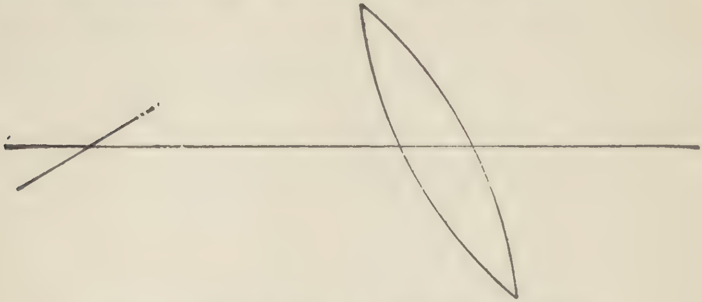


Fig. 81.—Focal line of a lens placed obliquely.

A pencil reflected or refracted obliquely by a spherical surface is also astigmatic by incidence. It is the same phenomenon which constitutes spherical aberration.

Let *cabd* (fig. 82) be an incident pencil parallel to the axis of a refracting spherical surface. Suppose that the pencil is

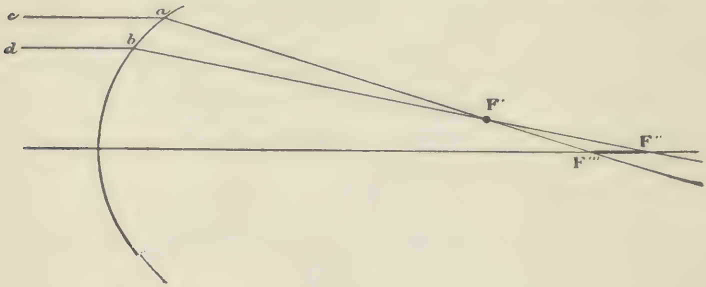


Fig. 82.

Astigmatism by incidence.—F′, first focal line; F″F‴, second focal line.

cylindrical, so that *ab* is the diameter of the small round spot which represents the aperture of the surface: *ab* is then one of the principal meridians and the diameter perpendicular to *ab* is the other. On account of the spherical aberration the ray *a*F′ meets the axis nearer the surface than the ray *b*F′. The first focal line, which is perpendicular to the plane of the paper, is at F′, for, if we imagine the entire figure rotating around the axis,

F′ describes an arc of a circle, a small part of which may be considered as a straight line, and it is easy to see that all the rays of our pencil must pass through this straight line (at least approximately). As, on the other hand, the rays must all meet the axis, F″ F″′ is the second focal line.—Here again the meridian of greatest refraction is that which contains the axis.

When the incidence is oblique, all the surfaces, the plane surfaces included, give astigmatism by refraction.

It is the same in the case of reflection, but then the plane surfaces are an exception. Ordinary mirrors are not exempt from this defect on account of the refraction through the thickness of the glass which is in front of the coating. The best images that we can obtain are those formed by reflection on a surface of mercury, especially when the layer is very thin: the pencil is not astigmatic at all. (1)

64. Astigmatism of the Human Eye. Historical.—This defect of the human eye was discovered by *Thomas Young* in 1801. He never noticed that his vision was defective, and claimed that he saw as well as most people. He proved the defect in his own eye by means of his optometer, and also by observing the forms of the circles of diffusion produced by a luminous point. He measured its degree by means of the optometer and expressed it, as we still do, by the difference of refraction of the two meridians. He had 1.7 D. of astigmatism against the rule. He proved that his astigmatism was not seated in the cornea, because, by performing his celebrated experiment of putting the eye under water and substituting a spherical lens for the cornea (see page 202), he found the same degree.—He attributed the astigmatism to the obliquity of the crystalline lens, which obliquity he thought much greater than it really is, and remarked that the defect could be corrected with glasses placed obliquely in front of the eye.

The astronomer *Airy*, a professor at Cambridge, was the first who corrected the defect by a cylindrical glass (1827). He had

(1) It is claimed, however, that we can still observe a trace of astigmatism, in this case, with the telescopes of the greatest magnifying power. This astigmatism might be due to the fact that the surface is not really plane on account of the spherical form of the earth.

high compound myopic astigmatism of the left eye, which he studied and measured by means of a luminous point.—Later, *Colonel Goulier* likewise studied this defect and prescribed cylindrical glasses to a certain number of patients.

It was only after the invention of the ophthalmometer by *Helmholtz* that the measurements of *Knapp* and *Donders* drew attention to this prevalent anomaly of the human eye. The works of these two investigators appeared almost at the same time, but those of Donders had greater influence. He was, in fact, the first to have cylindrical glasses put in the test case, which greatly contributed to their more general use. The methods used for the examination of patients were quite defective. The luminous point was especially used to find the meridians, and the refraction of each meridian was then measured by means of the stenopaic slit and spherical glasses.—A little later *Javal* introduced the examination by the star figure and cylindrical glasses.

65. Physiologic Astigmatism.—It is rare to find an eye completely free from astigmatism; but when the degree is slight, it scarcely affects the vision. We call this astigmatism physiologic. It is a disputed question at what degree we should begin to consider astigmatism pathologic; some have placed the limit at 0.5 D. or at 0.75 D., others at 1 D. or 1.5 D, In certain people we can improve vision with a cylinder of 0.75; others, on the contrary, experience no improvement, although they may have really the same degree of astigmatism. The aperture of the pupil, and especially the greater or less regularity of the astigmatic pencil, here play an important part. One of the best means of disclosing low degrees of astigmatism consists in observing the form under which a luminous point appears when placed at different distances. If the luminous point indicates a trace of astigmatism, we can generally also verify it by the star figure and a weak cylindrical lens, by placing the latter at first in the correct position and then in the contrary position. The patient then tells that the former position equalizes the lines better than the latter.

66. Corneal Astigmatism.—The principal seat of astigmatism is in the anterior surface of the cornea, which is not strange, since it is at this place that the principal change of index occurs. A deformity of one of the internal surfaces of the eye, which, at the anterior surface of the cornea, would produce considerable astigmatism, has only slight effect on account of the little difference of index of the media. The refraction is expressed, as we have seen, by $\frac{(n-1)\,1000}{R}$ (see page 16), that is to say, for the cornea, by $\frac{337.5}{R}$ and, for one of the internal surfaces, by $\frac{60}{R}$. The same deformity would, therefore, produce an effect five or six times less.

We may conceive also that, in the normal eye, astigmatism by incidence could scarely play any part, since the visual line passes approximately through the center of curvature of the cornea and through the middle of the pupil. It is otherwise in cases where there exists a considerable displacement of the pupil (corectopia), and especially in the case of an artificial pupil.—Under ordinary circumstances, therefore, it is the *form* of the anterior surface of the cornea that principally determines astigmatism; the examination of this surface thus plays an important part in the search for astigmatism.

67. Measurement of Corneal Astigmatism.—There exist different means of examining whether the cornea is astigmatic and of estimating the degree of its deformity (disc of *Placido,* keratoscope of *de Wecker* and *Masselon,* etc.); but, to measure it, one can scarcely think of using any other means than the ophthalmometer of *Javal* and *Schioetz,* which we have already described. The progress which it marks, compared with old ophthalmometers, consists especially in the facility with which we find the principal meridians by means of the difference in the level (*dénivellation*). If the arc is in a principal meridian, the images of the two mires must be on the same level and the black lines which are at the middle of the mires must be in the prolongation of each other. Outside the principal meridians there is a

difference in the level (*dénivellation*) greater in proportion as the astigmatism is more pronounced.

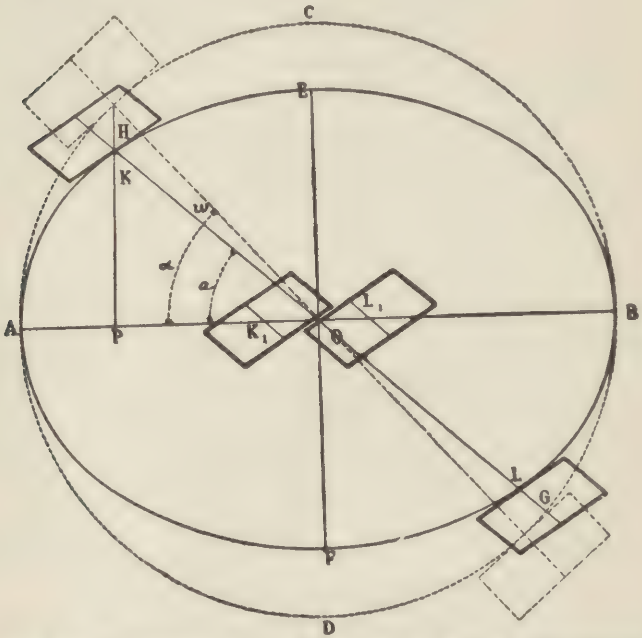


Fig. 83.—Explanation of the difference in the level (dénivellation).

To explain this phenomenon, let us examine a spherical cornea after having removed the doubly refracting prism from the instrument which then acts as a simple telescope. We then see only the images of the two mires, separated by an interval of about 3 millimeters. By rotating the arc these images describe a circle. Let ABCD, figure 83, be this circle to which the images of the mires always remain tangents. Let us replace the prism in position. Then the images are in the same meridian as the mires themselves, and as the doubling (*dédoublement*) of the prism takes place exactly in this meridian there is no difference in the level,—If we replace the spherical cornea by an astigmatic cornea, the vertical meridian of which is the more curved, the circle ABCD is replaced by the ellipse AEBF which is constructed as shown on page 140 by reducing the distance of each

point from AB in the proportion of the radii of the two principal meridians. By this construction the dotted diameter becomes the diameter KL, on which the images now are. The latter are, therefore, no longer situated in the meridian of the mires, and as the prism always acts in the direction parallel to this meridian, it follows that on obtaining contact the two images are not on the same level. Only when the arc is in one of the principal meridians the mires and their images are in the same plane and there is no difference in the level.

We can account for the difference between the image produced by a spherical cornea and that of an astigmatic cornea, by drawing on a sheet of paper a circle with two oblique diameters, perpendicular to each other, and observing the inverted image formed by a strong spherical lens held at some distance from the eye. The image is identical with the drawing; but if a convex cylinder with horizontal axis be added, the circle is replaced by an ellipse with the long axis horizontal, and the two diameters form between them obtuse angles above and below.

After having placed the ocular in focus for the spider thread, and then the instrument in focus for the eye, we begin by finding the meridian of least refraction. We place the mires in contact and make the arc rotate 90°. This done, the images of the mires partly overlap, and the number of gradations overlapped indicates the degree of astigmatism in dioptries.—If very exact measurements are desired, it is preferable to find each of the meridians separately, and to obtain contact in each of them. We read the refraction of each meridian on the arc, and the difference indicates the astigmatism.—We sometimes observe that the two principal meridians are not exactly perpendicular to each other; this is due to the relatively great distance between the mires; for, the principal meridians of a minute part of a surface are always perependicular to each other.—This is attributable to the fact that the meridians, instead of being plane sections of the cornea, possess a certain curvature.

68. Regular Corneal Astigmatism.—We distinguish between *direct* astigmatism or astigmatism with the rule, in which the

meridian of greatest refraction does not differ much from the vertical, and *perverse* astigmatism or astigmatism against the rule, in which the horizontal meridian is that of greatest refraction. If the direction of the meridians differs much from the horizontal and vertical directions, we say that the astigmatism is *oblique*.

Schioetz and *Nordenson* have compiled statistics on the direction of the corneal astigmatism in school children. Following are the results obtained by *Nordenson:*

Corneal astigmatism,	none	9	per cent.
— —	with the rule	77	—
— —	against the rule	1	—
— —	oblique	12	—

Thirty per cent. had astigmatism of at least 1 D., 2 per cent. an astigmatism over 1.5 D.—It seems that astigmatism against the rule becomes more frequent with age, and that astigmatism with the rule changes into astigmatism against the rule under the influence of an increase of tension. *Pfalz* and *G. Martin* have thus found astigmatism against the rule very common in glaucomatous patients, and the experimental researches of *Eissen* on rabbits' eyes confirm this result.

Except in post-operative cases, corneal astigmatism only very rarely exceeds the degree of 5 to 6 D.; astigmatism against the rule and oblique astigmatism are never so pronounced.—If there is a difference between the degree of the astigmatism of the two eyes of the same person, we generally find that the most astigmatic eye has the maximum curvature greater and the minimum curvature less than those of the other eye, but the difference is generally greater for the meridian of greatest refraction (*Javal*).

69. Relations Between Ophthalmometric and Subjective Astigmatism.—We have said that the first ophthalmometric measurements were made by *Donders* and *Knapp*. They noticed that there existed a certain difference between the ophthalmometric and subjective measurements. They attributed this difference

to an astigmatism of the crystalline lens which would act in a direction contrary to that of the cornea. Since then much has been said of crystalline astigmatism, but what has been said about it is purely hypothetical, for if I except some measurements which I have made with the ophthalmophakometer, and to which I shall refer later, I do not think that ony one has observed directly astigmatism of the crystalline lens. Now, the difference between ophthalmometric and subjective astigmatism may be attributed to many other causes. To assume nothing as to the nature of this astigmatism I shall call it *supplementary astigmatism.* According to most investigators the part which it plays is the following:

1° If there is no ophthalmometric astigmatism, we generally find a slight subjective astigmatism against the rule;

2° If the ophthalmometric astigmatism is against the rule, the subjective astigmatism is generally against the rule and greater;

3° If the ophthalmometric astigmatism is with the rule and of a value intermediate between 1 and 3 D., the subjective astigmatism generally differs only slightly from it;

4° If the ophthalmometer gives an astigmatism with the rule and greater than 3 D., the subjective astigmatism is also with the rule, frequently greater.

Javal tried to express the relation between subjective astigmatism (Ast) and opththalmometric astigmatism (Asc) by the empiric formula:

$$As_t = k + p.\ As_c,$$

in which formula k and p are two constants, k=0.5 D. against the rule and p=1.25.—This formula would give the following relation:

$$\text{As. opht. } \overbrace{2 — 1}^{\text{Against the rule.}} — 0 — \overbrace{1 — 2 — 3 — 4 — 5 — 6}^{\text{With the rule.}} \text{ dipotries}$$

$$\text{As. subj. } \underbrace{3 — 1.75 — 0.5}_{\text{Against the rule.}} — \underbrace{0.75 — 2 — 3.25 — 4.5 — 5.75 — 7}_{\text{With the rule.}} \text{ dipotries}$$

It is well understood that this permits of many exceptions, for supplementary astigmatism depends on so many factors, that

it is very difficult to give a general expression of its value. Among these factors I shall state the following:

1° *The Deformity of the Internal Surfaces.*—Although these deformities, as I have already remarked, play quite an important part in the literature, this question has, up to the present, been completely ignored. To give an idea of the part which they might play, I add the following table, which gives the results for some eyes I have measured:

	M*me* T.	D*r*. B.	M. V.
Thickness of cornea	1.15mm	1.06mm	1.31mm
Position of the anterior surface of the crystalline	3.54mm	4.24mm	3.66mm
Thickness of crystalline	4.06mm	3.98mm	4.25mm
Anterior surface of cornea:			
Radius. Horizontal meridian	7.98mm	7.78mm	8.29mm
— Vertical meridian	7.60mm	7.90mm	8.33mm
Horizontal refraction	47.24 D.	48.46 D.	45.48 D.
Vertical refraction	49.60 D.	47.72 D.	45.26 D.
Posterior surface of the cornea:			
Radius. Horizontal meridian	6.22mm	5.66mm	6.17mm
— Vertical meridian	5.55mm	5.11mm	5.87mm
Horizontal refraction	—4.73 D.	—5.19 D.	—4.77 D.
Vertical refraction	—5.30 D.	—5.76 D.	—5.01 D.
Anterior surface of the crystalline lens:			
Radius. Horizontal meridian	10.20mm	12.26mm	10.42mm
— Vertical meridian	10.10mm	10.09mm	9.33mm
Horizontal refraction	6.13 D.	5.10 D.	6.00 D.
Vertical refraction	6.19 D.	6.19 D.	6.70 D.
Posterior surface of the crystalline lens:			
Radius. Horizontal meridian	6.17mm	6.38mm	6.73mm
— Vertical meridian	6.24mm	7.11mm	8.49mm
Horizontal refraction	9.53 D.	9.22 D.	8.73 D.
Vertical refraction	9.42 D.	8.27 D.	6.93 D.
Astigmatism in Dioptries: (1)			
Anterior surface of the cornea	2.36 *d*	0.74 *i*	0.22 *i*
Posterior surface of the cornea	0.57 *i*	0.57 *i*	0.24 *i*
Anterior surface of the crystalline lens	0.06 *d*	1.09 *d*	0.70 *d*
Posterior surface of the crystalline lens	0.11 *i*	0.95 *i*	1.81 *i*
Complete system	1.40 *d*	1.05 *i*	1.62 *i*

(1) [Here *d* (direct) stands for astigmatism with the rule and *i* (indirect) for that against the rule.]—W.

Although we manifestly cannot draw general conclusions from the measurements of three eyes, I wish, however, to direct attention to some of these results. We observe in the first place that the vertical meridian of the posterior surface of the cornea presents a more pronounced curvature than the horizontal meridian. This condition is repeated in the three eyes to which I here refer, as well for the first, the anterior surface of which presents astigmatism with the rule, as for the other two in which it presents astigmatism against the rule. I have also met the same deformity in other eyes which I have measured, so much so that there is reason to believe that the condition is general. It is a deformity analogous to that which, in the case of the anterior surface of the cornea, produces astigmatism with the rule; but, as the posterior surface acts like a concave lens, this deformity produces astigmatism against the rule. It is probably for this reason that eyes, which have no ophthalmometric astigmatism, generally have subjective astigmatism against the rule. The influence of the posterior surface of the cornea must correspond partly with the term k of the formula of *Javal*.

As to the crystalline surfaces, we observe that the anterior surface presents in the three cases astigmatism with the rule, the posterior surface astigmatism against the rule. I do not know whether it is a coincidence or whether it indicates a general rule.

2° The *obliquity of the crystalline lens* must, after what we have said on refraction by lenses placed obliquely (page 143), produce astigmatism against the rule, but very little, at most a half dioptry, and perhaps less, if the special structure of the crystalline lens results in compensating the effect of its obliquity as certain authors (*Hermann*) have supposed.

3° Mention has been made of an *astigmatic accommodation of the crystalline lens,* which would have the effect of correcting the corneal deformity, and often even over-correcting it. In my opinion this astigmatic accommodation is not sufficiently demonstrated; I shall speak of it forthwith.

4° We must not forget the *influence of the distance of the correcting glass from the eye,* in consequence of which the concave

correcting glass is stronger, the convex glass weaker than the true astigmatism. This influence makes itself felt the more according as the glass is stronger, and, in order to calculate it, we must take into account not only the cylindrical glass, but also the spherical glass with which it is combined (*Ostwalt*) (1).—If certain authors have found that the subjective astigmatism with the rule frequently exceeds that found with the ophthalmometer (the factor *p* of *Javal*), it is due, perhaps, to the fact that they generally use concave cylinders.

5° Among the factors which play a part in supplementary astigmatism, the most important is probably the variation of the *astigmatism in the different zones of the cornea.* The peripheral zones frequently present a value, and sometimes also a direction more or less different from those of the central zones. This, among other things, follows from the measurements of the peripheral parts of the cornea made by *Sulzer;* but it is especially after I began to work with the optometer of Young that I frequently found considerable differences between the refraction of different parts of the pupillary space, and that I became convinced of the importance of these differences. There certainly exist some regularly constructed eyes, in which the astigmatism is nearly the same in the whole pupillary space, but most eyes are more or less irregular. Entirely regular astigmatism is only imaginary.—This explains also the hesitancy of many patients when tested with different cylindrical glasses. We have all met cases in which it is almost impossible to obtain a definite answer from the patient. Sometimes he prefers one cylinder, sometimes another somewhat different, and, at each new examination, he manifests a different preference. Most frequently if the patient hesitates, he has good reasons for doing so.—Examination with the luminous point (see chap. X), which has been much neglected, but which we have used for some time at the laboratory of Sorbonne, shows why the patient hesitates and why we frequently do not obtain a very encouraging result by correction.

---

(1) [See also an article by the translator in the *Archives of Ophthalmology,* Vol. XXII, No. 1, 1893, where this question is discussed fully.]—*W.*

70. Astigmatic Accommodation.—The question of astigmatic accommodation has been much discussed for some years past. It was *Dobrowolsky* who first expressed the idea that astigmatic patients could partly correct their defect by producing a deformity of the crystalline lens in a contrary direction, by an irregular contraction of the ciliary muscle. He thus supposed a *latent astigmatism* which could be made manifest by instilling atropine, exactly as in the case of hypermetropia.—Later, the idea was adopted by *Javal,* and pushed to its extreme conclusions by *G. Martin, Vacher* and others, who went so far as to find in this astigmatic accommodation the origin of a series of diseases: blepharitis, keratitis, migraine and even, in certain cases, cataract. Some time ago a reaction set in; most of the authors in later years, like *Eriksen, Sulzer* and especially *George Bull,* do not admit astigmatic accommodation.

The advocates of astigmatic accommodation based their belief especially on the change of the astigmatism observed on instilling atropine. The phenomenon is, in all probability, due to the fact that the astigmatism of the peripheral parts differs from that of the central part; in ordinary circumstances these parts are outside the pupil, but in consequence of atropinization the latter is dilated so as to allow the peripheral parts to come into play. The area of these peripheral parts is generally greater than that of the central art which corresponds to the pupil in ordinary circumstances. Suppose, for example, that the diameter of the pupil may be brought from 4 to 8 millimeters. The area of a circle being expressed by r_2n, that of the ordinary pupil is about 12 square millimeters and that of the dilated pupil about 50 square millimeters. The pupil has consequently increased by 38 square millimeters, or about three times its size. Thus much more light enters through these peripheral parts; and it is not surprising that this fact greatly influences the answers of the patient. All the observations of a change of astigmatism after instilling atropine prove nothing, therefore, in favor of astigmatic accommodation. It has been proposed to study the question by placing before the eye a diaphragm of the size of the undilated pupil, but I do not see how we could assure ourselves

whether the position of the diaphragm really corresponded with that of the undilated pupil.—The only observations in favor of astigmatic accommodation which could lay claim to some value, are those in which the observer, provided with a weak cylinder, begins by seeing distinctly one line of the star figure and ends by seeing all with the same distinctness. But the observations of this kind which have been published are by no means beyond all criticism. If any one desires to again perform this experiment he had better use a luminous point: after having placed a weak cylinder before the eye, it would be necessary to observe the different forms under which the luminous point would be seen at different distances (see the following chapter) and to repeat this examination after having worn the cylinder for an hour or two, to see if the figures had undergone any change.

The alleged astigmatic accommodation was always of a very low degree, 1 D. to 1.5 D. at most. Frequently, in order to discover it, a very persistent atropinization was necessary, lasting as much as fifteen days and even until symptoms of poisoning appeared. I think that frequently the patient, weary of the struggle, ended by answering all that was desired.

71. Post-operative Astigmatism.—If we examine the cornea eight days after the extraction of a cataract, we find an enormous astigmatism against the rule, sometimes reaching 12 or 14 D. The vertical meridian is flattened, probably in consequence of the interposition of an exudation between the lips of the wound; the phenomenon is more pronounced if there exists a hernia of the iris. This astigmatism diminishes gradually; it may disappear altogether, but generally one or two dioptries remain. For this reason it is prudent to postpone the selection of spectacles for two or three months after the extraction, or, if the patient desires to have them immediately, to warn him that it will be necessary to change them after two months. Contrary to what we would expect, the agreement between the subjective astigmatism and the ophthalmometric measurement is less than for the normal eye, which is due partly to the distance of the correcting glass from the eye (see page 153), partly to the fact

that the cornea very frequently retains a certain degree of irregularity after extraction. What we have said of the extraction of cataract applies also, but in a much less degree, to iridectomy and other operations performed on the cornea.

72. Keratoconus.—Apart from post-operative cases, we meet the highest degrees of corneal astigmatism in cases of keratoconus. (1) The apex of the cone does not generally coincide with the visual line, which gives rise to a strong astigmatism, the direction of which varies, following the direction of the apex of the cone. We observe at the same time that the images of the mires are

Fig. 84.—Keratoscopic images of a case of keratoconus.

very irregular. By removing the prism and placing the keratoscopic disc in its place, we easily find the direction of the look which brings the apex of the cone into the axis of the ophthalmometer; we then see the image of the keratoscopic disc quite

(1) The expression "keratoconus" is not very happy; the form of the cornea approaches in these cases that of a hyperboloid; we know, indeed, that this body closely resembles a cone with rounded apex.

small and frequently regular, round or oval; in every other position its form is ovoid (fig. 84).—The cases which *Javal* had first described under the name of *decentered eyes,* because he thought their deformity depended on an unusual size of the angle *a,* were affected with a light degree of keratoconus, as he has since acknowledged. Outside of cases of keratocnus, we quite frequently meet cases in which the images of the mires or of the keratoscopic disc present more or less pronounced irregularities, for example, in consequence of old lesions of the cornea. Frequently, however, we still succeed in making an ophthalmometric measurement which may give information useful for the choice of a cylinder.

73. Symptoms of Astigmatism.—The most important symptom of astigmatism is the diminution of visual acuity, which is a consequence of the want of distinctness of the image. Generally the images are a little deformed, but astigmatic patients are accustomed to this deformity and take no notice of it.

Asthenopia of Astigmatic Patients.—On account of their diminished acuity astigmatic persons are obliged to bring objects near them for the purpose of obtaining larger retinal images. They are, therefore, obliged to accommodate more than other persons, which is in itself a cause of astigmatism. But there are yet other reasons for it.

It may be asked how astigmatic persons see, that is to say, what part of the interfocal distance is it that they bring preferably on the retina. Following *Sturm* it was believed that, in cases in which they have their choice, they prefer to use the circle of diffusion so as to see all the outlines with the same degree of confusion. According to later researches (*Javal*) it is the vertical focal line that they use preferably. There are several reasons for this preference: one is that it is much more important in reading to see the vertical lines distinctly, the legibility of the letters depending especially on the distinctness with which the vertical lines are seen. Another reason is the importance which vertical outlines have for binocular vision. If one sees only the horizontal lines, there is nothing to indicate

for what distance the eyes must converge. For want of being able to use the vertical focal line astigmatic persons have recourse to the horizontal line, but very rarely to the intermediary part.

In cases of astigmatism *with the rule,* the degree of accommodation to be used depends, therefore, on the meridian of least refraction. Any one having compound hypermetropic astigmatism, simple hypermetropic astigmatism or mixed astigmatism is, therefore, in the same situation as a hypermetrope; he has the same reasons for having accommodative asthenopia. Persons having myopic astigmatism with the rule or against the rule (if it is not combined with hypermetropia) have less cause to suffer from asthenopia and seem, indeed, to suffer less. *George Bull* especially has laid stress on this explanation of the asthenopia of astigmatic persons.

74. Examination of Astigmatic Persons.—When, on examining the patient with spherical glasses, we do not find a satisfactory acuity we suspect astigmatism, unless the explanations of the patient give reason to suspect an internal disease of the eye. We then submit the patient to ophthalmometric examination, which, according to the rules that we have laid down, gives an approximate idea of the direction and degree of the subjective astigmatism. If we find a very low degree with the ophthalmometer we may generally come to the conclusion that the complaints of the patient need not be attributed to astimgatism. We then pass to the subjective examination; make the patient myopic two or three dioptries and move the star figure close enough for him to see one of the lines distinctly. *Under these circumstances, the patient sees distinctly the line which corresponds to the meridian of greatest refraction.* The direction of this line indicates, therefore, the direction of the axis of a convex cylinder; a concave cylinder must be placed perpendicularly to this direction. It is rare to find an appreciable difference between the direction indicated by the ophthalmometer and that thus found, unless in the case of a very slight ophthalmometric astigmatism which can have no bearing, in its position and value, on the

total astigmatism. We may then proceed to find the cylinder which equalizes all the lines, but the simplest way is to find directly the cylinder which gives the best visual acuity: we place before the eye the glass which corrects the spherical ametropia, joining thereto the cylinder indicated by the ophthalmometer, in the position found by means of the star figure. After having found how much the visual acuity is thus improved, we try whether a further improvement is obtained by making the glass rotate slightly in both directions and adding a +1 and —1 cylinder, being very careful to place the axis of the glass parallel to that which is already in the frame. According as the acuity gains by adding a one dioptry convex or concave cylinder, we replace the glass of the frame by the following number, and recommence the examination. With patients who are good observers, or when the astigmatism is slight, we may sometimes reach a greater degree of accuracy, by using a half-dioptry cylinder. When we have found the weakest cylinder which gives the best visual acuity, we verify the spherical glass by adding a + 1 spherical which ought to diminish the visual acuity and a — 1 spherical which ought not to increase it.

After having made the subjective examination, we examine the patient with the ophthalmoscope. I will mention farther on the ophthalmoscopic signs of astigmatism on which great stress was laid at a time when there were no other objective signs of this anomaly; they have become today almost mere curiosities, especially since skiascopy has assumed a merited importance. When we make use of it for verification, we place the correcting glass in a frame and examine by skiascopy whether the correction is complete. We can also use it to find out the direction of the axis and the value of the astigmatism, if we have no ophthalmometer.

Skiascopy with a luminous point especially enables us to find very exactly the direction of the axis by means of the luminous band, mentioned on page 141. In order that the phenomenon may be distinct it is necessary that the eye of the observer be placed in one of the focal lines, and that the mirror forms the image of the luminous source at the place of the other focal

line. The observer will then see luminous the meridian at the focus of which he is. Thus if the observed eye has a myopia of 2 D., combined with an astigmatism with the rule of 2 D., he will see a horizontal luminous band if he is placed at 50 centimeters and illuminates the eye with a concave mirror which projects the image of the luminous source at 25 centimeters. To see the band vertical he must place himself at 25 centimeters and examine with a plane mirror.—Generally it is necessary to dilate the pupil.

There are two points in particular on which I would lay great stress. First, the importance of the subjective examination which must always have the last word; it is only in cases in which it is impossible to obtain information from the patient, that we can attempt to give correcting glasses according to the data furnished by the objective methods. The reason is that, in most cases, the correction of the eye by a cylinder is not a simple optic problem. Most frequently the astigmatism is not the same in the entire pupillary space; that of the exterior zones differs more or less from that of the central zones; the best correcting glass is only a sort of guess, which neither the ophthalmometer nor skiascopy can assume to indicate exactly. It is well understood that these differences are usually not great, especially in the case of persons who consent to the correction, but they suffice, however, to make the subjective examination indispensable.

The other point which I would emphasize is that the prescribing of cylinders should not be abused. Since the invention of the ophthalmometer there is too decided a tendency to prescribe cylinders as soon as a diagnosis of astigmatism is made. Cylindrical glasses should not, in my opinion, be prescribed unless they produce a palpable improvement of the visual acuity; the wearing of glasses is always an annoyance for the patient, and cylindrical glasses more so than any, as well on account of the difficulty of wearing them in eye-glasses as on account of the errors in the direction of the axis which opticians sometimes commit, the difficulty of replacing a broken glass, etc.

If there are several cylinders which give the same acuity it is best to choose the weakest. If there is astigmatism of only one eye, we may allow the patient to say whether he will have it corrected or not; generally he does not gain much by the correction except in cases where there is a tendency to strabismus.

If we combine two cylinders of the same strength by placing the axes parallel, they act like a cylinder twice as strong; if we place the axes perpendicularly to each other, they act like a spherical glass, and if the axes form an acute angle with each other the effect is the same as that of a sphero-cylindrical combination, the spherical and cylindrical strength of which vary with the angle. As we can obtain no other effect with two cylinders than with one cylinder combined with a spherical glass, the bi-cylindrical glasses are now abandoned.

The variable cylindrical lens of *Stokes* was composed of one cylinder which remained fixed and another which could be rotated; we thus obtained a variable cylindrical effect, but the instrument had this disadvantage that the direction of the axis varied also. *Javal* remedied this by making the two cylinders rotate in opposite directions; but, in spite of this improvement, the lens of *Stokes* has never been of any practical utility, because of the spherical effect which varies at the same time as the cylindrical. (1)

We can always obtain the effect of a given sphero-cylindrical combination with the cylinder of contrary sign, by changing the spherical glass. A +5 spherical combined with a +3 cylindrical is thus equivalent to a +8 spherical with a —3 cylindrical. Really, there is need, therefore, of only one kind of cylinder; there is also now a tendency to prescribe only concave cylinders which are combined with convex sphericals to obtain the effect of convex cylinders. By placing the cylinder on the side of the eye we thus obtain a slight periscopic effect.

Periscopic glasses, which were invented by *Wollaston,* are con-

(1) [This last defect has been overcome in the optometer of the translator. In this instrument two spherical lenses are so moved that the spherical effect, produced by the rotaion of the two cylinders is always neutralized by the contrary spherical effect of the two spherical lenses. Thus a purely cylindrical action is obtained. See *Annals of Ophthalmology,* Vol. III, No. 1.]—*W.*

cavo convex menisci the concave side of which is next the eye. Their advantage consists in this that the peripheral parts of the visual field appear more distinct because the rays pass through the glasses less obliquely than in the ordinary case. This advantage also exists when the eye is motionless as regards the peripheral directions of the look. For some time the attempt has been made to replace cylindrical glasses by *toric* glasses, one of the surfaces of which is cut as a *tore*, the other as a spherical surface. They have the advantage of being periscopic, but their manufacture is difficult and up to the present they are not very popular.

Cases of exact correction of astigmatism are among the most agreeable which the oculist can meet, and it happens quite frequently that a normal acuity, or even higher than normal, may be obtained. Frequently the acuity remains under the normal, and there is a certain number of cases in which the effect of the glasses is nil or nearly so. Oculists are not in agreement as to the number of cases in which a good result may be obtained. *Schweigger* says that, in a considerable minority of cases of astigmatism the correction obtained by cylinders is quite satisfactory. Other authorities are less pessimistic.

Bibliography.—*Œuvres de Young*, edited by Tscherning, p. 125.—Airy. *Transactions of the Cambridge Phil. Soc.*, 1827, t. II et 1849, t. VIII.—Sturm. *Sur la théorie de la vision.* Reports, 1845.—Goulier. *Sur un défaut assez commun de conformation des yeux et sur les moyens de rendre la vue distincte aux personnes qui en sont atteintes.* Reports, 1865.—Knapp (H.). *Ueber die Asymmetrie des Auges in seinen verschiedenen Meridiansystemen. Arch. f. Ophth.*, VIII, 2, p. 185.—Donders (F. C.). *Astigmatismus und cylindrische Gläser.* Berlin, 1862.—Javal (E.) in de Wecker. *Traité des maladies des yeux*, II, Paris, 1863.—Javal (E.). *Sur le choix des verres cylindriques, Ann.* d'oc., 1863.—Javal (E.). *Mémoires d'ophtalmométrie.* Paris, 1891.—Schioetz (H.). *Ophtalmometrische und optometrische Untersuchung von* 969 *Augen. Arch. f. Augenh.*, 1885.—Nordenson (E.). *Recherches ophtalmométriques sur l'astigmatisme de la cornée.* Ann. d'oc., 1883. Bull (G.). *L'asthénopie des astigmates.* Bull. de la Soc. franç. d'ophtal., 1892, p. 128.

CHAPTER X

IRREGULAR ASTIGMATISM

75. General Remarks.—When we do not succeed in obtaining a normal visual acuity by means of spherical and cylindrical glasses, we generally attribute the cause of this failure to the retina—we diagnose amblyopia.—Sometimes, but, as a rule, quite rarely, the diminution of visual acuity is attributed to an irregular astigmatism, especially if it is visible by the deformities of the ophthalmoscopic or skiascopic images. But it is probable that the more we will study the optics of the eye, the more the diagnosis of amblyopia will give place to that of irregular astigmatism, which has served up to the present as the common term for all optic defects of the eye other than myopia, hypermetropia and regular astigmatism, that is to say, those which we can correct with test case lenses. For some time past the majority of works which have been published on the optics of the eye, have had for their object the improvement of the methods used to determine these defects as quickly and as exactly as possible. There is little probability that we can, for the moment, make progress of any importance in this direction; these methods are, at present, very well developed; it even seems to me that we bid fair to overstep the limit, in this sense that we can perceive a tendency to desire to determine these defects too exactly. Quarters of a dioptry are, indeed, superfluous for our test cases, and even half dioptries are only rarely indispensable, except for very weak ametropias. So long as it was supposed that the refraction was the same in the whole pupillary space, we could imagine the possibility of determining this refraction with great exactness. But since we know that there are in nearly all eyes optic differences between the different parts of the pupillary space, and since these differences may reach several dioptries, the correcting glass must be regarded

as a sort of approximation which we cannot determine with perfect exactness. It seems that the construction of the eye is such, that the visual acuity is about 2 for a perfect optic system; but many eyes have optic irregularities which lower the acuity to 1, to five-sixths, to three-fourths or still lower, and these irregularities are frequently still more pronounced in astigmatic eyes, which prevents complete correction.

The study of these irregularities seems, therefore, destined to play a certain part in future works on the optics of the eye. As I have already remarked, we can study them with the keratoscopic disc of the *Javal* and *Schioetz* ophthalmometer, and we can measure them with the optometer of *Young*, which necessitates, however, on the part of the observer a certain amount of work to accustom himself to the instrument. But the best means of studying these irregularities is the following.

76. Examination of the Eye with a Luminous Point.—We have already seen that the first authors who devoted their attention to the question of regular astigmatism, used the luminous point to find the meridians and to judge of the exactness of the correction. Later, the luminous point was replaced by the star figure, which is in more common use for finding the meridians, but which gives information only on the astigmatism which can be corrected by a cylindrical glass. The forms under which a luminous point is seen furnish, on the contrary, fuller information: there is no optic defect of the eye which is not shown in these figures, sometimes, it is true, under a form which it may be difficult to interpret. This is why we have undertaken this examination at the laboratory of Sorbonne. As object we use a very small opening (0.2 mm. to 0.3 mm.), made in a dark screen, and on which is concentrated the light of a lamp or daylight. The patient, rendered myopic, gradually approaches the luminuous point while observing the form under which the latter may appear. We can also place the patient at a fixed distance, at one meter, for example, and virtually change the distance of the luminous point by placing concave or convex glasses before the eye; the patient must avoid as much as

possible using his accommodation. We can thus examine the form of the refracted pencil throughout its whole extent, for, as far as the question at issue is concerned, it amounts to the same whether the luminous point be fixed while the retina is displaced, or whether, the retina being fixed, we displace the luminous point. Most of the time the patient sees circles of diffusion presenting pretty exactly the form of the pupil, which diminishes according as the luminous point approaches the focus. But near the latter, in front and behind, there is a part, the *characteristic part of the pencil,* where the circle assumes irregular forms. The round diffusion spots are alike in all; at most we find some slight differences due to the form of the pupil, to a different distribution of the brightness of the circles, or to entopic phenomena which I shall describe in the following chapter. But the characteristic part of the pencil differs so much in different persons that I have never met two eyes in which it was alike, except, perhaps, in the two eyes of the same person.

Fig. 85. Forms under which a luminous point is seen by a regular eye. After *Rée.*

77. Different Forms of Irregular Astigmatism.—We can distinguish several groups:

1° In an ideal eye the *characteristic part* of the pencil is reduced to a point. We sometimes meet eyes which do not differ much from this type, but they are rare, and all have an exceptional visual acuity (fig. 85). (1) It is besides clear that, all things equal, the better the eye the shorter the *characteristic point* of the pencil.

2° Eyes regularly astigmatic should see figures similar to those of figure 77, but eyes so regular scarcely exist. In low degrees of astigmatism we scarcely ever have distinct focal lines, and in strong degrees, where the focal lines are clearer, irregu-

Fig. 86. Regular astigmatism with spherical aberration. *After Rée.*

larities appear when the astigmatism is approximately corrected by a cylinder. The most regular astigmatic patients frequently see forms analogous to those of figure 86. The focal lines are thicker at the middle and the interfocal diffusion spot is not

(1) Figures 85, 86, 87, 89, 90, 91, 92 are borrowed from a work which M. Rée compiled at the laboratory of the Sorbonne (*Undersoegelse af Oeiet med et lysende Punct,* Copenhagen, 1896) and which has the shape of a small atlas showing the forms under which the eye sees a luminous point. But the question is far from being exhausted, and it would be desirable that some one should again take it up in a clinic. With some exceptions, the eyes of the persons examined by *M. Rée* were what we call normal eyes; but it is especially astigmatic persons, whose vision does not improve with cylinders, that should be examined.

Fig. 87.—Figures of a luminous point obtained by combining an ordinary strong spherical lens with a cylindrical lens (astigmatism with spherical aberration). After *Rée.*

Fig. 88.—A, forms which a luminous point presents to my right eye (obliquity in one meridian, the vertical).—B, appearance of the same figures if I cover the lower half of the pupil.—C, appearance of the figures if I cover the upper half of the pupil.

The figures *a* correspond to a distance of 60 centimeters; the figures *b* to 1 meter; the figures *c* to 1.50m and the figures *d* to infinity.

circular, but in the form of a lozenge. These forms are due to the combination of a regular astigmatism with a quite pro-

Fig. 89.—Eye with double obliquity. After *Rée.*

nounced spherical aberration, for we can obtain forms wholly analogous with a combination of a +20 sph. with a +6 cyl. of

Fig. 90.—Figures of the left eye of M. *Rée* (Obliquity in one meridian, the vertical). Curved focal line.

our test cases (fig. 87). It is for this reason that one is obliged to use an aplanatic lens to obtain figures of pure astigmatism. In the more irregular eyes we can generally find figures which represent more or less perfectly the focal lines, that is to say, there are two planes where the figures are more or less elongated so that their two long axes are perpendicular to each other; but these figures are far from being linear.

Fig. 91.—Curved focal line. After *Rée*.

3° It is not rare for the optic system of the eye to affect a certain obliquity, so that the figures are symmetrical in relation to a single axis (and not in relation to two axes, as in regular astigmatism). It is so in the case of my right eye (fig. 88) and also in that of *M. Rée* (fig. 90). These figures are, up to a certain point, analogous to those which are obtained with a lens placed obliquely.

4° Frequently we discover an obliquity in the two directions perpendicular to each other, so that the figures are not symmetrical at all (fig. 89).

5° An anomaly which is not at all rare consists in a certain curvature of the focal lines, due probably to the fact that the principal meridians of the cornea show an analogous curvature (figs. 90, 91).

Fig. 92.—Irregular eye (Diplopia). After *Rée.*

6° We quite frequently meet more irregular figures, those for instance of figure 92, belonging to an eye which has a rather pronounced diplopia.

78. Rules for Analyzing the Figures of the Luminous Point.—The figures are sometimes quite difficult to analyze. Here are some directions for this analysis:

1° We can always decide whether a part of a figure is formed by crossed rays or not, by covering a part of the pupil. If it is the homonymous part of the figure which disappears, this part is formed by rays which have already crossed the axis before reaching the retina; if it is the heteronymous part which disappears, the rays have not yet crossed the axis.—Sometimes we

can with advantage use cobalt glass (see page 134) for this analysis.

2° If the luminous point is beyond the *punctum remotum,* and if the observer notices a concentric brightness on a part of the diffusion spot, this part corresponds to a less refracting part than the remainder of the pupil; for, the focus of this part is nearer the retina and its rays are, consequently, less dispersed.

3° If, within the focus, the figures are elongated in one direction, downwards for example, they are elongated in the same direction beyond the focus, and the eye is more refracting in this direction. Thus in figure 95, A, in which the lower part of the surface is supposed to be more refracting, the part of the cone situated above the axis is everywhere larger. The diffusion spots are seen elongated downward (fig. 88).

4° The aberroscopic phenomena (page 123) always tell us in what direction the refraction increases or diminishes, starting from the center of the pupil.

Finally the optometer of Young permits a more exact analysis of these irregularities.

Let us take, for example, my right eye (fig. 88), and see how we can use these rules to analyze the figures. We observe that the upper part of the figure *d,* A, seen at infinity, has a greater brightness than the lower part. On covering the upper half of the pupil, this part disappears, while, if we cover the lower half of the pupil, this part does not change. We conclude from this, following rule 1°, that the whole figure is formed by rays that have crossed the axis, that is to say, that the whole pupillary space is myopic, and, following rule 2°, that the upper part is much less myopic than the remainder.—If I move nearer up to 1.50 m. from the luminous point, I see the figure *c* which resembles a luminous T written in a less luminous half circle. If I cover the upper half of the pupil, the vertical stroke disappears and the horizontal stroke becomes weaker. We conclude from this, following rule 1°, that the vertical stroke is formed by rays which have not yet crossed the axis. The point situated at 1.50 m. is, therefore, already situated within the

far point of this part, while it is situated beyond the far point of the lower part. All the figures are elongated downwards, which also shows (following rule 3°) that the pupil is more refracting below. The lines of the aberroscope are convex towards the middle, below and towards the two sides, while they are straight or slightly concave towards the middle above (fig. 93), which shows that the refraction diminishes towards the periphery above and increases in the three other directions.—Finally we find, by measuring with the optometer of Young, the refraction indicated by the diagram (fig. 94, A). The measurements confirm the other observations, unless it be that they disclose a slight degree of hypermetropia near the upper border of the pupil, which had escaped attention in the analysis of the figures. It follows that the course of

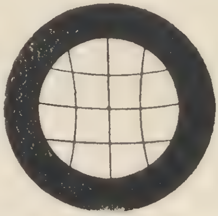


Fig. 93.—Aberroscopic phenomena of my right eye.

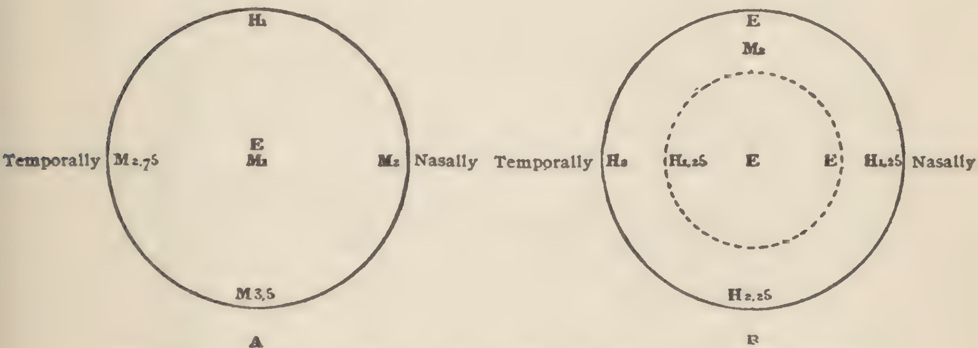


Fig. 94.—A, Diagram of the variations of refraction in the pupil (dilated) of my right eye.—B, diagram of the refraction in the pupil of Demicheri: the dotted circle indicates the normal pupil, the full circle the dilated pupil.

the rays must be nearly as I have illustrated them in figure 95; A corresponds to the vertical meridian, B to the horizontal meridian; the place marked 2 corresponds to figure 88, *c*.

As to the means to use for the correction of these defects, they

still remain to be discovered. The only information we can give for the present is that the forms mentioned under rule 3° could probably sometimes be corrected more or less effectively with glasses placed obliquely.—*Contact glasses* could evidently correct the greater part of these defects, which reside especially in the

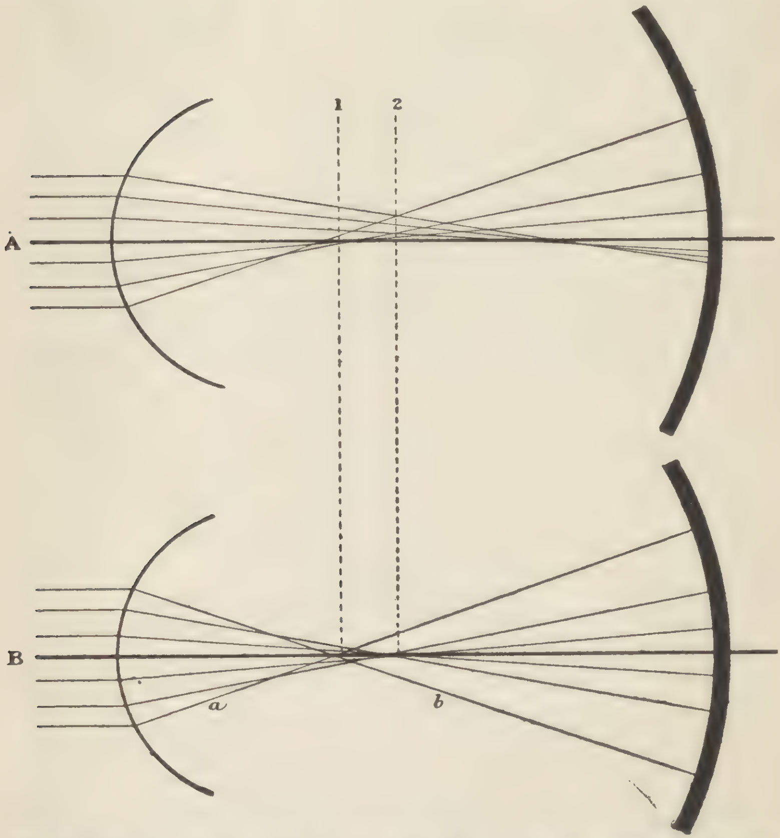


Fig. 95.—Course of the rays in my right eye: A, in the vertical meridian (obliquity); B, in the horizontal meridian (spherical aberration).

cornea. As the cornea scarcely tolerates contact *Sulzer* caused to be cut similar glasses, which are furnished with a rim by which they are supported on the sclera. Under this form, contact glasses are easier to wear, but they seem nevertheless to

cause a certain annoyance, which will probably prevent their use, except in special cases.

Bibliography.—Tscherning (M.). *Die monochromatischen Abweichungen. Zeitschrift f. Psych. u. Physiol. der Sinnesorg.*, IV, p. 456.—Rée (O. M.). *Undersoegelse af Oeiet. med et lysende Punkt.* (Danois). Copenhagen, 1896.

CHAPTER XI

ENTOPTIC PHENOMENA

79. Manner of Observing Entoptic Phenomena.—When we approach a luminous point, the circle of diffusion to which it gives rise increases in size. At the moment when the luminous point is at the anterior focus of the eye, the rays are parallel after refraction, and the circle of diffusion is the size of the pupil; on approaching nearer to it, the circle still increases.

In these circumstances we observe *entopic phenomena,* that is to say, shadows which the corpuscles situated in the refracting media of the eye project on the retina. If, instead of a point, we use a larger luminous source, the cone of the shadow becomes too short to reach to the retina, except the object is very near the latter. Another way of observing entoptic phenomena consists in placing ourselves at a great distance and observing the luminous point through a strong convex lens. In this case the displacements of the shadows take place in the direction contrary to that which we are going to point out later.—Among the entoptic observations I shall cite the following:

1° The luminous spot is limited by the shadow of the border of the iris; we can thus study, therefore, the irregularities of the latter. The pupillary contraction is very well observed on opening or covering the other eye.

2° We very frequently see small circles the centers of which are bright, and which have an apparent motion from above downwards, depending on the winking of the eyelids. They are produced by small specks on the anterior surface of the cornea, and which move in a contrary direction (fig. 96).

3° On winking the eyes we produce transverse striæ, due probably to the wrinkles of the epithelial layer. If we wink for some time, for example when keeping one eyelid closed while working with a microscope, or as artists frequently do in

order to obtain a better idea of the entire impression of a landscape, we can produce striæ which last for several hours and give rise to a very marked diplopia of the horizontal lines (fig. 97). *George Bull* especially has studied this question; according

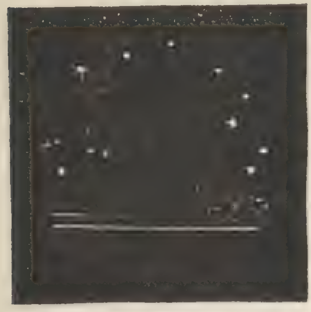


Fig. 96
After *Helmholtz.*

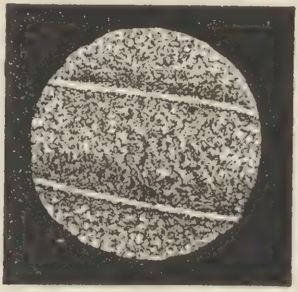


Fig. 97.—Striæ produced by winking the eyelids. (After *George Bull.*)

to him the phenomena are specially pronounced after reading for a long time in the horizontal position, and give rise to a peculiar annoyance which he has named tarsal asthenopia.

4° On winking the eyelids while looking at a distant luminous point, we observe long striæ which run upwards and downwards from the point. These striæ are due to the layer of tears which

Fig. 98.—Prismatic effect of the layer of tears.

is in the conjunctival sac, and which, near the border of the eyelids, assumes the form of a prism with a concave surface (fig. 98). This prism deflects the rays which meet it, and, as its surface is concave, the parts placed near the border of the

eyelid act as a stronger prism, which causes greater deflection of the rays: it is for this reason that we see a stria and not simply a second image of the luminous point. The upper eyelid deflects the rays upwards; it produces, therefore, the striæ which we see directed downwards. In fact, if we lower a screen placed near the eye, it is the stria directed downwards which disappears first. This phenomenon is not, porperly speaking, an entoptic phenomenon, but I mention it here because of its resemblance to those mentioned under No. 3°.

5° If we rub the eye, the luminous spot presents a speckled appearance, due to irregularities of the cornea; this appearance soon disappears (fig. 99).

6° We sometimes observe small round discs, sometimes bright and surrounded with a black border, sometimes dark with a bright (fig. 100), sometimes dark, with somewhat more lumin-quently see also the star figure of the crystalline lens, sometimes bright (fig. 100), sometimes dark, with somewhat more lumin-

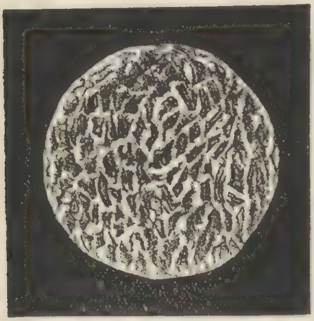


Fig. 99.—Speckled appearance of the entoptic field produced by rubbing the cornea. (After *George Bull*).

After *Helmholtz*.
Fig. 100.

ous borders. The crystalline opacities are outlined in the spot with great distinctness. An intelligent patient can thus follow step by step the development of his cataract, as we can see on the drawings which *M. Darier* has just published (fig. 101).

7° Nearly every one sees objects situated in the vitreous body; they become partly visible without further aid by simply looking at the sky, that is when they are very near the retina. They are sometimes mobile, sometimes fixed, but presenting in the latter case an apparent motion. If, for example, the shadow is seen a little above the point of fixation, the patient looks a little higher in order to fix it; but as the shadow is always seen above the point of fixation, it continues to direct the visual line higher and higher; and the shadow always flees before the look, for which reason the name *muscæ volitantes* has been given to this phenomenon. To make certain whether the motion is apparent or real, we can look at the sky through a window, on which we select a mark in order to assure fixation; after having made a rapid movement with the look, we fix this point. If the corpuscle is fixed, it should then remain motionless, but most frequently we see it descend slowly which indicates that the corpuscle really ascends.

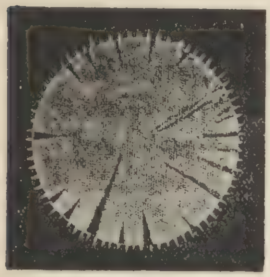


Fig. 101.—Incipient cataract, seen entoptically. (After *Darier*.)

8° We may use entoptic observation to study slight displacements of the eye as a whole, which it is very difficult to observe otherwise. To this end I have had constructed a small instrument, the entoptoscope (fig. 101*a*). It consists of a small plate of wood which we take between the teeth; on the plate is fixed a rod which carries a plate of copper having the form of the cap of a sphere. In the middle is pierced a very fine opening (1/10 mm.), which is on a level with the eye. In the concavity of the cap are stretched two threads, one horizontal and one vertical, placed in the form of a cross and forming cords with the cap. When we take the instrument between the teeth and look towards the sky we see the entoptic field occupied by the cross which is greatly enlarged. We select a point in the cross as a fixation point. The position of the cross is thus invariably dependent on that of the head; if therefore, in given circum-

stances, we observe a displacement of the cross in the entoptic field, it is because it is the latter, that is to say the eye, which suffers the displacement. We can thus prove that the eye is slightly displaced, a little upwards when we wink the eyelids, a little downwards when we open the eye very widely. When we lean the head to one side the eye undergoes a slight displacement in the direction of the weight, etc. The phenomena are especially striking when we instill eserine, because the field is then very small. The displacement of the cross may then reach a fourth or a third of the entire extent of the field.

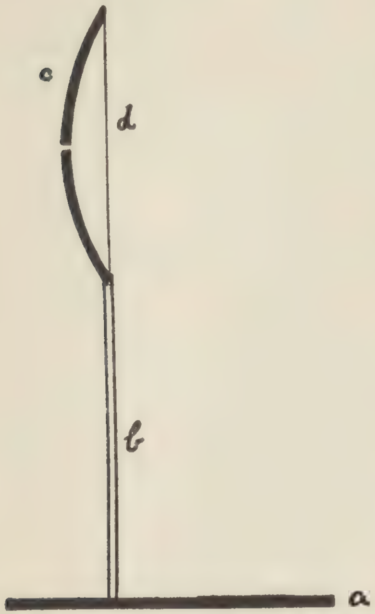


Fig. 101*a*.—Entoptoscope. *a*, planchette of wood; *b*, rod; *c*, copper plate, perforated; *d*, thread.

80. Analysis of Entoptic Phenomena.

a). OBSERVATION OF THEIR PARALLAX (*Listing*). — By fixing different points of the entoptic field, we observe that the entoptic phenomena are displaced in the field. If the corpuscle which gives rise to the shadow is behind the pupillary plane, the shadow moves in the same direction as the visual line (fig. 102, *a*, *b*). Taking the position *b*, the visual line is directed upwards; the shadow has descended to near the lower border of the field, but seems to have ascended (by the projection outwards). It is easy to see that we have the contrary parallax if the object is in front of the pupillary plane, and that it disappears if the object is in this plane. As the movement is greater in proportion as the object is more removed from the pupillary plane, we

can thus form an approximate idea of the position of the corpuscle.

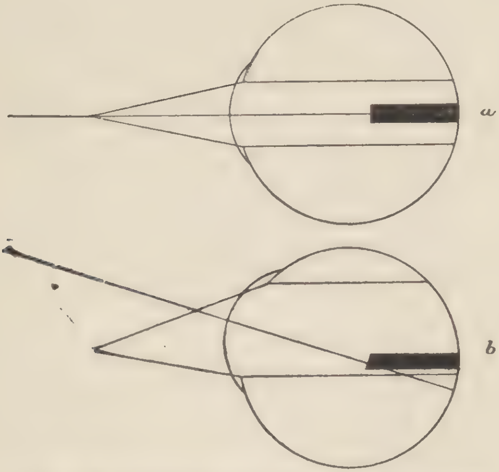


Fig. 102.—Parallax of the entoptic phenomena.

b). Measurement of the Distance of the Corpuscle from the Retina (*Brewster, Donders* and *Doncan*).—To *measure* this distance *Brewster* proposed to use two luminous points. We then see two circles of diffusion which partly overlap, and each corpuscle produces two shadows. We measure the distance between the two shadows of the same object and the diameter of the free part of one of the circles DE (fig. 103); the ratio between these two measurements is equal to the ratio between the distance of the object from the retina and that of the pupil from the retina.

Let A and B (fig. 103) be two luminous points which must be in the anterior focal plane of the eye, d the middle of the pupil, o the object, p and p_1 the shadows and c and c_1 the centers of the circles of diffusion. Since the points are in the focal plane, dc is parallel to op and dc_1 to op_1, therefore: $\frac{pp_1}{cc_1} = \frac{op}{dc}$, and figure 103 b shows that $cc_1 = \mathrm{DE} = \mathrm{R} + a$ if R is the radius of the circle of diffusion.—We can make measurements by using as a luminous source a sheet of white paper strongly illuminated. We look through two stenopaic openings towards this sheet

and we notice the places where the shadows are projected as well as the borders of the circles (Donders). *Doncan* made the measurements *a double vue* by comparing the entoptic phenomena with a scale seen with the other eye.

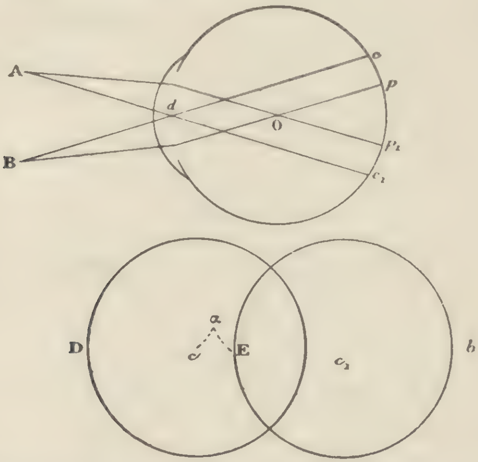


Fig. 103.—Determination of the position of an entoptic object. After *Brewster*.

c). Examination of the Refraction of the Object.—So far, we have treated the entoptic phenomena as shadows, and the objects which produce them as opaque bodies. Most frequently, this is not the case, as they are more or less transparent; but their refraction is different from that of the surrounding parts, whether their surface has a different curvature, or whether their index is different.

It is easy to see (fig. 104) that the more refracting objects must concentrate the light so that the entoptic image becomes luminous and surrounded by a dark border; this is the case with the images of the corneal specks.—On the contrary, if the object is less refracting than the surrounding parts, the image is dark, with a more luminous border. The difference is specially marked in the case of the star figure of the crystalline lens, which, in some people, appears dark, in others luminous, thus indicating that the refraction of the corresponding parts is

sometimes greater, sometimes less than that of the surrounding parts.—If we make the experiment by placing ourselves at a great distance, and making the eye strongly myopic, we should have the phenomena inverted.

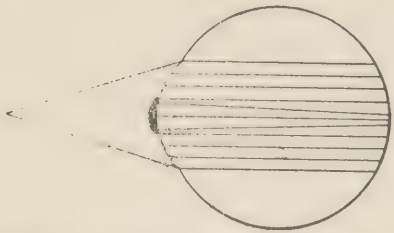


Fig. 104.—The drop on the cornea causes convergence of the rays which pass through it so that we see a luminous center surrounded by a shadow.

In the experiment which we have just noted (fig. 104), the dark border is due to the fact that part of the rays which should illuminate it are made to converge towards the middle of the entoptic image, by the interposition of the corpuscle. This border is always diffuse and frequently somewhat pronounced; it must not be confounded with the diffraction ring which surrounds the images along the border of the pupil when the luminous point is very small. This ring, which sometimes may be double or triple, is always very thin and very distinct.

81. Entoptic Observation of the Vessels of the Retina (*Purkinje*).—*a*). If, in a dark room, we hold a candle at some distance from the eye while we look directly in front, we see the retinal vessels greatly magnified projected on the dark portion of the room. They appear dark (of a deep blue) on a somewhat more luminous ground (orange).—If we move the candle towards or away from the visual line, the vessels seem displaced in the same direction; if, on the contrary, we move the candle around the visual line, the vessels seem to move in the direction opposite to that of the candle. The *fovea* appears without vessels: in my eye it offers a kind of starlike appearance; in others

(*Burow*) it appears as a luminous disc, limited by a crescent-shaped shadow.

The explanation of these phenomena has been given by *H. Müller*. By refraction there is formed at *a* (fig. 105) a retinal image of the candle; the part of the retina thus illuminated sends diffuse light in all directions. The vessel *v* intercepts the rays *av*, so as to form the shadow *b* on the sensitive layer of the retina; it is this shadow that we see (the retina is represented too thick on the figure; really the shadow is very near the vessel). Illuminated directly, the vessel also form a shadow

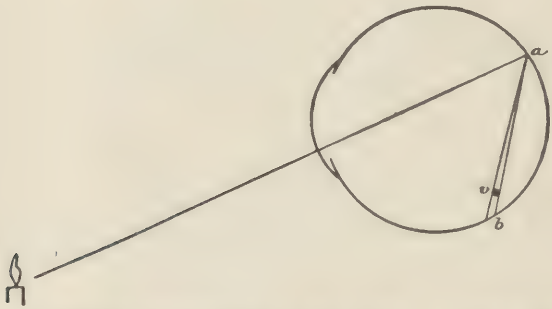


Fig. 105.—Entoptic observation of the vessels. (After *H. Müller*.)

on the sensitive part situated behind it; but this shadow is not usually perceived, because it is always formed at the same place (and because the sensitive layer has thus become accustomed to it) or, perhaps, because the part of the retina which is behind the vessel, being always covered, is never fatigued and consequently remains much more sensitive, so that the little light which passes through the vessel produces as strong an impression on this part as the full light on the remainder of the retina.

It seems that the vessels form in ordinary circumstances negative scotomata, like the spot of Mariotte, although it may be difficult to observe them, except near the papilla, because of the instability of the fixation (see chap. XVIII).

b). We concentrate with a convex lens the light of a flame on the sclera, as far as possible from the border of the cornea. By

bringing the focus somewhat on the sclera, we see dark vessels on an orange ground. The vessels move in the same direction as the luminous focus. On concentrating the light on the internal part of the sclera we succeed in seeing the luminous focus itself under the form of a red sun near the external border of the visual field.

The explanation is analogous to that of the preceding case. The light of the image of the flame, formed on the sclera, passes through this membrane and the choroid, and disperses in the interior of the eye where it forms vascular shadows at unusual places.—*H. Müller* measured the distance *ab* (fig. 106), separating two successive positions of the luminous focus, and the displacement *aB* of the shadow of a vessel corresponding to this displacement of the light. With these data, he calculated that the vessel should be 0.17 to 0.33 mm. in front of the sensitive layer. This experiment seems to prove that it is the layer of the cones and rods that is the sensitive layer, for the distance of the small vessels near the macula from the layer with the cones is very nearly the same (0.2 to 0.3 mm.).

Another phenomenon, also due to the influence of the light which passes through the sclera and the choroid, is observed when we place ourselves near the luminous source, a window for exampe, so that one eye may be illuminated while the other is in the shade. After a little while we then observe, on closing the eyes alternately, that the white objects seen with the illuminated eye present a greenish tint, while they appear reddish to the other eye. The light which passes through the sclera and the choroid is colored red by the blood of the latter membrane. This red light "fatigues" the retina of the illuminated eye, which has the effect of making white objects assume a greenish tint. The other eye sees them red by contrast.

When we read in full sunlight, we sometimees see the letters vividly colored red. The phenomenon is probably of the same kind as the preceding. The red light, which passes through the membranes of the eye, comes to be added to the light which passes through the pupil. It is not sufficiently great to percep-

tibly change the tint of the white paper, brightly illuminated by the sun, but it colors red the black letters, which send back only very little of the white light.

c). Looking at the sky through a stenopaic opening, we see very distinctly pictured the granulated ground and the delicate vessels which surround the macula; but the stenopaic opening must be kept in continuous motion, otherwise the phenomenon disappears. If we look at the sky without the stenopaic opening, the shadow of the vessel is too short to reach the sensitive layer. The same phenomenon is frequently observed when working with the microscope: when we illuminate the field with daylight, we see the vessels by placing the eye at the ocular and giving it a to-and-fro motion. The *muscæ* of the vitreous body may also be very well observed in this way.

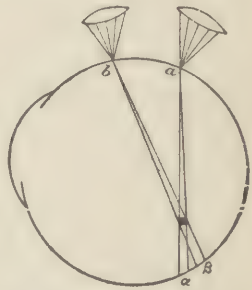


Fig. 106.—Entoptic observation of the vessels by illumination of the sclera.

When making this experiment, as well as the preceding one, we sometimes see the vessels become luminous; this is due to the fact that the parts of the sensitive layer on which the shadow falls, in ordinary circumstances, are now exposed to the light, which acts much more strongly on these parts than on the remainder.

82. Other Entoptic Phenomena.—*a*). Looking towards the sky, we very frequently see bright points which seem to move lively and then to disappear, giving place to others (*Purkinje*). The phenomenon is often more pronounced if we look through a cobalt glass. This phenomenon is explained by the pressure which is exerted on the sensitive layer by a globule of blood which is stopped in a very narrow capillary. (1)

(1) [Another and very probable explanation of this phenomenon assumes that we observe in the little bright bodies some relatively empty capillary spaces, produced by small temporary local stoppages of the circulation in the capillaries of the retina. See the paper by the translator in the *Ophthalmic Record*, February, 1900.]—*W.*

b). By compressing the eye for some time, we can see the retinal vessels and even notice the blood globules magnified about 50 times. The retinal vessels appear bluish; but, before perceiving them, we see those of the chorio-capillary membrane, red on a black ground (*Vierordt, Laiblin*). It seems that this experiment, which *Young* had already made, would not succeed with everybody.

c). A pressure localized on a small part of the sclera gives rise to a *phosphene* which, like every other retinal impression, is projected in the opposite direction. Making the experiment in darkness, we notice that the phosphene has the form of a feebly luminous disc, surrounded by a bright border, corresponding to the inflection of the retina. With very prominent eyes Young succeeded in producing a phosphene corresponding to the macula: exteriro objects which were in the position of the phosphene were still visible, but presented very pronounced deformities.—If we exert on the eye a pressure sufficiently strong and uniform, the entire visual field is darkened in consequence of the anemia of the retina.

d). On making, in a dark room, rapid movements with the eyes, we observe two luminous circles corresponding to the places of entrance of the optic nerves and due to the traction produced by these nerves during the movement.

e). On making an effort of accommodation in a dark room, we sometimes see a very large luminous circle, which is attributed to the traction which the ciliary muscle exerts on the interior membranes of the eye during accommodation (phosphene of accommodation of *Czermak*). I did not succeed with this experiment.

f). A weak electric current makes visible at the moment of closure the dark papilla on a blue ground, if the current is ascending; whitish blue on a dark orange ground if the current is descending: on opening the current we have the phenomena reversed. If the current is strong, we see all the colors of the spectrum mixed.

g). On looking towards the sky through a Nicol prism, we see the *brushes of Haidinger*, an indistinct cross, one of the

arms of which is yellow, the other blue; the phenomenon rotates with the nicol. Some persons can see the phenomenon, but less pronounced, without a nicol.

h). *Phenomena of Diffraction in the Eye.* Looking towards a very intensely luminous point we see it surrounded with an infinity of very fine, many-colored radiations, the whole of which is known under the name of *ciliary corona.* Its extent varies with the intensity of the luminous point. If the latter is very bright (a reflected image of the sun) the diameter of the corona may reach 8 degrees or more. The cause of the phenomenon is, in all probability, to be found in the fibrous structure of the crystalline lens.

Besides the ciliary corona most people see around the entire luminous source a somewhat vivid diffraction ring A, presenting the colors in the well-known order: red outside, blue inside. The diameter of the ring (blue) is about 3 degrees. The space which separates it from the luminous source is filled with the ciliary corona.

Druault and *Salomonsohn* have recently described a second, larger ring B (6 to 7 degrees in diameter), which seems to appear in every one when the pupil is dilated. It presents the colors in the same order as the first, but it is more irregular, and composed of radial striæ. Making these observations with monochromatic light, the ciliary corona presents itself under the form of a luminous dust, which is concentrated towards the periphery so as to form the two rings which I have just described. Quite near the luminous source we see one or two black, very fine rings, due to diffraction by the border of the pupil.

The ring A is probably due to the epithelial cells of the cornea, and analogous to the rings which we observe on looking through a glass plate covered with grains of lycopodium. On covering a larger and larger part of the pupil with a screen, we see the entire ring become indistinct and disappear at once. *Schioetz* has shown that on exposing the cornea to the action of distilled water for some time, as in the experiment of Young, page 202, we observe a pretty system of rings, the first of which

corresponds almost to the ring A. We must note, however, that *Druault,* on looking through a dead cornea, showed the existence of a ring, the dimensions of which scarcely differed from those of the ring A, and which was undoubtedly due to the endothelium of the membrane of *Descemet:* he could remove the entire epithelium of the anterior surface without producing the least change in the ring, which would, on the contrary, disappear as soon as he touched the endothelium.

The ring B, which was previously described by *Donders,* is due to the crystalline fibres which act as a grating. If we cover a part of the pupil with a screen, we see a part of the ring disappear while the remainder does not change. *Druault* succeeded in reproducing the phenomenon with dead crystalline lenses.

The rings which glaucomatous patients see resemble these rings, but are generally larger (10 to 11 degrees). As the size of the rings is inversely proportional to that of the corpuscles which produce them, it is probable that the origin of the glaucomatous rings is to be found in the deepest layer of the corneal epithelium, the cells of which are much smaller than the superficial cells (*Schioetz*). Placing a drop of blood in the conjunctival sac we obtain a very pretty ring (diameter 7.5 degrees for the yellow) surrounded by a second paler ring. The space between the first ring and the light is not black, as for the other rings here described, but yellowish or maroon (*Druault*). These rings seem analogous to those sometimes seen by persons affected with conjunctivitis.

i). I recently described a kind of entoptic phenomenon which I observed in the following circumstances. We surround a lamp with a transparent shade, made of some layers of colored tissue paper, for example. We place ourselves at some meters distance, and interpose an opaque screen, in which has been cut a vertical slit, between the lamp and the eye; the distance of the screen from the eye may vary between 30 cm. and several meters. We close the left eye and fix with the right eye a point on the screen, situated near the right border of the slit. To begin, we hold the head so that the eye may be in darkness. Then we move the head so that the eye enters into the luminous

pencil which passes through the slit while maintaining fixation at the same place. At the same moment we see the phenomenon appear under the form of two blue arcs, feebly luminous, but bright, which go from the slit towards the position of the blind spot by turning around a fixed point (fig. 106*a*). The phenomenon lasts only a moment; an instant later the arcs become narrow, the interior which was black is filled with a blue glow, and the whole disappears, to reappear again with the least motion of the eye. To see the phenomenon with the left eye it is necessary to fix the left border of the slit.

Fig. 106*a*.—Entoptic phenomenon.

According to a communication from *D. Crzellitzer* the phenomenon was described by *Purkinje* in a publication which I have not at my disposal. It seems very prevalent; among persons whom I have examined in this regard, I have met only a single one who has not been able to see it. The form of the arcs recalls the course of the nerve fibres at this place. The appearance resembles that of certain phosphorescent bodies, by the bluish color and by the impression which it gives of being feeble and yet bright at the same time; we again find the same appearance for different other phenomena which we observe in darkness, for instance the after image of Purkinje (see page 292), the trace which the impression of a red coal leaves on the retina, and so forth.

Bibliography.—*Œuvres de Young*, edited by Tscherning, p. 71, 140 and 168.—Purkinje (J. E.). *Beiträge zur Kentniss des Sehens*, 1819, p. 89, *et neue Beiträge*, 1825, p. 115.—Listing (J.). *Beitrag zur physiologischen Optik. Göttingen*, 1845.—Doncan (A.). *De corporis vitrei structura.* Utrecht, 1854.—Brewster (D.). *Transactions of the Royal Soc. of Edinb.*, XV, 377.—Muller (Heinrich). *Verh. der med. physik. Gesellschaft zu Würzburg.* IV, V.—Haidinger. *Ueber das directe Erkennen des polarisirten Lichts und der Lage der Polarisationsebene. Poggend. Ann.* 1844. —Darier (A.). *De la possibilité de voir son propre cristallin Ann. d'oc.* t. CXIV, p. 198, 1895.—Schioetz (H.). *Om nogle optiske equeskaber ved Cornea.* Christiana, 1882.—Druault (A.). *Sur la production des anneaux colorés autour des flammes. Arch. d'ophtalmol.*, Mai, 1898, *et Compte rendu du Congrés d' Utrecht*, 1899.—Salomonsohn (H.). *Ueber Lichtbeugung an Hornhaut und Linse. Arch. f. Anal. u. Phys.*, 1898. Tscherning (M.). *Eine Selbstbeobachtung. Kl. M. f. A.* June, 1898.—Tscherning (M.). *Compte rendu du Congrés d' Utrecht*, 1899.

CHAPTER XII

ACCOMMODATION

83. Measurement of the Amplitude of Accommodation.—We have defined the amplitude of accommodation as the difference between the distances of the *far point* and the *near point,* measured in dioptries. It expresses the value of a convex lens which, added to the eye, would form an image of the *near point* at the position of the *far point.*

For the determination of the *near point* it is necessary, on account of the relation between accommodation and convergence, which I shall discuss later, to close the eye which we are not examining. In order to reach the highest degrees of accommodation, the patient is sometimes obliged to squint inwards, and, if both eyes are open, the need of seeing single will prevent him from attaining the limit of his accommodation. In clinics, we generally confine ourselves to determining the shortest distance at which the patient can read fine print. It is necessary, for this determination, to use very small letters, otherwise the patient may still read them within the *near point,* although seeing the letters only indistinctly.—Another method consists in determining the strongest concave glass through which the patient can see distant objects distinctly (the table of visual acuity), since the concave glass forms an image of them so much nearer in proportion as it is stronger. We can also use optometers, that of *Badal* or of George Bull (1) for example.

The determination of the *near point* is always uncertain, because we can never know whether the patient makes a maximum effort or not. It succeeds especially poorly in persons of little intelligence, in children, etc. Anyhow, to determine it exactly is generally of little practical importance; if we desire an exact

(1) The optometer of *Bull* resembles externally that of *Young,* enlarged, but the principle is different. We look through a lens of 6 D. without slits, and the line is replaced by a series of small dominoes. The patient must simply tell the most distant and nearest of these dominoes that he can see distinctly.

measurement, we can instil eserine, but we thus obtain an amplitude slightly higher than that which the patient would attain, even when trying his best.

For scientific researches it may sometimes be important to know exactly the amplitude of accommodation. We can then determine it with the optometer of *Young,* if the observer is master of his accommodation, that is to say, if he can make an effort of accommodation without fixing a near object. If not, the best means is to offer a hair in a ring, and to see how close we can move it to the eye before it appears dim. We may with advantage add this ring to the optometer of Young.

The amplitude of accommodation diminishes in a very regular manner with age. According to *Donders,* the diminution begins to make itself felt at the close of infancy. It is so regular, at least beginning at 25 or 30 years, that we can frequently determine the age of the patient to almost within one or two years, by means of the optometer of *Bull,* for example. At the age of 47 or 48 years this diminution begins to manifest itself in emmetropes, by the appearance of *presbyopia.* In hypermetropes the presbyopia makes its appearance sooner; it appears later in low myopia, and myopes of a high degree never become presbyopic, although the amplitude of accommodation diminishes in them as in every one else. In emmetropes it is very rare to find an exception to the rule laid down above, unless the pupil is very small. If, therefore, a patient reads without glasses when over 50 or 55 years old, he must be myopic, if the pupil is of the ordinary size.

Presbyopes do not suffer from accommodative asthenopia; when reading they are obliged to hold the book farther away, especially in the evening; the manner in which they hold the book, far from their eyes and near the lamp, is very characteristic.

As to the choice of spectacles, it is clear that if we fix on a distance for work of 33 centimeters we are never obliged to give to an emmetrope glasses of a greater strength than 3 dioptries. But it is frequently useful, especially when the acuity is diminished, to choose a shorter distance for work, for ex-

ample 25 centimeters, corresponding to 4 dioptries. We frequently notice a tendency to give somewhat stronger glasses, which, however, cause only slight inconvenience. Thus, the series

50	55	60	65 years
+ 1	+ 2	+ 3	+ 4 dioptries

is, perhaps, a little strong, especially for high degrees.

Paralysis of Accommodation.—We meet this disease especially in children who have had diphtheria. If we learn that the child has not been able to read for some time past, although it sees perfectly at a distance, and if we do not find hypermetropia, we may be almost certain that it has had diphtheria. The diagnosis of paralysis is verified when the child reads well with the proper convex glasses. Generally there are no other symptoms of ocular paralysis, among others no mydriasis. We prescribe convex glasses almost until the recovery, which generally takes place in a space of two or three months.

The second form of paralysis which we occasionally meet is that which forms part of a more or less complete paralysis of the third pair. It is usually accompanied by mydriasis and frequently by paralysis of external muscles. It seems, however, that it may exist without any complication.—In glaucoma and cyclitis, the diminution of the ampiltude of accommodation is frequently one of the first symptoms.

Spasm of Accommodation.—There have been described two forms of spasm of accommodation. 1° As we have seen, one has been accustomed to diagnose spasm of accommodation when one found a weaker refraction after the instillation of atropine. The existence of this supposed spasm, which is always of a very low degree (0.50 to 1.50 D.), is very doubtful, since the diminution of refraction, after the instillation of atropine, may often be attributed to the weaker refraction of the peripheral parts of the optic system of the eye.

2° We sometimes observe in hysterical patients a true spasm of accommodation, extending most frequently to the entire amplitude, and not to a small part, as in the preceding case.

These cases are rare; they give rise to a transient myopia, which is generally complicated by monocular diplopia.

84. Mechanism of the Accommodation. Historical, A.—Theoretically, the eye could accommodate itself by one of the following mechanisms:

a. Increase of Curvature of the Cornea.

b. Increase of Curvature of the Crystalline Lens.

c. Elongation of the Globe.

These three hypotheses have found their adherents, as also have the two following which are theoretically impossible:

d. Advance of the Crystalline Lens.

e. Contraction of the Pupil.

As to the hypothesis *d,* we must note that, even if the crystalline lens would advance so as to touch the cornea, this advance would not suffice to explain any considerable amplitude of accommodation.—The accommodative contration of the pupil was discovered by *Scheiner.* By looking through an opening a little smaller than the pupil, it is easy to convince one's self that this contraction is not sufficient to explain accommodation.

Apart from these five theories, there have been proposed still others, much less plausible. *Kepler,* who was the first to propound the problem of the mechanism of accommodation, supposed an advance of the crystalline lens, whilst *Descartes* was the first to suppose an increase of curvature of this organ.

The theory of the change of curvature of the cornea found support in the measurements of this curvature made by *Home* and *Ramsden* towards the end of the last century.—The discussion continued until towards the middle of the century, and the false hypotheses on the nature of accommodation have even resulted in two beautiful discoveries. The theoretical researches of *Sturm* on the form of the astigmatic pencil were, indeed, undertaken to prove that accommodation did not exist: this author thought that distant objects were seen with the posterior

part and near objects with the anterior part of the focal interval. On the other hand, when *Arlt* discovered that myopia depended on the elongation of the globe, he was guided by a false idea on accommodation. He thought that the action of the external muscles produced an elongation of the globe, when one is forced to see close at hand; and, as it was known that myopia was a consequence of near work, he concluded that myopia must be produced by an elongation of the globe. On making an autopsy on some excessively myopic eyes, he proved the lengthening of the globe in these cases, and believed that he had thus confirmed his hypothesis. We now know that this form of myopia does not depend on near work, and that accommodation is not obtained by an elongation of the globe, but by an increase of curvature of the crystalline lens!

The question was decided by the observation of the changes of the images of *Purkinje* during accommodation, which prove that accommodation is effected by an increase of curvature of

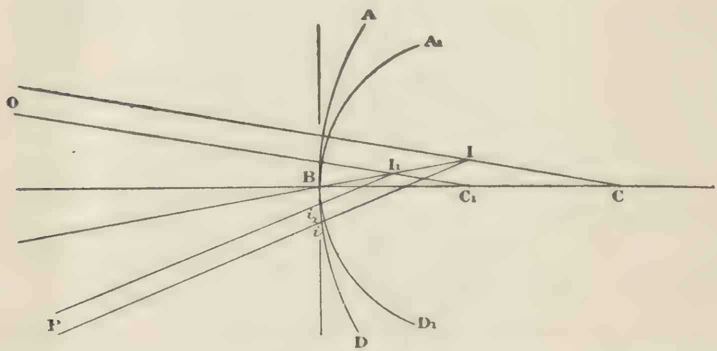


Fig. 107.—Centripetal movement of the catoptric image of the anterior surface of the crystalline lens during accommodation. (Discovered by *Cramer*.)

the anterior surface of the crystalline lens. The discovery was made in 1849 by *Max Langenbeck,* but attracted scarcely any attention; it was only after the beautiful researches of *Cramer* (1851-52) that the truth was definitely accepted. Cramer con-

structed an instrument which he called *ophthalmoscope,* with which he could conveniently observe the catoptric images of the crystalline lens, and it was easy for him to show that that of the anterior surface made, during accommodation, a quite extended centripetal movement. This fact has been verified by all those who have examined the catoptric images during accommodation; it is due to the increase of curvature of the anterior surface.

Let ABD (fig. 107) be the surface in a state of repose and C its center, A_1BD_1 the surface in a state of accommodation and C_1 its center, O an object (a lamp placed at a great distance). To find the position of the image, we draw OC (OC_1) (supposing that these are the apparent surfaces we need not take into account the corneal refraction). The image must be on this straight line, at an equal distance between the surface and center, at I for the surface in repose, at I_1 for the surface in a state of accommodation. The observing eye sees the images projected in the pupillary plane, it sees I at i and I_1 at i_1; it sees the image, therefore, make a centripetal movement during accommodation. It is the same, whatever may be the position of the observing eye; there is only one point where it does not see motion, viz., when it is on the prolongation of line II_1: in this case the two images I and I_1 overlap, and there is no apparent displacement. The line II_1 passes through the point B, the place where the two surfaces touch. This point B, towards which the apparent movement of the image takes place, whatever may be its position in the pupil, is usually situated a little outside the center of the latter; generally it is found almost on the optic axis of the eye.—Recently, *Coronat* again described the centripetal movement, whence he erroneously inferred a see-saw movement of the crystalline lens.

The question of knowing by what change the eye accommodates itself for near vision being solved, it remained to be discovered by what means the change was effected. *Cramer* attributed the change to the contraction of the iris; he thought that the iris in the state of repose was greatly swollen in front, and

became flattened during accommodation by a simultaneous contraction of the sphincter and dilatator. He thought that it thus exerted a pressure on the peripheral parts of the crystalline lens, and that the ciliary muscle, contracting at the same time, exerted a traction on the choroid, which pushed the vitreous body forward. In this way the crystalline lens, subjected to a pressure in its whole extent, except on the pupillary part, became swollen at this place. Several other theories, conceived after that of *Cramer,* also involved the participation of the iris in the act of accommodation; they were necessarily abandoned when *Graefe* published his celebrated case of complete aniridia, of traumatic origin, in which the amplitude of accommodation was intact.

A short time after the discovery of *Cramer,* and without being acquainted with his work, which was published only in the Dutch language, *Helmholtz* made the same observation. He used as his object the distance between two lamps (or a lamp and its image formed by a mirror). During accommodation, the distance between the two images diminished considerably, which is easy to understand, since a sphere forms an image smaller in proportion as its radius is less.

Helmholtz confirmed, moreover, the observation made previously by *Hueck,* according to which the anterior surface of the crystalline lens advances a little during accommodation. He measured the thickness of the crystalline lens, which he found a little greater during accommodation than in a state of repose. He also measured two dead crystalline lenses, and found their thickness greater than that of the living crystalline lens in a state of repose. He further concluded that there was a slight increase of curvature of the posterior surface of the crystalline lens during accommodation.

The following are the numbers which he adopted for his schematic eye, compared with those which he found for the dead eye:

	SCHEMATIC EYE		DEAD EYE	
Radius of the anterior surface..	10mm	6mm	10.16mm	8.87mm
— — posterior surface.	6mm	5.5mm	5.86mm	5.89mm
Thickness	3.6mm	4mm	4.2mm	4.31mm
Focal distance	43.71mm	33.79mm	45.14mm	47.44mm
Total index	1.4545		1.4519	1.4414

Later, he supposed for the schematic eye an index of 1.4371, which would give for the living eye in repose a focal distance of 50.62 mm. and for the eye in accommodation 39.07 mm.

To explain the mechanism of accommodation Helmholtz announced the following hypothesis, which he gave, however, only as probable: in a state of repose the crystalline lens is kept flattened by a traction exerted by the zonula. When the ciliary muscle, of which he considered the anterior extremity as fixed, contracts, it draws the choroid slightly forward, which relaxes the zonula. Having become free, the crystalline lens then swells by its own elasticity, approaching the spherical form.

This hypothesis does not seem to have been at first generally accepted. (1) *Hencke,* and other authors, tried to explain the phenomena observed by other hypotheses. After having discovered the supposed circular fibres of the ciliary muscle, *H. Müller* thought that this muscle changed the form of the crystalline lens by a direct pressure, an idea which was abandoned when it became known that the ciliary body never touches the crystalline lens.

On the other hand, the hypothesis of *Helmholtz* was strengthened by the experiments which *Hensen* and *Voelkers* performed on dogs. They thrust very fine needles into the eye a little behind the ora serrata; on stimulating by the electric current the ciliary ganglion, they saw the free extremity of the needle describe a movement backwards, which proves that the choroid

(1) See Donders. *Anomalies of the Refraction of the Eye.* London, 1864.

is drawn forwards. The phosphene of *Czermak,* which had also been seen by *Purkinje,* also indicates a traction forwards of the interior membranes of the eye. By examining eyes on which an iridectomy had been performed, *Coccius* also established during accommodation, phenomena which could militate in favor of the hypothesis of *Helmholtz* (swelling of the ciliary processes, at least apparent diminution of the diameter of the crystalline lens, and an increase in the width of its *border,* that is to say, of the very peripheral part which is seen black with the ophthalmoscope).

Thanks to these observations, thanks also to the ever increasing fame of *Helmholtz,* his theory ceased little by little to be disputed, and his followers, more loyal than the king, proclaimed as certain what he had himself, with much reserve explained as probable. (2) Thus, *Mauthner* declared the question of accommodation definitely solved by the theory of *Helmholtz.*

Before explaining the mechanism of accommodation as I intend to, I must add some remarks to the historical explanation which we have just read, and which is classical, because there have been authors who have expressed ideas on accommodation in my opinion more correct than those in vogue up to the present time. First, I will make an objection. If it is true that the crystalline lens, in repose, is kept flattened by a traction exerted by the zonula, we should expect to find the dead crystalline lens, taken from the eye in its capsule, in a state of maximum accommodation, or perhaps even still more swollen, since it is no longer exposed to any traction. The followers of *Helmholtz* have, indeed, strongly insisted on the fact that

(2) Great men are, indeed, too reserved through fear of their followers. *Helmholtz* formed the idea of comparing the cornea to an ellipsoid, and although he said intentionally that the cornea does not resemble such a surface, this idea has so taken root that it will be difficult to eradicate it. It is so also with his ideas of accommodation; if we take the trouble to compare the cautious terms which he used, with the mode of expression of his followers, we shall see the difference. The participation of the posterior surface of the crystalline lens in accommodation, which everybody considers as certain, had for *Helmholtz* merely the character of a grand probability.—Measuring his three living eyes, he found for the crystalline lens a thickness about ½mm. less than that of dead crystalline lenses; and he added: "On the other hand, it seems to me very improbable that I have committed an error of a ½mm. making these measurements." In the modern treatises we read, on the contrary: "If we remove the crystalline lens of the eye of a young person, we see it immediately assume a spherical form," etc.

he found the dead crystalline lens thicker than the living crystalline lens in repose, although the difference does not seem to exceed the limit of error (see page 85); but, if we take the trouble of examining his numbers (page 199), we shall see that his dead crystalline lenses were by no means in a state of accommodation. He measured in all three living eyes and found, as radii of the anterior surface of the crystalline lens in repose, 11.9 mm., 8.8 mm. and 10.4 mm., while for the dead eyes he found 10.16 mm. and 8.87 mm. His autopsies, therefore, by no means tell in favor of his hypothesis.

It is so also in the case of the measurements which *Stadfeldt* undertook recently. He measured eleven living human crystalline lenses in a state of repose, with the ophthalmometer; the radius of curvature of the anterior surface of the crystalline was on an average 10.6 mm., while the average of the same radius of the six dead crystalline lenses, taken from the eye in the capsule and measured with the ophthalmometer of Javal, without being exposed to any traction, was 11.4 mm.

85. Mechanism of Accommodation. Historical, B.—It was *Young* who first demonstrated that accommodation was effected by an increase of curvature of the crystalline surfaces. Moreover, he had more exact ideas on what happened during accommodation than those which are actually now in vogue. He wrote his celebrated treatise *on the mechanism of the eye* in 1801, and it is truly astonishing that nearly a century should have passed before his book was understood and before we came to know as much as he. Before proving that the accommodation is effected by an increase of curvature of the crystalline lens, he begins by showing that there can be question only about an increase of curvature, either of the cornea or of the crystallin lens, or of a lengthening of the globe, and he eliminates, as theoretically impossible, the other hypotheses which had been proposed.—Let us now pass to his analysis.

a. Accommodation is not Effected by an Increase of Curvature of the Cornea.—*Young* proved this thesis by a

series of experiments, several of which closely approach our modern ophthalmometric methods. Observing the corneal image he did not discover the least change during accommodation; he obtained, however, a very visible change by exerting a pressure on a peripheral part of the cornea, and this change of curvature is much less considerable than that which would be necessary to explain accommodation.

It is evident that a change of the cornea sufficient to explain accommodation would have been very visible. *Young,* who experimented with his own eyes, was at this time 27 years old, and his amplitude of accommodation measured about 10 D. Actually, we can easily measure a quarter of a dioptry.

His most conclusive experiment consisted in putting the eye under water (fig. 108): he took a weak objective of a microscope which had very nearly the same refraction as the cornea, filled the tube with water, and placed it before his eye also plunged into water. In these conditions, the action of the cornea, which was surrounded by the liquid on both sides, was eliminated and replaced by that of the objective. Now in this experiment the amplitude of the accommodation remained intact.

Fig. 108.—Method of putting the eye under water. (After *Young*.)

b. ACCOMMODATION IS NOT EFFECTED BY AN ELONGATION OF THE GLOBE.—To prove this fact *Young* employed a method which he could use because he had very prominent eyes. He turned the eye inwards as much as he could, and applied against its anterior surface a strong iron ring; then he thrust the ring of a little key on the external side between the eye and the bone, until the phosphene reached the fovea. The rings were kept at a fixed distance. Placed between the iron ring and that of the key, the eye could not lengthen. He should therefore, if

accommodation was effected by a lengthening of the globe, either find it abolished, or see in every case the phosphene, due to the pressure, extend over a much greater surface. But in these conditions the accommodation remained unaltered, and the width of the phosphene did not change.

c. Persons Operated on for Cataract have Lost all Trace of Accommodation.—By measuring with his optometer persons operated on for cataract, *Young* easily succeeded in proving this fact.

d. He then explained the direct proofs of the increase of curvature of the crystalline lens. It was to these experiments that I alluded when I said that he had, on accommodation, ideas which are ahead of our own time. I again performed these experiments some years ago, and it was by starting from them, by repeating them and adding others to them, that on the mechanism of accommodation I have come to form ideas which differ materially from those which have been current up to the present.

It was impossible for *Young* to describe clearly the mechanism of accommodation, because at that time the non-striped muscle fibres were unknown, which kept him from suspecting the contractility of the body known later as the ciliary muscle; he was thus led to postulate the contractility of the crystalline lens, an hypothesis which he soon abandoned. His researches in this direction necessarily could not but remain fruitless.

The ciliary muscle was discovered, at the same time and separately, by *Bowman* and *Bruecke* (in 1846). Ideas on the structure and function of this muscle have varied considerably. Sometimes the anterior extremity, sometimes the posterior extremity has been considered as fixed; sometimes the mobility of both extremities was taken for granted (*Donders*), sometimes both were considered fixed. The oldest descriptions seem to be the best, especially that of *H. Müller;* most of the modern works seem influenced by the hypothesis of *Helmholtz*. Ac-

cording to *H. Müller,* we must distinguish between a longer superficial part (fig. 109) composed of longitudinal fibres which are inserted in front on the sclera, near the canal of *Schlemm,* and which are lost behind in the choroid, and a deep part, also composed in greater part of longitudinal, but shorter, fibres, and not going so far either in front or behind, as the superficial fibres. These fibres are not inserted in the sclera. The deepest layer is composed of oblique or even circular fibres. *Müller* thought that they formed a true sphincter, but the existence of such a sphincter is by no means proved; after holding for some time a circular direction, these fibres seem to change their course and to continue in the deep longitudinal fibres. It seems that at least a part of the deep longitudinal fibres ends thus; others seem to end free, without insertion, in the part of the muscle which goes towards the anterior chamber.

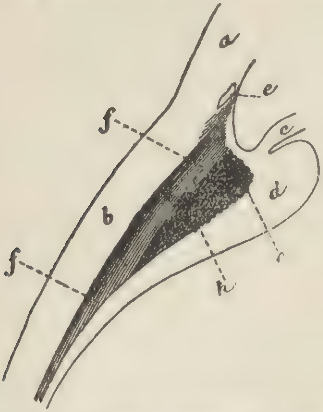


Fig. 109.—Ciliary muscle of man. (After *H. Müller.*)
a, cornea; *b,* sclera; *c,* iris; *d,* ciliary process; *e,* canal of Schlemm; *f,* longitudinal fibres; *g,* circular fibres; *h,* transitional fibres of the ciliary muscle.

By dividing a hardened eye into two halves by a longitudinal section, we easily discover the small white triangle of the ciliary muscle. If we then exert a traction upon the iris in order to separate the ciliary body from the sclera, we do not tear the muscle from its insertion near the canal of Schlemm, but we divide it into two leaflets, both of which end, behind, in the choroid. In the fresh eye there also always remains a part of the muscle adhering to the sclera as *Mannhardt* had already observed. When making this experiment we produce an appearance which forcibly recalls the ciliary muscle of certain animals

(the cat, for example, fig. 110), in which the muscle is divided in front into two parts separated by a prolongation backwards of the space of *Fontana.*

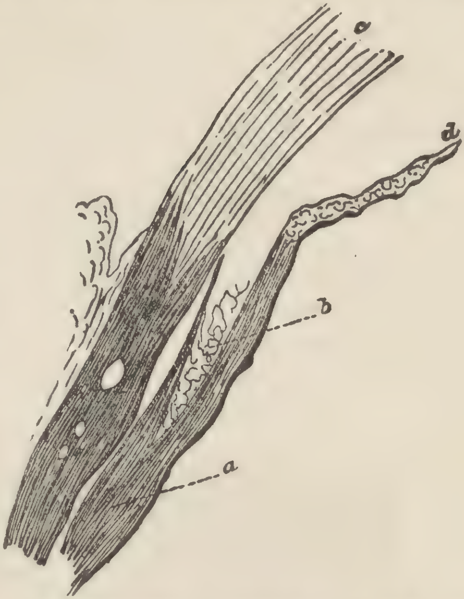


Fig. 110.—Ciliary part of the eye of a cat. *a,* Ciliary muscle dividing in front into two leaflets; *b,* canal of Fontana; *c,* cornea *d,* iris.

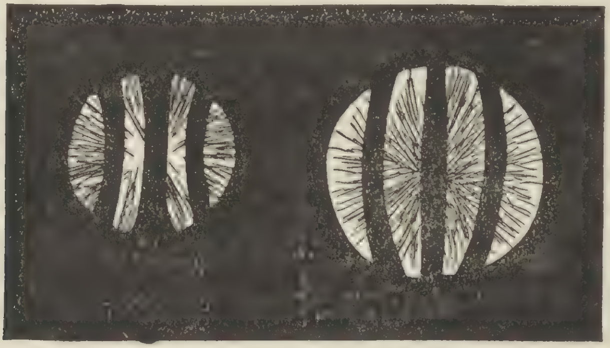
Among the authors who have reached a result different from that of *Helmholtz,* I shall mention *Mannhardt,* who, by a study of the comparative anatomy of the ciliary muscle, reached the conclusion that it is the posterior extremity of the muscle which should be considered as fixed, and that accommodation must be produced by a traction exerted by the ciliary muscle on the zonula. He was vigorously attacked by *H. Müller,* and his work scarcely attracted attention because it could not be considered that a traction on the zonula could produce an increase of the curvature of the crystalline surfaces. We cite, moreover, the remarkable observations of *Foerster* (1864), according to which the tension diminishes in the anterior chamber during accommodation. He observed several patients in whom he performed paracentesis so that the iris and crystalline lens were nearly in contact with the cornea. When the patient made an effort of accommodation, the middle of the cornea became depressed to assume its old form by the relaxation of the accommodation. It must be noted, however, that the phenomenon persisted after instillation of atropine. In persons having a corneal fistula he obtained an

almost immediate effect from atropine by placing a drop in the conjunctival sac and making an effort of accommodation, the liquid being sucked into the anterior chamber by the diminution of tension. These beautiful observations, which *Arlt* declared equivalent to physiologic experiments, are scarcely explicable by the theory of *Helmholtz*.

86. Personal Experiments.—Finally I come to my own experiments on accommodation: the first (1°) are derived from the statements of *Young*.

1° *The amplitude of accommodation diminishes towards the periphery of the pupil.*

a. Aberroscopic Phenomena.—We have already seen that with the aberroscope (see page 122) most persons see the



I II

Fig. 111.—Change of aberroscopic phenomena during accommodation. I, Repose. II, Accommodation.

shadows concave towards the periphery. But, on making an effort of accommodation, the form of the shadows change: they turn their concavity towards the middle, which indicates that the refraction increases towards the middle (fig. 111). After what we have said on page 118 it follows that the central refraction must have increased more than the peripheral refraction.

Some people in a state of repose see shadows straight or slightly concave towards the middle. In such people this de-

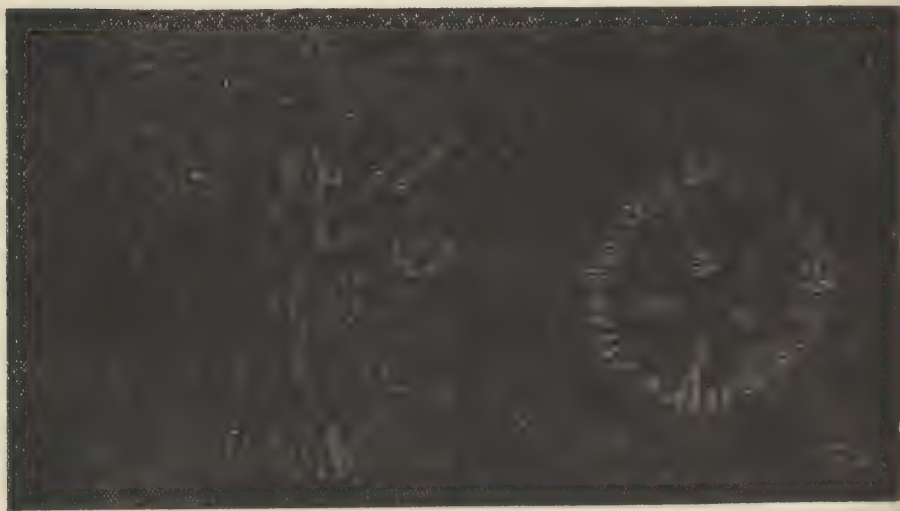
formity becomes still more pronounced during accommodation.

b. Change of the Circle of Diffusion.—If we observe a distant luminous point, after having made the eye myopic, it appears under the form of a luminous disc, the brightness of which is generally uniform or concentrated at the middle. During accommodation we see it change its appearance; we see a feebly luminous disc surrounded by a bright border. According to the explanation given on page 117, this observation means, like the preceding one, that the spherical aberration is overcorrected during accommodation, that is to say, that the central accommodation is greater than the peripheral accommodation. Although accommodation may increase the refraction of the eye by many dioptries, the circle of diffusion increases only slightly,

Fig. 112.—Appearance of the luminous point (right eye of Professor *Koster*, treated with cocaine).

at least when the pupil is dilated. Figure 112 shows the appearance of the circle of diffusion of an emmetropic eye; rendered 8 D. myopic by a convex lens, this eye sees the circle of diffusion represented by *a*, figure 113, while *b*, same figure, represents the form under which it sees a luminous point by making an effort of accommodation of 8 D. without a lens. The pupil

was dilated. The explanation of the phenomenon is easy: let us imagine the pupil and circle of diffusion divided into corresponding zones; it is clear that if the accommodation is everywhere the same, all the zones of the diffusion circle ought to increase, while, if the accommodation diminishes towards the periphery, the outside zones increase little or nothing and the central zones, on increasing, come to partly cover the peripheral



a *b*

Fig. 113.—The same eye as in figure 112.

a, Appearance of the luminous point, the eye being rendered myopic 8 D. with a convex lens (Repose). *b*, Appearance of the luminous point, without lens, the eye accommodating 8 D.

Measured with the optometer of *Young*, the central accommodation was 8 D.; the peripheral accommodation (at 2.5mm from the axis) was 3.3 D.

zones. This is the reason why the circle of diffusion is surrounded during accommodation with a bright border, without increasing much in diameter.

c. Measurement with the Optometer of Young.—The optometer of *Young* enables us to measure directly the difference betwen the central accommodation and peripheral accommodation.

We measure the central accommodation with the two nearest slits (see page 122), which we place as nearly as possible at the middle of the pupil, and the peripheral accommodation with the triangular plate which we lower just enough to be able to see see the two lines. In this way we prove that *at the border of the pupil* (supposed to be five millimeters) *the amplitude of the accommodation is only half the central accommodation* or still less. If, after having dilated my pupil *to the utmost* (with a mixture of cocaine and homatropine), I use an interval of 7 millimeters, my accommodation which, at the middle of the pupil, is 2.5 D. to 3 D., diminishes nearly to zero (0.2 D.) on the borders. Here are some measurements:

	Central amplitude (interval 0.75 mm.).	Peripheral amplitude (interval 5 mm.).
Young	9.8 D.	4.2 D.
Koster	8 D.	3.3 D.
Demicheri	7.5 D.	3.7 D.
—	6 D.(1)	3 D.
—	4 D.(1)	2 D.
Mme T.	6.7 D.	3.8 D.
Tscherning	3 D.	1.25 D.

We find still more considerable differences between the central and peripheral accommodation, by placing the two slits sometimes at the middle of the pupil, sometimes near the borders:

	AMPLITUDE OF ACCOMMODATION		
	Temporal border.	Center.	Nasal border.
Demicheri (Homatropine)		6 D.	2 D
— —	0	4 D.(1)	1 D.
Mme T.	5 D.	6.7 D.	5 D.
Tscherning (Homatropine)	0.25 D.	3 D.	0

d. SKIASCOPIC EXAMINATION.—Observations *a* and *b* are easy to make, but they require that the observer be young, that his pupil be well dilated and that he be master of his accommodation; observations with the optometer of *Young*, as well as those with the ophthalmometer, which I shall describe forthwith, are quite delicate and require special instruments. But we possess

in skiascopy with a luminous point a very convenient means of studying the nature of accommodation. To make the observation we select a child or a young person whose pupil is well dilated with cocaine. It is better to select a person whose pupil is well dilated, who is almost emmetropic, and who has not too much aberration in a state of repose. We place the lamp, surrounded with its perforated screen, at one side of and a little behind the observed person and we project light on his eye by means of a concave mirror, which forms the image of the opening at 15 to 20 cm. from the observed eye, in which position we place a mark of fixation. As long as the observed person does not accommodate, the condition of *Jackson* is not fulfilled, and we see the pupil entirely illuminated, but at the moment when the observed person fixes the fixation mark the ring of over-corrected aberration appears with all desirable distinctness. The phenomena is especially striking if we compare the appearance of the accommodated eye with that of the non-accommodated eye,

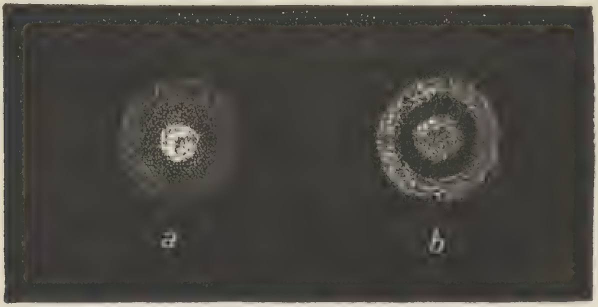
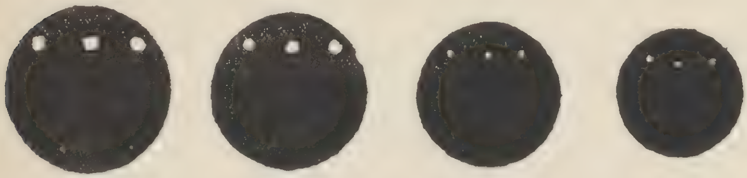


Fig. 113*a*.—*Skiascopic examination of accommodation.* *a*, Appearance of the emmetropic eye made myopic with a lens of +5 D. *b*, Appearance of the same eye, accommodating 5 D. without lens.

made myopic with a convex glass (fig. 113*a*). We have observed (page 119) that we see luminous, under these circumstances, the parts of the observed pupil which send light into the observing eye. Placed at 50 cm. the existence of the ring indicates, therefore, that there are, towards the borders of the pupil, parts, the myopia of which does not exceed 2 D., for otherwise the rays

proceeding from these parts would have already crossed the axis, and would not enter into the observing eye. To determine



a b_1 b_2 b_3

Fig. 114.—Reflection images, on the anterior surface of the crystalline of my right eye, of three lamps placed on a horizontal line, *a*, in a state of repose; b_1 b_2 b_3, in different stages of accommodation. Highest accommodation 3 D. with cocaine.

the degree of aberration produced by accommodation, we approach nearer and nearer the point of fixation; the ring becomes thinner and thinner, but it is rare that it disappears completely before the accommodation attains a very high degree. I have thus shown that a central accommodation of 8 D. accompanied a peripheral accommodation of 2 D. in a case in which the pupil was very large. The condition was, therefore, still more pronounced than in the cases which I examined with the optometer. The phenomena may present themselves a little differently if the positive aberration is very pronounced in a state of repose, but on making the calculations we obtain the same result.

2° *During accommodation the anterior surface of the crystalline lens increases in curvature at the middle, while it is flattened towards the periphery.*

I place the arc of the ophthalmophakometer horizontally, and attach three incandescent lamps to it, so that they are on the same horizontal line and just far enough apart for all three images formed by the anterior surface of the crystalline lens to be visible in the pupil. I direct the look of the observed person so that the three images are situated near the upper border. In a state of repose they are arranged in a straight line (fig. 114*a*) or following a curve slightly concave towards the center (fig. 115 A); during accommodation, they form a curve

convex towards the middle (fig. 114 b_1, b_2, b_3, 115 B), and the curvature of which is more pronounced in proportion as the accommodation is greater.

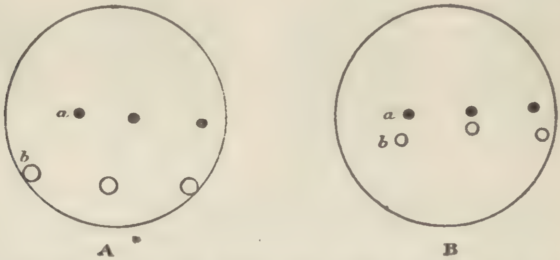


Fig. 115.—Reflection images of the right eye of Mme T.—A, in a state of repose; B, during accommodation (after a drawing of Professor *Koster*).—*a*, corneal images; *b*, images of anterior surface of the crystalline lens. Accommodation of 6 D.

It is easy to see that this phenomenon indicates a greater curvature at the middle than towards the periphery: indeed, let us suppose for an instant that we have added three other lamps, which would form their images near the lower border of the pupil, and let us consider as objects the distances between the two lamps situated on the same vertical line. We would thus have

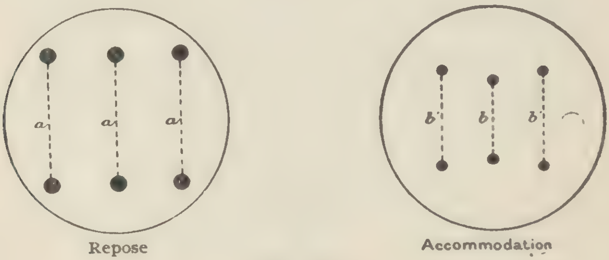


Fig. 116.

three equal objects, the images of which would be of the same size in a state of repose (*aa*, fig. 116), which indicates that the curvature is the same everywhere; but, during accommodation, the image (*b*, fig. 116) of the middle is considerably smaller

than the other two, b_1 b_1, which indicates that the curvature is greater at this place.

We observe an analogous phenomenon on the cornea, in cases of keratoconus. The keratoscope of *De Wecker* and *Masselon* is formed by a white square on a black ground. On examining a case of keratoconus with this instrument, and causing the look to be so directed that the apex of the keratoconus coincides with the axis of the instrument, we see the sides of the image of the square assume the form of curves turning their convexity towards the middle (fig. 117).

We might think, from these phenomena, that the curvature of the peripheral parts increases during accommodation, but less

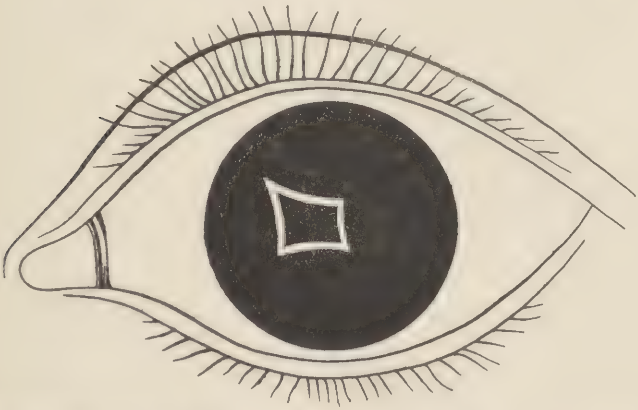


Fig. 117.—Deformity of the corneal image of a white square in a case of keratoconus. (After *Masselon*.)

than that of the central part. Nothing of the kind: the peripheral parts undergo a real flattening which causes, however, an increase of refraction. To understand this fact, which might appear paradoxical, we must recall what I have said on page 17 on refraction by surfaces of the second degree. Outside of the axis, it is the *normal* and not the radius of curvature which, for refraction (and also for reflection), plays the part of the radius of the sphere, supposing that the luminous point (or, in the case of reflection, the observing eye) is on the axis.

In figure 118, BDE represents a curve of the second degree, AF its axis, BH the radius of curvature at the point B, BG the *normal* at this point and the dotted curve a circle drawn with BG as radius. The luminous ray AB is refracted in the direction BF, exactly as if the surface were replaced by the circle BE.

The measurements which we have made with the optometer of Young enable us to calculate approximately the form of the surface, and the calculation will explain at the same time what I have just said. Let us suppose that all the accommodation is effected by the anterior surface, and let us take the experiment of *Demicheri* as an example. He had, at the middle, an accom-

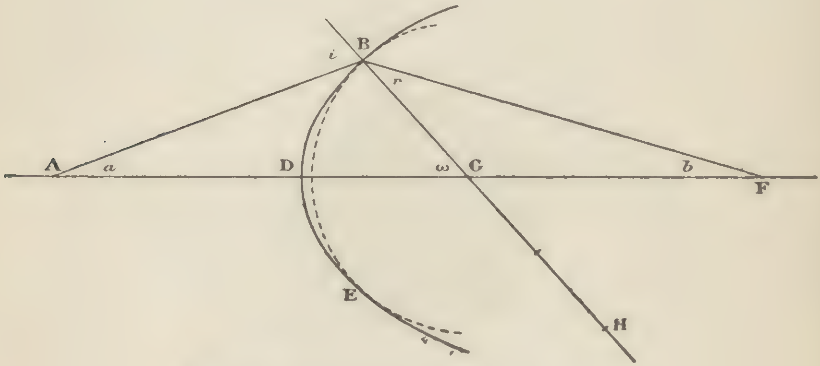


Fig. 118.—Refraction by a parabolic surface.

modation of 7.5 D., at 2.5 mm. from the axis an accommodation of 3.7 D. Let us suppose 10 millimeters for the radius of the anterior surface in a state of repose and 1.06 for the index of the crystalline lens in relation to the aqueous humor. We express the refraction of the surface by the inverse of the anterior focal distance $\frac{1}{F}=\frac{n-1}{R}=\frac{0.06}{0.010\text{ m}}=6$ D. During accommodation the central refraction increased 7.5 D.; the refraction of the surface would be, therefore, at this place 13.5 D. Whence we obtain the radius po by the formula $\frac{n-1}{po}=\frac{0.06}{po}=$ 13.5 D., which gives $po=4.44$. At 2.5 mm. from the axis the accommodation was 3.7 D., the refraction of the surface in a state of accommodation

6 D. $+3.7$ D.$=9.7$ D., and the normal N, at this place, would be found by the formula $\frac{n-1}{N}=9.7=\frac{0.06}{N}$, which gives $N=6.1$ mm. We can then find the radius of curvature p, at this place, by the formula $p=\frac{N^3}{po^2}$, which holds good for all surfaces of the second degree. It gives $p=12$ millimeters. We see that the surface is already flattened at this place during accommodation, and it is manifestly flattened still more farther towards the periphery. If a small part of the accommodation is effected by the posterior surface, as is probable, the flattening of the anterior surface towards the periphery must be still greater, for it is probable that the part of the accommodation which is due to the posterior surface diminishes relatively much less quickly towards the periphery. Supposing that the portion of the accommodation due to the posterior surface be 1 D., as well at the center as near the border of the pupil, we would have for the anterior surface $po=4.8$ mm., $p=14.2$ mm. The surface would have the form of a quite flattened hyperboloid (fig. 119), the apex of which would correspond very nearly with the optic axis of the eye, and would be found a little outside the visual line. It is interesting to observe that among all the surfaces of the second degree having $po=4.8$ mm., it is this hyperboloid which most nearly approaches the form of the surface in a state

Fig. 119.—Deformity of the crystalline surfaces during accommodation. The full curves indicate the shape in a state of repose, the dotted curves the accommodative shape. (Accommodation 7 D.)

of repose. Accommodation is effected, therefore, by a minimum deformity.

3° By placing the cursor A of the ophthalmophakometer above the telescope, and requesting the observed person to look towards the latter, we observe, when he makes an effort of accommodation, the following phenomena (fig. 120):

I. The image of the anterior surface of the crystalline descends quickly towards the corneal image, and is finally hidden behind the latter. It is this displacement which has been described by *Cramer*. Towards the end of this phase the pupillary contraction begins.

II. This movement ended, the small image of the posterior surface of the crystalline descends in its turn by a slow and

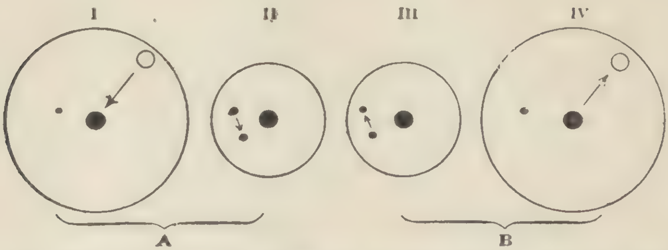


Fig. 120.—The four apparent phases of accommodation. ● Corneal image. —O Image of the anterior surface of the crystalline.— • Image of the posterior surface of the crystalline. A, accommodation; B, relaxation.

abrupt movement. Its displacement is much less than that of the large image; and, while the latter moves in a straight line, the small image is displaced in a curve with its concavity turned towards the middle. The pupillary contraction is greatest during this phase.

III. When the observed person relaxes his accommodation, the small image again ascends to resume its old place with a quick movement, as if moved by a spring.

IV. This movement ended, the large image re-ascends in its turn; its movement is rather slow, and as if hesitating.

The accommodative phenomena seem, therefore, to take place in two steps.

During the displacement of the small image, the large one is concealed behind the corneal image, so that we cannot see

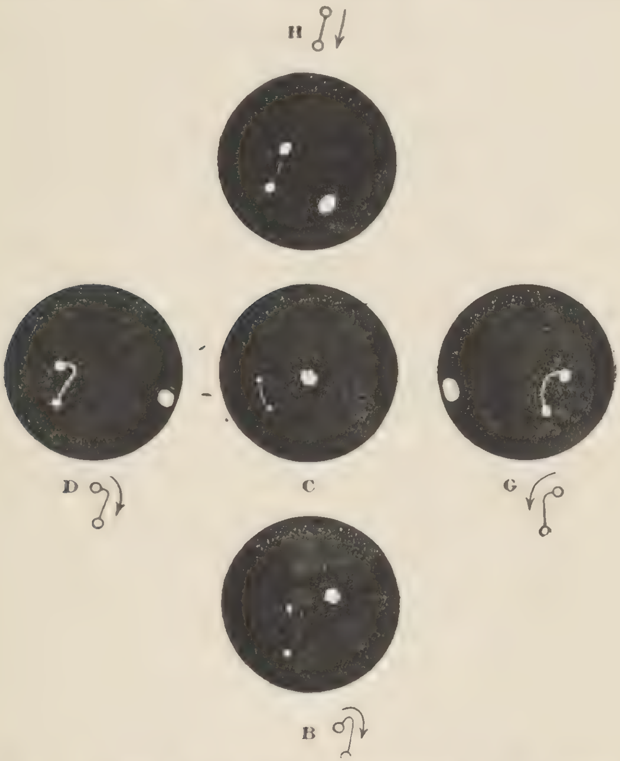


Fig. 121.—Right eye of Mme T.—Displacements of the image of the posterior surface during accommodation, observed with the ophthalmophakometer. C, by fixing the telescope; D, by looking to the right; G, by looking to the left; H, by looking upwards; B, by looking downwards.—The large white spot is the corneal image, the two small white spots indicate the position of the image of the posterior surface of the crystalline in a state of repose and during accommodation. The arrows indicate the direction of the displacement which takes place when an effort of accommodation is made.

whether it is displaced or not; it is not easy to find a direction of the look such that we can follow the two crystalline images

during the entire accommodative displacement. Sometimes they are concealed behind the corneal image, sometimes behind the iris. I have, however, succeeded in doing so by using two lamps, one for each image; in this way, we can satisfy ourselves that the large image undergoes a slight displacement downwards at the same time as the small one, but this displacement of the large image is concealed by the corneal image when we perform the experiment as I have just described. It is especially easy to observe the displacement downwards of the large image, if the direction of the look of the observed person is such that the image in repose is placed near the internal or external border of the pupil. The movement of *Cramer* then takes place in a horizontal direction. Having reached the end, the image makes a bend, becoming displaced a little downwards, but this latter displacement is much less than that of the small image. I may add that the small image is displaced downwards, whatever may be its position in the pupil (fig. 121), which indicates that the cause can be sought neither in the increase of curvature of the surface, nor in a displacement forwards or backwards of the crystalline lens. But this displacement downwards of the image is combined with a quite small centripetal displacement, which also takes place whatever may be the position of the image in the pupil, and which is probably due to a slight recession of the posterior surface.

The observation has again been made by *Hess* and *Heine*. They have found that the displacement of the small image takes place downwards, whatever may be the position of the head; if we lean the head on the right shoulder, the displacement of the small image takes place towards the side which is downwards, that is to say, for the right eye towards the temporal border of the pupil, for the left eye towards the nasal border. I was able to verify this observation, which seems to indicate that the change takes place under the influence of the weight. *Hess* also observed that an entoptic figure, situated on the posterior surface of the crystalline lens, is displaced downwards by a maximum accommodation, whatever may be the position of the head.

4° *Other Phenomena Accompanying Accommodation.*—We

have seen that *Hueck* discovered a slight advancement of the anterior surface; *Helmholtz* confirmed this observation. It is possible that we may sometimes meet such a displacement, although the experiment of *Helmholtz* did not succeed very well with me, and although I am not sure that his observations do not admit of another explanation. In the eye with which I have made my experiments, the anterior surface did not advance; the part corresponding to the pupil did not change its place, but the part covered by the iris receded with this membrane. There is formed during accommodation, at the anterior surface of the iris, a circular depression (fig. 122), the peripheral border of which, corresponding to the ciliary body, rises in a peak, while the central border presents a very gentle slope, corresponding to the anterior surface of the crystalline lens. I commend this observation, which was already made by *Cramer,* but which has often been regarded as proving an enlargement of the anterior chamber in the angle of the iris; it is easy to see that the most peripheral parts of the posterior partition of the anterior chamber do not recede. The phenomena are not

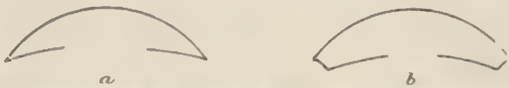


Fig. 122.—Change of the anterior chamber during accommodation; *a,* repose; *b,* accommodation.

always equally pronounced, but we can nearly always find at least a trace of them in young subjects. We can make the observation by oblique illumination, but the use of the magnifying glass (monocular) is not to be recommended; binocular vision is necessary in order to properly account for the change in the level of the iris. When the phenomenon is quite pronounced, we thus obtain a quite distinct idea of the conical form which the anterior crystalline assumes during accommodation.

As to the posterior surface of the crystalline lens, its changes are less manifest. We have seen that the catoptric phenomena seem to indicate a slight increase of curvature. The posterior surface remains very nearly in its place during accommodation;

sometimes, however, we observe phenomena which seem to indicate that it recedes a little.

The much-discussed question of knowing whether the thickness of the crystalline lens changes during accommodation is very difficult to decide, because the change, if it exists, does not exceed the limit of an error of observation. Influenced, perhaps, by the observation of *Helmholtz,* I had thought an increase of thickness established. Recently I took up the subject anew in collaboration with Professor *Koster;* we went to much trouble without being able to reach a definite result.

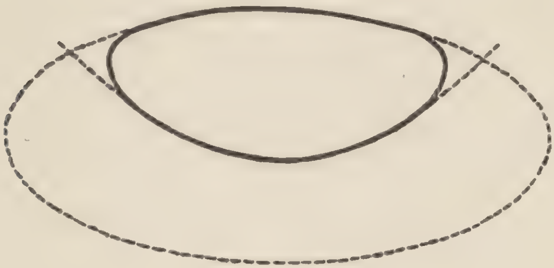
87. The Author's Theory of Accommodation.—After the observations which I have just described in the preceding paragraph, and which can be briefly expressed by saying that *accommodation is effected by the temporary formation of an anterior lenticonus,* the hypothesis of *Helmholtz* does not seem tenable; for

Fig. 122*a*.—*Reflection images on the anterior surface of the dead crystalline lens.* A, at the center; B and C, towards the borders.

it is not easy to conceive how such a mechanism could produce a flattening of certain parts of the crystalline lens and at the same time an increase of curvature of the other parts.

I have already observed that the curvature of the anterior surface of the crystalline lens of the dead eye corresponds with that of the living crystalline lens in a state of repose, and not at all with the accommodated crystalline lens. But the difference between the dead crystalline lens and the accommodated crystalline lens is still more striking, if we consider not only the curva-

ture at the middle, but the form of the entire surface, because the anterior surface of the accommodated crystalline lens is flattened towards the borders, as I have just explained; in the dead eye the curvature, on the contrary, increases considerably towards the borders, the surface having the form of an ellipsoid



A

B

Fig. 122*b*.—A, the dead crystalline lens; B, the accommodated crystalline lens. The dotted lines indicate the form of the surfaces of the second degree, to which the majority of crystalline surfaces most nearly approach.

of revolution around the short axis. This fact, which was already established by *Krause,* (1) is especially very striking if we examine the eye with the ophthalmometer, as I explained on

(1) Helmholtz seems to have been led into error by the celebrated measurements which Jean Louis Petit had made at the commencement of the eighteenth century. Most of the measurements of Petit are very exact, but those of the curvatures of the surfaces are without any value. He had a series of copper plates cut in the form of arcs of circles of different radii. His only means of determining the curvature of the surfaces of the eye consisted in finding the arc of the circle, which seemed to him to conform to the surface. The measurements of Krause are astonishingly good if we consider the manner in which he made them. He cut a fresh eye in two, along the axis, placed one-half of it in water under a micrometer and examined with a microscope of little magnifying power.

page 74. The most usual way is to remove the prism, and observe the image of the keratoscopic disc. As long as the ophthalmometer is placed in the direction of the axis of the crystalline lens, the images of the circle are round, but, if we displace the instrument so as to form the image near the border, it changes into an ellipse with the long axis vertical. Comparing figure 122*a* with those on page 75, we see that the deformity of the surface is quite the contrary of the conical form.—Following are the radii of curvature from 5° to 5° of an eye measured by Holth, compared with those which I have calculated for the eye of Demicheri in maximum accommodation:

	Age	0°	5°	10°	15°	20°
Dead eye............	28	12.4mm	12mm	11mm	9mm	7mm
Accommodated eye....	25	5.6mm	5.9mm	7.0mm	18.0mm	79.2mm

We see that we can scarcely suppose a more pronounced difference (fig. 122*b*). I, therefore, set myself to study the physical qualities of the crystalline lens, by using especially the lenses of horses, which are very large and consequently easily handled, and I have found that we cannot consider the crystalline lens as a simple elastic body in the sense of *Helmholtz*. The contents of the crystalline lens are composed, in the adult, of two parts, the nucleus, which cannot change its form, and the superficial layer which, on the contrary, possesses this faculty to a very high degree; its consistence is very nearly that of a solution of very thick gum. I call this layer the *accommodative layer* in order to show that it is due to it that the eye can accommodate itself. According as age advances, the nucleus increases while the accommodative layer diminishes and with it the amplitude of accommodation. The whole is surrounded by a capsule which is inextensible or very nearly so (*Hocquard*).

It has always been supposed that a traction exerted on the zonula must flatten the crystalline surfaces, while a pressure exerted on the borders would have, on the contrary, the effect of increasing their curvature. Nothing of the kind: a pressure exerted on the borders has, on the contrary, the effect of flattening the surfaces, while a traction exerted on the zonula increases

the curvature of the surfaces at the middle, while flattening them towards the periphery.

To verify this fact we take the crystalline lens from the eye of an ox or a horse, which must not be too old, with the capsule and zonula of *Zinn*. It is easy to see that by compressing the borders the surfaces are flattened; to observe the effect of traction we take hold of the zonula on both sides, very near the crystalline lens, and, by pulling, we can, on looking at the crystalline lens sideways, see that the anterior surface assumes a hyperbolic form (fig. 123). But we obtain a better idea of the deformity by studying the catoptric images. We place the crystalline lens with the anterior surface uppermost on a table and fix above it, at some distance, an opaque ring on which we have stretched a sheet of transparent paper; by illuminating this sheet of paper we see the catoptric image of the ring formed on the anterior surface of the crystalline lens as a black circle. We can also replace the ring by a big lens. The size and distance of the ring must be chosen so that the image may be sufficiently large, and placed so that the image may be centered with the crystalline lens. Then, by exerting a traction we see the circle change into an oval, the short axis of which corresponds with the direction of the traction, which proves that the curvature increases in that direction. The experiment succeeds the more easily the larger the ring. If we place the ring so that its image is near the border of the crystalline lens, we see it lengthen in the direction of the traction, which indicates a flattening in this

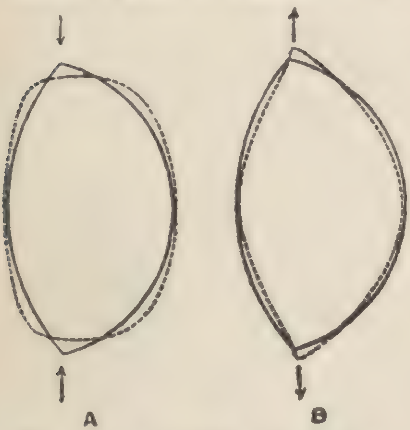


Fig. 123.—Crystalline lens of the ox twice enlarged: The dotted line indicates the form which the crystalline lens assumes: A, by a lateral pressure; B, by a traction exerted on the zonula. The arrows indicate the direction of the forces.

direction. *Dr. Crzellitzer* has recently constructed an instrument by means of which we can exert a traction on the zonula in all directions at once, and with which we can still better imitate accommodation. Instead of the ring we may use two candles placed so that their images are in the direction of the traction; on stretching we see them make a centripetal movement analogous to the movement discovered by *Cramer,* but much less extended. Indeed, on the one hand, it is probable that these animals (1) have not a very well developed accommodation, and on the other hand, it must not be forgotten that in the eye the displacement appears nearly doubled by the magnifying action of the cornea. The experiment can be considered only as an imitation of accommodation on a large scale; but the fact that we can obtain an increase of curvature by a traction exerted on the zonula is beyond doubt.

Furthermore, we should scarcely expect any other result. I have several times emphasized the fact that the nucleus has a much more pronounced curvature than the surfaces of the crystalline lens, and moreover, that it cannot change its form unless we crush it. Glancing at figure 124, we readily understand that by exerting a traction on the zonula the peripheral parts must flatten, while at the middle the curvature increases on account of the greater resistance and curvature of the nucleus. And the result will be the same if there is no nucleus, as is the case in young people, only if the curvature and resistance increase towards the center. The

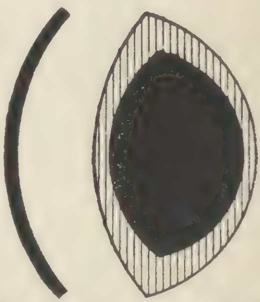


Fig.124.—Optic system of the eye of the ox (magnified twice).

(1) Dr. *Stadfeldt* later verified the results with human crystalline lenses, which he placed in a cork ring, fixing two opposite parts of the zonula with very fine needles. He measured the curvature of the surfaces with the ophthalmometer of *Javal* and *Schioetz,* and then determined the position of the focus, or rather that of the focal lines, with a microscope. In consequence of the traction, he always caused astigmatism, the maximum of curvature corresponding to the direction of the traction. On a crystalline lens belonging to a person aged 38 years, he thus produced an astigmatism of the anterior surface of 4 D. The posterior surface was only very slightly influenced.—The astigmatism disappeared with the traction.

increase of curvature of the central layers is visible on any preparation of the crystalline lens. The increase of resistance finds its optic expression in the increase of index towards the center.

By traction on the zonula we have obtained changes analogous to those which we observe during accommodation, and it seems to me that the structure of the ciliary muscle lends itself very well to the production of such traction. We have seen that it is composed for the most part, of longitudinal fibres, that the most superficial fibres are inserted in front on the sclera, near the canal of Schlemm, while the middle fibres end free near the surface which lies towards the anterior chamber, and that the deepest fibres are combined with the oblique and circular fibres which, perhaps, form their terminations. The muscle has the form of a little triangle, the external surface of which rests on the sclera, while the internal surface is turned towards the vitreous body and the anterior surface towards the anterior chamber. During contraction the antero-external angle remains fixed, the antero-internal angle recedes, as we can see directly in the anterior chamber, and the posterior extremity advances as the experiments of *Hensen* and *Voelkers* prove. The recession of the anterior part exerts on the zonula the traction which produces the deformity of the anterior surface; the advancement of the posterior extremity exerts on the choroid a traction which has the effect of sustaining the vitreous body and indirectly the crystalline lens, so that the latter does not recede under the influence of the traction. As far as the actual result is concerned, it matters little to which of the two actions we attach the greater weight. Let us conceive, for example, a moment when the anterior extremity may be fixed: the result of the contraction of the muscle would be that the crystalline lens, on account of the traction exerted on the choroid, would be pushed a little forward, which would produce also a traction on the zonula, which would suffice for the deformity of the crystalline surface. It may be that there exist, in this relation, individual differences as the dis-

agreement between the observations of *Helmholtz* and my observations seems to indicate. (1)

I think that this theory explains quite satisfactorily the greater part of the phenomena which accompany accommodation. It explains, in the first place, the deformity of the anterior surface; the direction of the zonula in the living eye is such that the effect of the traction must act almost exclusively on the anterior surface. It explains also the change of level of the iris and the diminution of tension of the anterior chamber (by the recession of the peripheral parts of the crystalline lens and iris).

The phenomena observed by *Coccius* are probably due to an optic illusion. Holding the crystalline lens of a horse in front of a red ground we see this color through the whole crystalline lens, except at a quite narrow border where the red rays undergo total reflection. By exerting a traction on the zonula, this border enlarges at the expense of the transparent part, which makes one think of a diminution of the diameter of the crystalline lens.

We have not succeeded, up to the present, in explaining satisfactorily the singular phenomena which I observed when the accommodation attained its maximum (page 216). I had attributed them to a displacement downwards of the crystalline lens, due to an unequal traction on the zonula. But since *Hess* and *Heine* have shown that the displacement takes place following the weight, this explanation must of necessity be abandoned. *Hess* supposes that the crystalline lens falls downwards by the relaxation of the zonula, as stated by *Helmholtz,* but apart from the fact that the hypothesis of Helmholtz must be rejected for other reasons, it is not easy to any longer suppose, in view of the manner in which the crystalline lens is fixed on the vitreous

(1) According to certain authors (*Arlt, Iwanoff*), the ciliary muscle differs in myopes and hypermetropes. If this is so, we might, perhaps, find the predisposition to myoia in a special structure of the ciliary muscle. It is, indeed, clear that the more the superficial fibres are developed the greater must be the traction exerted on the choroid, and this traction has evidently for its object the protection of the sclera against the increase of tension during accommodation. If the posterior extremity of the muscle were fixed, the sclera would be exposed to this tension every time one would accommodate. In view of this relation, it may be interesting to observe that the eye which I examined, in which the anterior surface of the crystalline lens did not advance during accommodation, is myopic about 6 D., and that that one of the three eyes of Helmholtz which showed the least advancement was slightly myopic.

body, that it can fall downwards unless the anterior part of the vitreous body is displaced also. The fact that the movement of the small image is much wider than that of the large one (1), indicates in every case that there can be no question of a displacement directly downwards, but rather a see-saw movement downwards and backwards.—Among other explanations which might occur to us, that of a deformity due to a displacement of the crystalline mass in the interior of the capsule would perhaps be the most probable.

As to the contraction of the pupil which accompanies accommodation, it is evident that it has the effect of eliminating the peripheral parts of the crystalline lens, which, by reason of their flattening, would render the image too poor. We know also that when the pupil is dilated with an alkaloid which has little or no effect on the accommodation (cocaine or homatropine), near sight diminishes relatively more than far sight; this phenomenon is often attributed to a diminution of the amplitude of accommodation, but at least with cocaine I have only very rarely been able to prove a real diminution of this amplitude. We must note, however, that eyes which have a strong spherical aberration correct this aberration by accommodation; these eyes may, therefore, see relatively better near at hand than far away, when the pupil is dilated.

When, in a paracentesis, we allow the aqueous humor to escape, we know that the crystalline lens and the iris come to be applied against the cornea, without this membrane noticeably changing form. In all probability, the crystalline lens is then in the state of highest accommodation, because it could not make such a movement without exerting a strong traction on the zonula. While performing paracentesis on a rabbit's eye, *Mannhardt* claims that he saw also the accommodative displacement of the images of *Purkinje*, by means of the ophthalmoscope of *Cramer*. It becomes probable, therefore, that the pupillary con-

(1) A slight displacement of the look downwards would give analogous phenomena. When the eye makes a movement, the displacement of the images is in direct relation with the distance of the center of curvature of the surface in question to the center of rotation of the eye. The displacement of the small image is relatively large because the center of curvature of the posterior surface of the crystalline lens is situated very far forward in the eye.

traction, which accompanies the escape of the aqueous humor, is accommodative. But the pupillary contraction accompanies the escape of the aqueous humor even in a dead eye; by introducing the point of a *Pravaz* syringe into the anterior chamber, it is easy to dilate or contract the pupil at will by injecting or removing the liquid. This contraction is, therefore, purely mechanical, and it then becomes probable that the accommodative contraction of the pupil is so also, although this mechanism is not yet clearly elucidated.

Bibliography.—Petit (J. L. *Mémoire sur e crystallin de l'oel de l'homme.* Hist. de l' Académie des Sciences, 1730.—Krause (C.). *Poggendorf's Annalen,* 1834-36.—Max Langenbeck. *Klinische Beitrage zur Chirurgie und Ophthalmologie.* Göttingen, 1849.—Cramer (A.). *Het Accommodatievermogen der Oogen.* Haarlem, 1853. Translated into German by Doden. Leer, 1855.—Helmholtz (H.). *Ueber die Accommodation des Auges.* Archiv für Ophtalmologie, 1, 2.—Gräfe (A.). *r Fall von acquirirter Aniridie als Beitrag zur Accommoda, tionslehre.* A. f. O. VII, 2, p. 150.—Brücke (E.). *Anatomische Beschreibung des menschlichen Augapfels.* Berlin, 1847.—Bowman (William). *Lectures delivered in the London Royal ophthalmic hospital.* Moorfields, 1847.—Müller (Heinrich). *Ueber einen ringförmigen Muskel am Ciliarkorper des Menschen und über den Mechanismus der Accommodation.* A. f. O., III, p. 1.—Mannhardt. *Bemerkungen über den Accommodationsmuskel und die Accommodation.* Arch. fur Opht., IV, 1.—Hueck (A.). *Die Bewegung der Krystallinse.* Leipzig, 1841.—Coccius (A.). *Ueber den Mechanismus der Accommodation des menschlichen Auges.* Leipzig, 1867.—Förster (R.). *Zur Kenntniss der Accommodationsmechanismus.* Kl. M. f. A., 1864 p. 368.—Rochon-Duvignaud. *Recherches sur l'angle de la hambre antérieure et le canal de Schlemm.* Paris, Steinheil, 1892.—Tscherning (M.). *Etude sur le mécanisme de l'accommodation.* Arch. de phys., January, 1894.—*L'optometre de Young et son emploi.* Arch. de Phys., October, 1894.—*Recherches sur les changements optiques de l'oeil pendant l'accommodation.* Arch. de phys., January, 1895.—*Théorie des changements optiques de l'oeil pendant l'accommodation.* Arch. de phys., January, 1895.—Crzellitzer (A.). *Die Tscherningsche Accommodationstheorie.* Gräfe's Archiv., XLII, 4, 1896.—Stadfeldt (A.). *Die Veränderung der Linse bei Traction der Zonula.* Kl. M. f. A., December, 1896.—Crzellitzer (A.). *Zonularspannung und Linsenform.* Bericht der Heidelberger Gesellschaft, 1896.—Hess (C.). *Arbeiten aus dem Gebiete der Accommodationslehre.* Gräfe's Archiv., 1896-99.—Heine (L.). *Die accommodation Linsenverschiebungen im Auge.* Gräfe's Archiv, 1897.—Tscherning (M.). *The Theory of Accommodation.* Ophthalmic Review, April, 1899.—Tscherning (M.). *La surcorrection accommodative de l'aberration de sphéricite de l'oeil.* Journal de Physiologie, March, 1899.

CHAPTER XIII

OPHTHALMOSCOPY

88. Methods of Illuminating the Fundus of the Eye.—It has been known from the remotest times that the pupil of certain animals (dog, cat, etc.) can appear luminous. The phenomenon was thought to be analogous to the production of light by the glow-worm (*phosphorescence*); in reality it is due to the existence of the *tapetum,* a part of the choroid the retinal surface of which is strongly reflecting and has a metallic reflex; its purpose is not very well elucidated. As to the human pupil, it has been known for a long time that it may, in very rare cases, appear luminous after the development of an anterior tumor of the eye (*amaurotic cats'-eye*). *Beer* also remarked the ocular glow in certain cases of aniridia.

Towards 1850 *Cumming* and *Bruecke* discovered the method of making the pupil of the normal eye appear luminous, and *Helmholtz* in 1851 achieved the great invention of the ophthalmoscope which was destined to revolutionize ophthalmology.

Like every other object the fundus of the eye sends back light when it is illuminated. Let A (fig. 125) be a luminous point

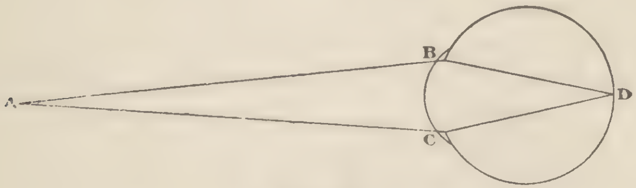


Fig. 125.

for which the eye is accommodated. This point sends into the eye the cone ABC, the rays of which reunite at D. This point, being illuminated, sends the rays in all directions; those contained in the cone ABC emerge from the eye to meet at a point A. Generally, therefore, the eye can send back light to a point

which has first sent the light to it, and if in ordinary circumstances the pupil of the eye appears black, it is because the pupil of the observing eye, being black, cannot send light back into the observed eye. In order that it may appear luminous, a luminous source must be placed in front of the observing eye; this is what we do by means of the ophthalmoscope.

Following are the different circumstances in which we can see the pupil luminous:

a. The pupil of *albinos* is seen red because the fundus of the eye is illuminated by the light which has passed through the sclera. If we cover the eye with a screen pierced by an aperture corresponding to the pupil, the latter appears black.—By concentrating a bright light on the sclera by means of a lens, we can make the pupil of a normal eye luminous, especially if the person has a fair complexion.

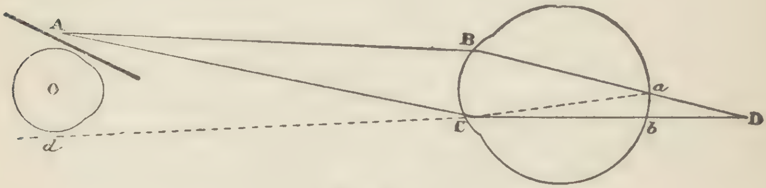


Fig. 126.

b. If, in the case of figure 125, the eye is not exactly focused for the luminous point, the latter illuminates on the retina a circle of diffusion (*ab*, fig. 126). This circle sends back the light not only in the direction of the luminous point, but also in neighboring directions: thus the point *a* sends back the cone B*a*C which, outside the eye, takes the direction ABC*d*, so that the observing eye *o* may be placed in this cone. Placing a lamp at some distance from the observed eye and sighting near the border of the flame, from which we shelter ourselves by a screen, we can frequently see the pupil luminous, especially if it is a little large and if the patient does not fix the flame.

The experiment succeeds more easily if the observed eye is strongly ametropic, because then the rays, having emerged from the eye, soon diverge greatly, so that the observing eye may

easily find a place in the luminous cone. If the eye is not ametropic we can make it so by means of a strong lens or by putting it under water, or, as *Bellarminoff* has lately done, by placing a plate of glass in contact with the cornea so as to eliminate the refracting power of this membrane. By this latter means we can make the fundus of the eye visible for several persons at once.—In the case of *amaurotic cat's-eye,* the presence of the tumor in the interior of the eye makes the latter strongly hypermetropic, so that the fundus becomes easily visible.

c. Principle of the Ophthalmoscope of Helmholtz.—Let AB (fig. 127) be a plate of plane, parallel glass and L a lamp which sends light towards this plate. The greater part of the light passes through the plate, but a part is reflected towards the observed eye, D. It enters this eye and illuminates the retina. The latter sends back light towards the plate: a part of this light is reflected towards the lamp L, but the greater part passes through the plate and enters the observing eye C, which, consequently, sees luminous the pupil of the observed eye. To com-

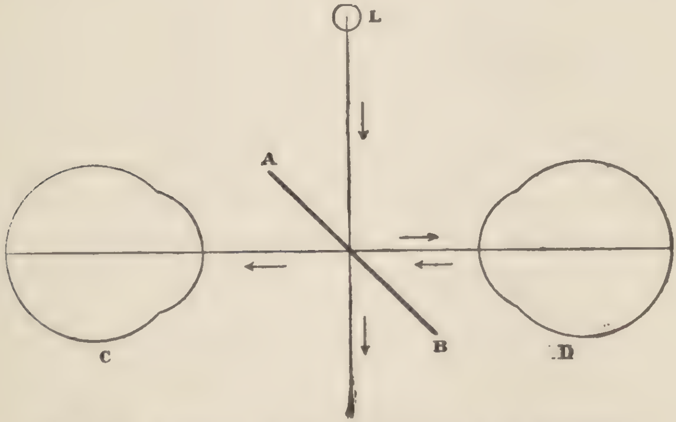


Fig. 127.—Principle of the ophthalmoscope of *Helmholtz.*

pensate for the loss of light which, proceeding from L, passes through the plate, *Helmholtz* used several plates, placed one behind the other.

d. Principle of the Ordinary Ophthalmoscope.—We obtain a more intense illumination by means of a silvered mirror; the observer looks through a small portion from which the coating has been removed or which has been perforated.—As a concave mirror concentrates the light it illuminates better than a plane mirror, and the latter better than a convex mirror. (1) Generally it is useful to have a good illumination; but we sometimes see better the very delicate changes in the fundus of the eye by using a weak illumination, and very delicate opacities of the vitreous body or of the crystalline lens disappear if the illumination is too strong.

The ophthalmoscope is the only practical means of illuminating the eye. Nevertheless, a different method may sometimes prove serviceable. We place the lamp behind the observer so that the light reaches the observed eye by glancing along the head of the observer; we concentrate the light on the eye with a lens. When the pupil is dilated we can thus see the fundus of the eye feebly illuminated, and we often distinguish details situated far forward in the vitreous body (tumors of the ciliary body, detachments, etc.).

89. Examination by the Erect Image (*Helmholtz*).—The conditions for seeing the pupil luminous were known, before *Helmholtz,* by the researches of *Cumming* and *Bruecke,* and *Babbage* seems to have already illuminated the pupil with a mirror from a small portion of which the coating was removed for observation purposes; but none of these scientists thought of studying the conditions under which this ocular glow can form an image of the fundus of the eye.

When preparing the lectures, in the course of which he was

(1) The clearness of the retinal image of the flame which is formed in the observed eye is the same in all cases, but the image is larger when we use a concave mirror than when we use a plain or convex mirror.—One can verify this for oneself by putting one's eye in the place of the observed eye. The image of the flame which one then sees in the mirror corresponds to the illuminated part of the retina; it is larger in the case of the concave mirror than with the plane or convex mirror.—Placing the flame behind the mirror, one sees, in the same circumstances, the opening as a luminous circle which corresponds to the part of the fundus of the eye which the observer can see at once (ophthalmoscopic field).

to illustrate for his class the methods of making the pupil appear luminous, *Helmholtz* proposed to himself the problem to be solved, not a difficult task for an experienced physicist. He easily succeeded in solving it theoretically, and then constructed the first ophthalmoscope by combining some glass plates with the lenses of a test case; after some days of hard work he succeeded in seeing the fundus of the living eye which no one had ever seen before him.

Helmholtz used examination by the *erect image*. Suppose that the observer is emmetropic (if he is not he must correct his refraction): he can then see the fundus of the eye of another emmetrope without any further aid, since the rays emerging from the observed eye are parallel. If the observed person is not emmetropic he must be made emmetropic. We, therefore, look for the strongest convex glass or the weakest concave glass with which we can see the fundus of the eye distinctly; this glass indicates at the same time the refraction of the eye; but the observer must cultivate the habit of not using his accommodation, otherwise the results will be false.—The refraction which we find with the ophthalmoscope ought to be in agreement with that found by subjective examination. It must be noted, however, that the glass of the ophthalmoscope is generally a little farther away from the eye examined than a glass placed in a frame. We find, therefore, as by the subjective method, too low a number for hypermetropia, too high a number for myopia, and the error is more pronounced in the case of an ophthalmoscopic examination on account of the greater distance. For low degrees of ametropia it is insignificant; for high degrees, especially of myopia, it is sufficient to make the determination fallacious. Latent hypermetropia is generally disclosed by ophthalmoscopic examination because in the dark room the patients do not fix.

MAGNIFICATION.—To obtain a numerical expression of ophthalmoscopic magnification, we may compare the retinal image, formed in the observing eye, of an object (the papilla of the fundus of the examined eye) with the retinal image which the observing eye would have of the same object, placed free in air,

at the working distance of the observer. We often make this distance 20 centimeters.

Let us suppose that both eyes, that of the observer and that of the observed person, are emmetropic.

Let O=AB (fig. 128) be the object of the fundus of the observed eye; we draw the rays AC and BD parallel to the axis. These two rays will intersect at the anterior focus Φ_1, and all the

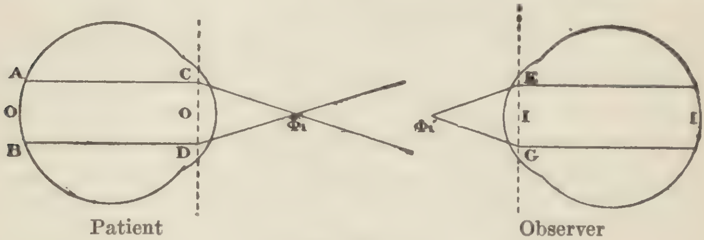


Fig. 128.

other rays proceeding from A and B are parallel to either of these; among other rays Φ'_1E and Φ'_1G which, prolonged, pass through the anterior focus of the observing eye. After refraction in this eye these rays are parallel and determine the size of the image, I. Designating by F_1 the anterior focal distance of the observed eye, by F'_1 that of the observing eye, the two similar triangles $CD\Phi_1$ and $EG\Phi'_1$ give the relation:

$$\frac{I}{O}=\frac{F'_1}{F_1}$$

We see that, if the optic systems of both eyes are alike, I is equal to O. The papilla of the observed eye forms in the observing eye an image equal to itself. By placing the fundus of the eye free in the air at the working distance, equal to 20 centimeters, the retinal image I_1 of the object O (fig. 129) would be found by the formula

$$\frac{O}{I_1}=\frac{200}{F'_1}.$$

By multiplying this formula by the preceding one, we obtain the magnification in the erect image:

$$G=\frac{I}{I_1}=\frac{200\text{ mm}}{F_1}.$$

By supposing 15 millimeters for F_1, the magnification would be about 13, but this number is arbitrary, since the working distance has been chosen arbitrarily.

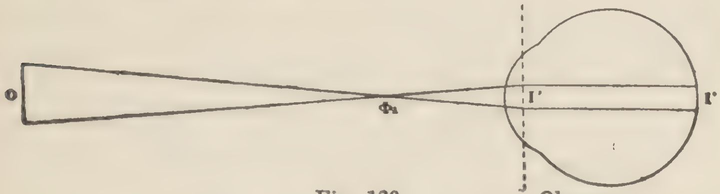
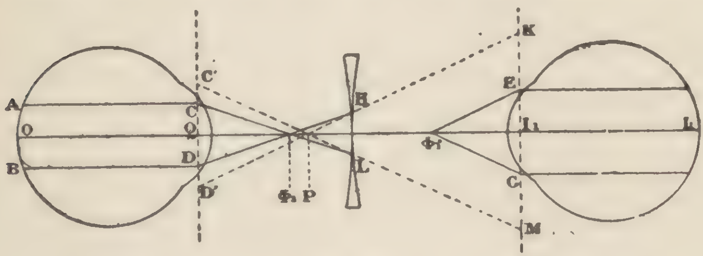


Fig. 129. Observer

If the observed eye is myopic, the magnification is greater, supposing that the correcting glass is beyond the anterior focus of the observed eye, as is always the case. The construction is the same as in the preceding case, but on meeting the concave glass the rays $C\Phi_1$ and $D\Phi_1$ (fig. 130) are made more



Patient Fig. 130. Observer

divergent. The rays Φ'_1E and Φ'_1G which are parallel to them diverge, therefore, more than in the preceding case, which makes the image I_1 greater. If there is a case of myopia of curvature the magnification is still greater; the point Φ_1 is, in fact, situated nearer the observed eye, which causes the rays HK and LM, and consequently also the rays Φ'_1E and Φ'_1G to diverge still more. In the hypermetropic eye the reverse takes place. It follows that in an astigmatic eye we see the papilla elongated in the direction of the meridian of greatest refraction.

Ophthalmoscopic Field.—According to Helmholtz we find the ophthalmoscopic field, that is to say, the aggregate of the

parts of the fundus of the eye, visible simultaneously by joining by straight lines the middle of the pupil of the observing eye to the borders of the pupil of the observed eye, and by making these straight lines undergo the same refraction in the observed eye as if they were rays. Figure 131 shows that the field is greater in the hypermetropic eye, smaller in the myopic eye, if the observing eye is beyond the anterior focus of the observed eye, as is always the case. As it is the border of the pupil of the observed eye which limits the field, we increase it by instilling atropine.

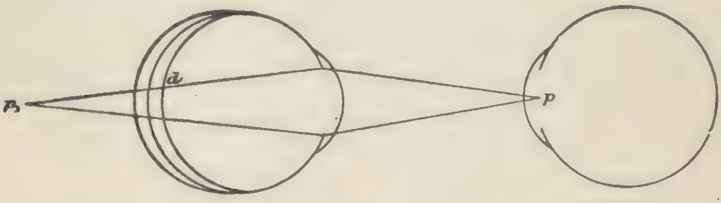


Fig. 131.—Construction of the ophthalmoscopic field.

This is an instance of inverse constructions which we frequently use in geometric optics; to know what points of the fundus of the observed eye can send back rays into the pupil of the observing eye, we reverse the problem by imagining the pupil of the observing eye luminous and finding what parts of the fundus of the observed eye it can illuminate. The result is the same on account of the reversibility of the optic processes. In reality the field is a little larger than that which we have found by our construction, since we have reduced the pupil of the observing eye to a point; from the point d, situated outside the field, some rays could still enter the observing eye through the lower parts of the pupil. To have the field complete it would be necessary to construct, not the image p_1 of the center of the pupil p, but the image of the entire pupil or rather of the opening of the ophthalmoscope, formed by the optic system of the observed eye. We would thus obtain a larger field, but the parts near the border would be very slightly illuminated.

90. Examination by the Erect Image. Observations.—To tell the size of intra-ocular objects, it is customary to compare them with the diameter of the papilla; we thus say that the width of a staphyloma is the fourth or half of the diameter of the papilla. The attempts which have been made to obtain more exact measurements by means of a micrometer (*Donders, Leroy*) have not given practical results.

The refraction is usually the same for the entire fundus of the eye. According to *Young,* if we suppose a sphere drawn around the eye with the distance of the far point as radius, the position of the retina is such that it is everywhere found at the place where the best images of objects situated on this sphere would be formed. A certain degree of astigmatism by incidence is inevitable for the peripheral parts; but the retina is here found between the two focal lines almost at the place which would correspond with the circular diffusion spot.

Thanks to this arrangement, we can use the papilla for the determination of refraction by the erect image; generally its refraction scarcely differs from that of the macula. There are exceptions to this rule, however. For instance, I examined on consultation a young man in whom a myopia of 4 D. was indicated, while a colleague, very experienced in determination by the erect image, and myself found, each for himself, emmetropia by the ophthalmoscope. It was later established beyond doubt that the patient had really a myopia of 4 D. Then, asking ourselves whether the myopia might not be due to a spasm of accommodation, we resorted to a treatment by atropine, but without changing the result. Analogous differences seem quite frequent in cases of excessive myopia, by reason of the elongated form of the globe.

A difference between subjective and ophthalmoscopic refraction may therefore be due: 1° to a greater distance of the correcting glass from the observed eye (see page 234); 2° to the fact that a latent hypermetropia may become manifest in the darkness; 3° to the fact that the papilla may have a different refraction from the macula; 4° to stimulation.

To judge of the depth of a papillary excavation we can

measure the difference of refraction between the edge and pit of the excavation, keeping in mind that a difference of one dioptry corresponds to almost a third of a millimeter. We can measure by the same process the tumefaction of the papilla in cases of optic neuritis, the distance of an opacity of the vitreous body from the retina, etc.

Another means of judging whether one point is situated in front of another consists in making slight movements of the head (with the ophthalmoscope). We shall then see the nearer point make a movement in a contrary direction in relation to the other point (*parallax*).

The magnification of 13 which we have found for the erect image has nothing to do with the apparent size of the papilla, which depends on the distance to which we project the image without knowing it. When we begin to use the ophthalmoscope, the papilla frequently appears very small, and generally its size seems to vary for different observers. I have noticed a phenomena of the same kind when looking at a luminous point (see page 165). If the point is very distant the circle of diffusion appears very large to me. But if I observe a luminous point placed at the focus of a lens of 20D., held in front of my eye, the point appears extremely small, and this although the retinal image ought to be exactly the same in both cases. Accommodation is often charged with playing a part in this optic illusion, but we must observe that it takes place even if every trace of accommodation be excluded. It rests on an unconscious conclusion relatively to the distance of the object (see chapter XXII).

The macula is usually difficult to see: most frequently the pupil must be dilated. The fovea is sometimes visible as a dark spot with a small whitish point in the middle; its place is marked in every case by the peculiar manner in which the vessels come from all sides to disappear in its vicinity. We never see a trace of the yellow color which is so striking in the dead eye; certain authors have, therefore, considered this yellow coloration as a phenomenon due to changes after death, and this idea seems confirmed by an observation which I have

made. We generally suppose that if we do not see the yellow color of the macula, it is because the yellow light is drowned by the red light reflected by the blood. I, therefore, thought that we should be able to see it by illuminating the eye with a strong sodium flame. The blood does not reflect this light or reflects it only slightly, and the appearance of the fundus of the eye recalls that of photographic illustrations of ophthalmoscopic images; we see the vessels black on a gray ground, but the macula, which we should expect to find illuminated, remains at least as dark as in ordinary ophthalmoscopy.

The red color of the fundus of the eye is due to the vessels of the choroid; wherever the choroid is defective we see the white background of the sclera, in cases of coloboma for example. It is curious that we never see a trace of the retinal purple with the ophthalmoscope. In the normal state the retina is completely transparent; we see only its vessels. Sometimes we can, however, distinguish it as a grayish veil in the parts near the papilla. If the black pigment be strongly developed, the fundus of the eye appears of a uniform deep red. If it is but slightly developed, the fundus has often a marble or spotted appearance due to the meshes of the vascular network of the choroid.

Most normal eyes have a physiologic excavation or cup of the papilla which has the appearance of a whitish spot. It is then easy to see, by the erect image, that the bottom is more myopic than the border; we see indistinctly the vessels of the excavation when those of the borders appear distinct and vice versa, at least when the excavation is a little deep. The physiologic cup never reaches the borders of the papilla. We can be certain that an excavation is pathologic only when it reaches the borders everywhere.

We frequently perceive in the normal eye a *pulsation of one or several of the large veins.* During the systole the tension of the globe increases enough to compress the large veins near their starting place where the intra-venous tension is weakest.

At the moment of diastole the tension of the globe diminishes, the pressure ceases and the veins empty themselves. (1)

The *pulsation of the arteries* is nearly always a sign of glaucoma; the tension of the globe is so high that the arteries remain empty, except at the moment of systole.

The papilla is generally limited by a very thin white border, sometimes surrounded by an incomplete black border, formed by the pigment of the choroid. The white border is called the scleral border; it is attributed to the visibility of the sclera between the choroid and the papilla. Sometimes it is larger and mistaken for an incipient staphyloma.

One can see the red fundus of one's own eye by looking in a mirror held before a flame. A luminous pencil passes through the opening of the ophthalmoscope, enters the eye, is reflected by the retina, emerges from the eye, meets the mirror, and is again reflected towards the retina. If the course of the rays permit, for example if the eye is emmetropic and the mirror plane, we may even distinguish the details. We see at the same time the catoptric image of the cornea as a large circle of diffusion.

Auto-ophthalmoscopes have been constructed as well as ophthalmoscopes, by means of which several observers can see simultaneously the fundus of the eye.

Another way of examining oneself consists in observing with one eye the image of the other formed by a looking-glass; we can in this way perform ophthalmoscopy of the left eye with the right eye by the inverted image, and we can, with a small concave mirror placed not far from the eye, observe the images of *Purkinje*, etc. It was by working thus with my own eye that

(1) [Lately Dr. S. Türk has studied this question again in a number of persons with irregular heartbeat (arythmia).

These observations prove that the venous narrowing is independent of the entrance of the arterial pulse wave into the eye, and he infers that the cardiac systole produces not the narrowing, but the dilatation of the veins. He further shows that this venous pulsation cannot be caused by a rhythmic interference with the exit of the blood from the vena centralis retinæ because a dilatation, caused in this way, ought to be propagated opposite to the direction of the blood-current. He, therefore, considers this phenomenon caused by a propagation of the arterial pulse wave through the capillaries into the veins which is accounted for by the relatively high extravascular pressure in the eye (*Engelmann's Arch. f. Physiol.*, 1899).]—*W.*

I observed for the first time the conical deformity of the anterior surface of the crystalline lens during accommodation (page 210).

91. Examination by the Inverted Image.—This examination was introduced into oculistic practice by *Ruete* in 1852. It was especially adopted and developed by the Berlin school (*Graefe*), while the Vienna school (*Jaeger*) especially used the erect image. As the Berlin school held for a long time a more influential position, examination by the inverted image was for a long time more used than the other. The two methods, however, merit a place side by side. The inverted image gives a less magnification and a larger field: it is, therefore, very useful for studying the general appearance of the fundus of the eye, while the erect image serves especially for the study of the details and for the determination of refraction.

Examination by the inverted image is made by holding a strong convex lens (most frequently +13) at a distance from the eye almost equal to its focal distance. This lens forms a real and

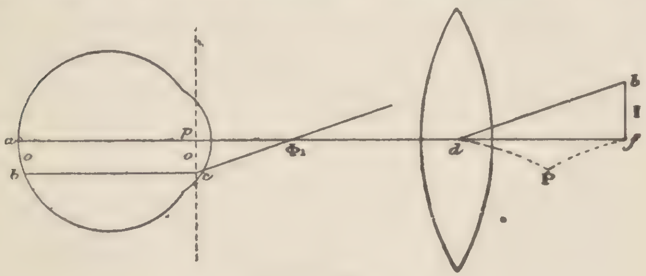


Fig. 132.

inverted image of the fundus of the eye, situated on the other side of the lens, in the vicinity of its second focus. It is this image that the observing eye sees when accommodating, or, which is better, by looking through a convex lens of about 4 D., placed behind the mirror. If the examined eye is emmetropic, the rays leaving the eye are parallel and the image is formed at the focus of the lens; if it is myopic the image is a little nearer, if hypermetropic a little farther than the focus. In the latter

case the observer is frequently obliged to move back a little in order to see the image distinctly.

MAGNIFICATION.—If we use a lens of +13, the magnification is about five times for an emmetropic eye. Let $ab = O$ (fig. 132) be an object in the fundus of the observed eye. We draw the ray bc parallel to the axis: it passes, after refraction, through the anterior focus of the eye Φ_1, and the other rays coming from b are parallel to it, since the eye is emmetropic. One of these rays db' passes without refraction through the optic center of the lens, and it is on this ray db' that the image b' of b is formed, in the focal plane of the lens. The two triangles $pc\Phi_1$ and dfb' are similar: we have, therefore, $\frac{b'f}{pc} = \frac{df}{p\Phi_1}$, that is to say, the magnification is equal to the relation between the focal distance of the lens and the anterior focal distance of the eye. The anterior focal distance of the eye being 15 millimeters and that of the lens 77 millimeters, the magnification is $\frac{77}{15}$ or about 5. We can increase the magnification by using a weaker lens, but the image at the same time moves away from the lens so that the observer is obliged to move back, which makes this way of increasing the image of little practical value. In cases of persons operated on for cataract it may be useful to use a stronger lens (+18) to obviate the necessity of moving away.

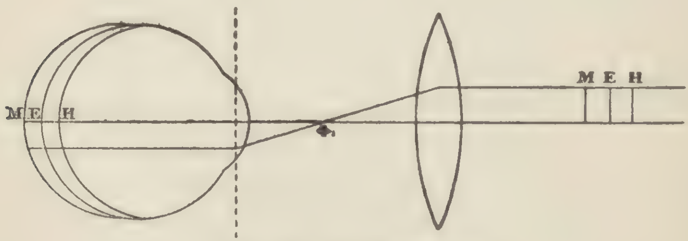


Fig. 133.—After *Bjerrum.*

INFLUENCE OF REFRACTION OF THE EXAMINED EYE ON THE MAGNIFICATION.—A glance at figure 133 suffices to show that if we place the lens so that its focus coincides with the anterior

focus of the eye, the magnification is the same whatever may be the refraction of the examined eye (principle of *Badal*). (1)

If the lens is nearer the eye, as is generally the case, the magnification is greater in the hypermetropic eye, less in the myopic eye (fig. 134). For this reason the papilla of the astigmatic eye

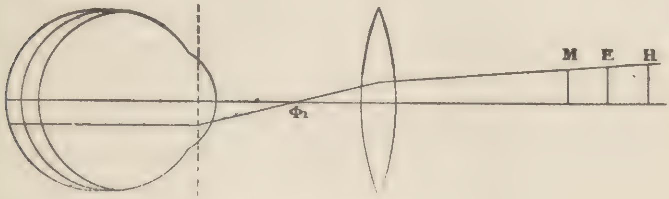


Fig. 134.—After *Bjerrum.*

is seen elongated in the direction of the meridian of least refraction; by moving the lens away the other meridian is elongated and finally that which corresponds to the meridian of greatest refraction is seen to be the greater just as by the erect image.

Ophthalmoscopic Field.—In order that the field may be as large as possible, the lens must be at a distance from the eye almost equal to its focal distance. Under these circumstances the image which the lens forms of the pupil of the observed eye is very large and fills the entire lens; the iris disappears from the field.

We construct the field as for the erect image, by supposing the center (P, fig. 135) of the pupil of the observing eye luminous and finding what part of the fundus of the eye it could illuminate. In drawing figure 135, it has been supposed that the image P_1 of the center of the pupil of the observer coincides with the nodal point K of the observed eye, so that the "rays" A*a* and B*b* suffer no refraction: *ab* is therefore the field, and we

(1) This is exact only if the ametropia is axial. In case of a myopia (hypermetropia) of curvature, the anterior focus is situated near the eye in proportion as the refraction is greater.—Repeating the construction of figure 133, we see that by making the focus of the lens coincide with the anterior focus of the eye the magnification is greater in the case of myopia.—The astigmatic eye has two anterior foci, one for each principal meridian; to obtain the same magnification in both meridians, the focus of the lens must be nearer the eye than the more distant anterior focus.

note that it does not depend on the pupil of the observed eye, since the cone AP_1B does not touch its borders. The field is limited only by the borders of the lens; it is therefore preferable to use a large lens as they do in England. If we move the lens nearer or farther away, so that a larger part of the cone AP_1B coincides with the pupil, it may happen that the latter may be too small, so that the iris intercepts the most peripheral rays. The field is then limited by the iris of the observed eye, which may be seen through the lens. If the pupil is small, it may be difficult to hold the lens exactly at the proper place for the iris to disappear; this is why dilation of the pupil is advantageous.—It must be noted, furthermore, that a small part of the

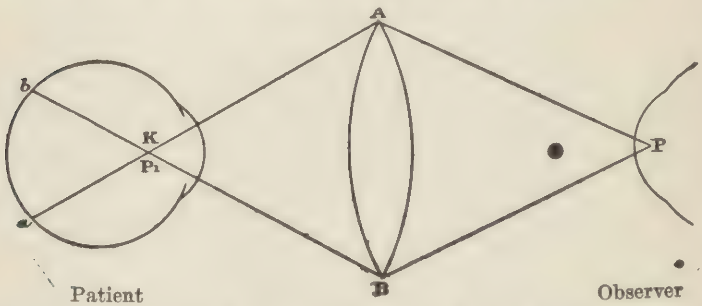


Fig. 135.—Construction of the ophthalmoscopic field by the inverted image.

field is well illuminated. If we use a concave mirror of 20 centimeters focus, as is customary, we see at the fundus of the eye a quite distinct image of the flame (because the image formed by the mirror is almost at the focus of the lens so that the rays which meet the eye are almost parallel); it is only the part of the field which corresponds to this image that is illuminated; the remainder is in darkness.—The illuminated portion may be increased by using a plane mirror, but the illumination is then less bright.

We can see the inverted image without any lens if the patient is myopic more than 6 D.; by moving the head from side to side, we make sure that the vessels are displaced in the contrary direction, for we can also see the fundus of the hypermetropic eye (by the erect image) at a sufficiently great distance. The

visual field is very small and the magnification often so great that one vessel may fill half of the field. The existence of this image is sufficient to establish the diagnosis of a strong myopia. —It is often difficult to examine the high degrees of myopia by the erect image, and by the inverted image the enlargement is sometimes not sufficient. We can then use this image which the myopic eye itself produces, by magnifying it; we make no change from the ordinary way of examining with the inverted image; it is only necessary to move the lens far enough away for the image to be formed between the lens and the observed eye. The lens then produces an enlarged virtual image of this inverted image, which is also inverted and situated farther behind; to see it distinctly it is often necessary to place oneself very near the lens, especially if one uses a convex glass behind the mirror. We can thus obtain an enlargement nearly as great as by the erect image (*Demicheri*).

We can use the examination by the inverted image for the determination of the refraction of the eye, by measuring the distance from the observed eye at which the inverted image is situated, since this distance varies with the refraction of the eye. This method, which was proposed by *Schmidt-Rimpler*, has never become very popular.

The appearance of the fundus of the eye is very nearly the same with both methods. We must except the macula, however, which, by the inverted image, often presents itself under a special form, as an oval spot, with the long diameter horizontal, a little larger than the papilla; this spot is dull, a little darker than the rest, and surrounded by a bright circle, corresponding to the convexity of the border of the fovea, which acts as a kind of convex mirror. Analogous reflexes often appear also on other parts of the retina, especially in young subjects.—Differences of level are observed by the parallactic displacement which is obtained by subjecting the lens to a slight to-and-fro movement.

92. Ophthalmoscopic Examination of the Refracting Media.— To examine the transparency of the refracting media it is pre-

ferable to use a weak illumination; we use preferably a plane mirror or even a convex mirror. *De Wecker* recommended the use of the plates of *Helmholtz* for this examination. We see, indeed, the shadows which the opacities produce by intercepting a part of the rays sent back by the fundus of the eye. If the fundus is strongly illuminated, and if the obstacles are not completely opaque, they allow a part of the light to pass and the shadow is less complete.—It is useful to use a strong magnifying glass for this examination in order that we may place ourselves very near the eye. Otherwise many of the small corpuscles may escape in the examination.

It is quite rare for these opacities to be visible by the light which they themselves reflect. It may happen, however, that we can see the red color of hemorrhages situated far forward in the vitreous body, or the white color of certain opacities, especially when using the light in such a manner that it falls very obliquely along the head of the observer. In cases of *synchisis scintillans* the observing eye receives light regularly reflected by the surfaces of the small crystals situated in the vitreous body.

93. Skiascopy.—This method of examining ocular refraction was discovered by *Cuignet,* who described it under the ill-chosen name of keratoscopy. It was *Parent* who specially developed the method, and it was he who first gave the correct explanation of it.

The observer takes his place at one meter from the patient, whose eye he illuminates with a plane mirror; by rotating the mirror around a vertical axis we see the luminous spot on the face of the patient move in the same direction. The illumination of the pupil follows the same direction. whether the patient be hypermetropic, emmetropic or very slightly myopic.—If the myopia is over 1 D., the pupillary light is displaced in the contrary direction, and if the myopia is equal to 1 D., we do not see the light move in the pupil. The luminosity diminishes uniformly in the entire extent of the pupil to disappear suddenly.

The examination of figure 136 shows that the retinal image moves in the same direction as the mirror. If the observed person is hypermetropic, emmetropic or myopic less than 1 D., it

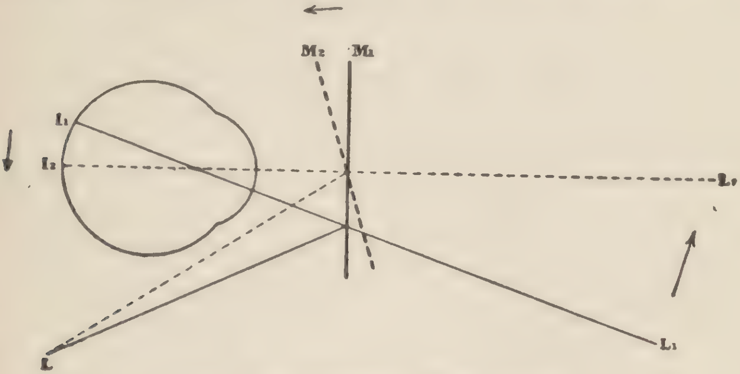


Fig. 136.—Skiascopy. Plane mirror.

L, lamp; M_1, first position of the mirror; L_1, image which it forms of the lamp; I_1, retinal image.—M_2, second position of the mirror; L_2, image of the lamp; I_2, retinal image.

is the erect image that the observer sees. The light seems to him to move on the retina, as it really does. If, on the contrary,

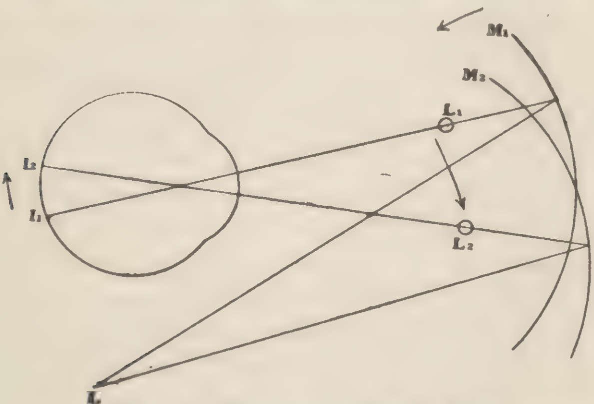


Fig. 137.—Skiascopy. Concave mirror.
The letters have the same significance as in figure 136.

the myopia is greater than 1 D., he sees the light move in the contrary direction, because the light comes to him from the in-

verted image which he observes.—To determne the degree of ametropia, we place before the eye of the patient stronger and stronger glasses, until the shadow covers the entire pupil at once; the patient has then a myopia equal to 1 D.

If we use a concave mirror we see, as in the preceding case, the luminous spot move on the face of the patient in the same direction as the mirror. But the retinal image of the flame moves in a contrary direction: we see, indeed, on figure 137, that the image of the flame (L_1 L_2) formed by the mirror goes in a direction contrary to that of figure 136, whence it follows that it is the same for the retinal image. The observer also sees the ocular glow move in an opposite direction if the observed person is emmetropic, hypermetropic or myopic less than 1 D. and in the same direction if the myopia is greater than 1 D.

Skiascopy is important in the search for astigmatism if we do not dispose of it with an ophthalmometer. If the mirror be moved in the direction of one of the principal meridians, everything happens as in a non-astigmatic eye. But if the movements of the mirror take place in another meridian, the shadow is seen to move in a direction which forms an angle with that of the mirror. This is due to the elliptical form of the diffusion spot. If we draw an ellipse with oblique axes on a sheet of paper, and observe it through a smaller circular aperture, while giving it a horizontal movement, it is almost impossible not to give way to the illusion that the motion takes place in an oblique direction. —We then find the motion to give the mirror in order that the displacement of the ocular glow takes place parallel to that of the mirror. We then determine the refraction of the principal meridians in the ordinary way.

When the ametropia is considerable, the glow is quite feeble and the boundary between the light and shade is curved. If on the contrary the eye is almost corrected, we see the glow very bright and its border is very nearly straight.

The explanation of this fact, which has given rise to a lively discussion, is quite simple. As the lamp (or its image formed by the mirror) is far from the observed eye, there is formed in the emmetropic eye a small pretty distinct retinal image of the

flame (fig. 138, A). As all the light is concentrated on this small image, it is quite bright and although it is small, it nevertheless fills the field because the latter is also very small, as it is easy to see by using the construction we have given for

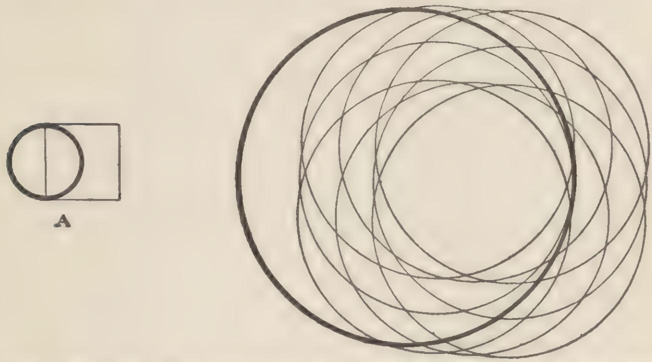
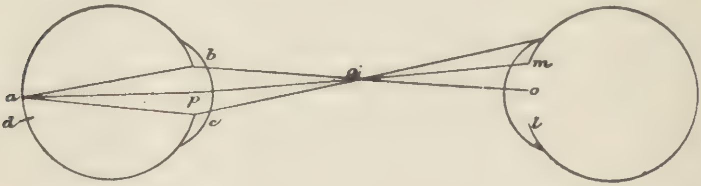


Fig. 138.—The thick circle indicates the limits of the skiascopic field: A, in an emmetropic eye; B, in a strongly ametropic eye. The square in A represents the image of the flame; in B, it changes into a large spot composed of circles of diffusion.

the ophthalmoscopic field. The right border of the ocular glow corresponds with the border of the retinal image of the flame. In the ametropic eye the field is large, and the retinal image is displaced by a diffusion spot, much larger and consequently not so bright. Each point of the distinct retinal image is replaced by a circle of diffusion of the same form as the pupil of the observed eye; as the latter is generally round, the spot also takes on a round form (fig. 138, B) more pronounced in proportion as the ametropia is greater. It is easy to prove the exactness of this explanation: if we use as luminous source a very long, bright line, the border of the ocular glow remains straight, even in the case of strong ametropia, because the superposition of the circles of diffusion cannot then produce a round form. Likewise, if we give the pupil a triangular form, by placing a stenopaic opening of this form before the eye of the observed person, the shadow retains also its rectinlinear border, for the supposition of triangular diffusion spots cannot give a round form to the diffusion spot.

But in neither case does the observer see a distinct image, because his eye is accommodated for the pupillary plane of the observed eye, while the image which he observes is in front of (M) or behind (H) this plane. And as it is not focused for the image, the latter is seen vaguely, each point being represented by a circle of diffusion, the border of which, as always, corresponds with the border of the pupil of the observer.



Patient **Fig. 139.** **Observer**

THEORY OF LEROY.—The explanation which *Leroy* has given of skiascopy, and which is widely accepted, especially in Germany is in thorough agreement with that of *Parent* which I have just explained. Let *a* (fig. 139) be an illuminated point of the retina

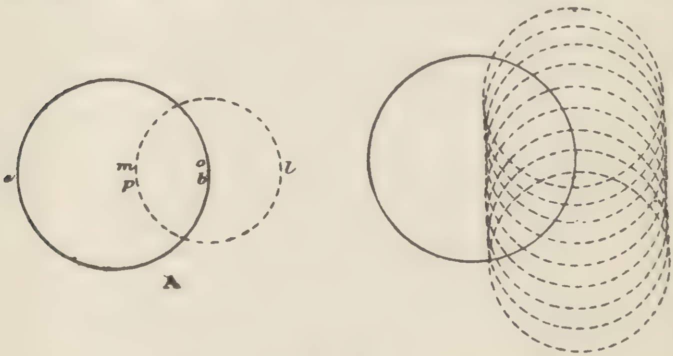


Fig. 140.

of the observed eye, supposed to be myopic, and *a'* its image. From the observed eye then starts the luminous cone *ba'c,* of which the part *a'mo* enters the observing eye. This eye sees luminous the part of the pupil which sends rays to it, that is the part *bp,* while *pc* is dark because the rays which come from

this part are intercepted by the iris of the observer. This Leroy somewhat subtly expressed by saying that the shadow is produced by the iris of the observer. We can imagine the pupil of the observer projected through a' on the pupil of the observed person (fig. 140, A) ; the part of this latter which it would cover would appear luminous. In regard to the theory of Parent, we would say that the observer sees the point a but dimly, that is to say as a diffusion circle the border of which, as we know, corresponds to the border of the pupil of the observed eye.

The two theories are therefore two different ways of saying the same thing. But were the curved form of the shadow explained by the form of the pupil of the observer it would be wrong, because the phenomena do not change if the observer looks through a triangular aperture placed in front of his pupil. The form of the pupil of the observer plays no part, for in reality it is not a luminous point which is found on the retina,

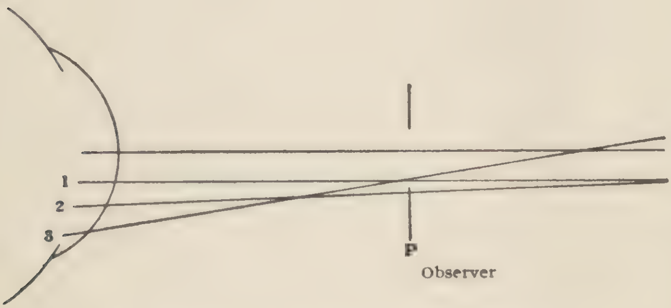


Fig. 141.—Theory of the paracentral shadow.

as the theory of Leroy supposes, but an image of the flame of which *ad* (fig. 139) is a section. The border of the image which we use is, therefore, a straight line perpendicular to the plane of the paper, and it would be necessary to repeat the construction of Leroy for each point of this straight line. We would thus obtain a series of projections of the pupil of the observer, which would delimit the part of the pupil of the observed eye which appears luminous (fig. 140, B). It is easy to see that

the form of each diffusion circle has no influence on the form of the border of the shadow.

Paracentral Shadow.—When one is near correction, one often sees the shadow move irregularly. *Bitzos* has described a paracentral shadow: a part of the pupil, near the center, appears dark, while the borders are still illuminated. This phenomenon indicates that the refraction is not the same everywhere in the pupil; it frequently makes impossible a very exact determination of the refraction.

We must not, therefore, expect a very exact determination by skiascopy, as is the case also for subjective measurement and determination by the erect image, simply because the very idea of ocular refraction does not permit of very great exactness.

Here is the explanation of the paracentral shadow. Let us suppose an eye emmetropic, but with a strong spherical aberration so that the peripheral parts of the pupil may be myopic. The rays coming from a luminous point of the retina would then have the direction indicated on figure 141. An eye, the pupil of which would be at P would receive rays 1 and 3 and would see luminous the parts corresponding with the pupil, while at 2 the pupil would appear dark, since the ray 2 would not enter the pupil. The observing eye would, therefore, see a bright center separated from equally bright borders by a dark ring. If P be displaced a little downwards, it would receive all the rays drawn on the figure, but some on the other half would not enter it, which would give the phenomenon of paracentral shadow. This shadow is, therefore, nothing else than the manifestation of spherical aberration. We have seen that the appearance which indicates aberration consists of a luminous ring towards the borders of the pupil, separated from the central light by a dark zone; tilting the mirror slightly the central light becomes partly joined to the ring and the dark part assumes the form described by *Bitzos*.

I have several times emphasized the advantages which skiascopy with a luminous point presents for the study of optic anomalies of the eye. It also lends itself very well to the ordinary measurement of refraction. At the critical moment when

the movement of the light changes its direction the far point of the observed eye coincides with the pupil of the observer. As, on the other hand, the principle of *Jackson* demands that the image of the luminous source coincide with the far point one is led to use a plane mirror and to place the flame, surrounded by its opaque screen, quite near the eye of the observer. But, in order to observe the luminous band of astigmatism and the ring of aberration, we must place the lamp by the side of and a little behind the patient.

Bibliography.—Cumming (W.). *Medico-chirurgical transactions.* XXIX, p. 284.—Brücke (E.). *J. Müllers Archiv für Anatomie und Physiologie,* 1847, p. 225.—Helmholtz (H.). *Beschreibung eines Augenspiegels zur Beobachtung der Netzhaut am lebenden Auge.* Berlin, 1851.—Ruete (Th.). *Der Augenspiegel und das Optometer.* Göttingen, 1852.—Coccius (A.). *Ueber die Anwendung des Augenspiegels, nebst Angabe eines neuen Instruments.* Leipzig, 1853.—Cuignet. *Keratoscopie.* Recueil d'opht., 1873-74.—Parent. *Diagnostic et détermination objective de l'Astigmatisme.* Recueil d'opht., 1881.—Leroy (C. J. A.). *Le phénoméne de l'ombre pupillaire.* Rev. gen. d'opht., 1887, p. 289.—Bellarminoff. *Neues Verfahren den Augenhintergrund zu besichtigen.* Münch. med. Wochenschrift, 1888.—Bitzos (G.). *La Skiascopie.* Paris, 1892.—Demicheri (L.). *Examen ophtalmoscopique à l'image renversée sur les yeux fortement myopes.* Ann. d'oc., 1895.

The theory of the ophthalmoscope is found explained in several treatises on ophthalmoscopy. The following small book is to be recommended on account of its brevity and clearness:

Bjerrum (I.) (of Copenhagen). *Instructions pour l'emploi de l'ophtalmoscope.* Translated by *Grosjean.* Paris, Steinheil, 1894.

CHAPTER XIV

THE PUPIL

94.—To properly understand the working of a dioptric intrument, we must not only know the position and power of the refracting surfaces, but also the size and position of its diaphragm. I have already referred to the difference between the size and position of the apparent pupil and the real pupil, and observed that the pupil is generally displaced a little to the temporal side. Its size varies in different people; generally it diminishes with age, and finally becomes quite small in old people. As a rule it is larger in myopes than in hypermetropes, at least in appearance, for the anterior chamber of myopes is often deeper, which makes the pupil appear larger. In cases of complete amaurosis, the pupil is immovable and very large, except when the amaurosis has a spinal origin, in which case the pupil is often greatly contracted.

The pupil contracts and dilates under many different influences; these movements are very complex and, for the most part, still imperfectly elucidated. All agree on the existence of the sphincter, while that of the dilator is disputed, although physiological observations make its existence probable. The movements of the pupil are under the influence of the motor oculi and the great sympathetic. Cutting the motor oculi produces a dilatation of the pupil, much less, however, than that which may be produced by atropine. The contractions which accompany accommodation and incidence of light cease at the same time, as well as accommodation itself. The contraction which accompanies incidence of light is, therefore, produced by a reflex action between the retina and the optic nerve on the one hand and the oculo-motor on the other. It must be noted, however, that *Brown-Sequard* produced a contraction of the pupil by concentrating light on an enucleated rabbit's eye, according to which experiment the light would also have a direct influence

on the muscles of the iris. An irritation of the oculo-motor produces a contraction of the pupil, an irritation of the great sympathetic at the neck produces, on the contrary, a marked dilatation, while the cutting of this nerve contracts the pupil.

95. Action of Mydriatics and Myotics.—The instillation of a drop of a solution of *atropine* (0.5 per cent.) produces a marked dilatation of the pupil; it paralyzes its movements as well as the accommodation: the effect generally lasts eight days. If we use a much-diluted solution, the effect does not last so long and the action on accommodation is much less pronounced. To explain why the dilatation by atropine is much greater than that obtained by cutting the motor oculi, it is supposed that it acts at the same time by irritating the terminal fibres of the great sympathetic.

Homatropine (0.5 per cent.) dilates the pupil, but it generally does not act to any extent on the accommodation if the solution is pure. (1) Its effect lasts twenty-four hours.

Cocaine (5 per cent.) dilates the pupil, but does not act on the accommodation; at least I have not been able to find any effect on my own eye. (1)

A mixture of homatropine and cocaine dilates the pupil still more than either one of these alkaloids by itself . Such a mixture is recommended, therefore, for investigations of accommodation, the more so because the pupil is dilated some time before accommodation begins to diminish. *Scopolamine* ($\frac{1}{5}$ per cent.) produces complete paralysis of accommodation, with a very marked dilatation of the pupil which we can further increase by adding cocaine.

With a solution of *eserine* (0.5 per cent.) we obtain a very great contraction of the pupil, and the accommodation reaches its maximum. I have obtained with eserine a little greater amplitude than I could produce spontaneously. It is doubtful whether eserine acts directly on the sphincter, or whether the contrac-

(1) Other observers maintain the contrary; the differences are perhaps individual; perhaps due to the fact that they use different preparations.

tion of the pupil is analogous to that which always accompanies accommodation.

96. The Movements of the Pupil.

1° *The pupil contracts under the influence of light* (reflex by the optic nerve). It is not alone the light which strikes the retina of a particular eye, but also that which enters the other eye, which causes the contraction. The pupils are equal in size, even if one eye is exposed to a much stronger light than the other. If the pupil does not contract when the light strikes the retina of the same eye, and does contract when it strikes that of the other eye, we may infer a complete amaurosis of the eye in question. In complete darkness the pupil reaches its maximum dilatation, so that the iris is often not visible (1) (*Cohn, Cl. Dubois-Reymond*). This fact has been demonstrated by taking photographs of the eyes in complete darkness: we illuminate them with mixtures of powders, the light of which does not continue long enough to give the pupil time to contract. It is not easy to reconcile this observation with every-day experience, which shows that the reaction of the pupil to light depends on the oculo-motor, the cutting of which produces only a medium dilatation.

It is manifest that the object of this contraction of the pupil is to regulate the quantity of light that enters the eye.

2° *The pupil contracts during accommodation.*—To examine the functions of the pupil we must see whether it contracts: *a*) when the light strikes the retina of the same eye; *b*) when the light strikes the retina of the other eye; *c*) when the patient makes an effort of accommodation. We know that accommodative contraction may exist without the reaction to light, and *vice versa* (*Argyll Robertson*). The accommodative contraction has this peculiarity that even the most peripheral parts of the iris show a centripetal movement, which is not generally the case for the reaction to light (*Hueck*).

The object of this contraction is to eliminate the action of the

(1) If the iris is not visible at all, it is an apparent phenomenon, due to refraction through the cornea, for if we plunge an eye, the pupil of which is dilated to this extent, in water, the iris becomes immediately visible (*Stadfeldt*).

peripheral parts of the crystalline lens, which do not sufficiently accommodate.

3° *The pupil contracts when the aqueous humor escapes.*—I have already remarked that this contraction is also observed after death (*Arlt*), so that it must be considered as a purely mechanical phenomenon, which we may identify with accommodative contraction . I have made some experiments to elucidate the nature of this contraction; before describing them it is important to speak of the posterior chamber, the existence of which has been disputed.

On examining an eye by oblique illumination, we easily see that the border of the iris is in contact with the crystalline lens. We also see this very well by examination with the third image of *Purkinje,* which I have mentioned page 50, or by examining an eye affected with mature cataract. If we remove the crystalline lens from the eye, or if it be dislocated, the iris shows at each movement of the eye the trembling known as *iridodonesis; Helmholtz* and others were led to infer from these facts the non-existence of a posterior chamber; there exists, nevertheless, a small space filled with liquid between the crystalline lens, the ciliary body and the peripheral parts of the iris. We sometimes see in perfect eyes a slight trembling of the peripheral parts of the iris when the eye makes a movement.

The observation of *Arlt,* showing that we still see the pupillary contraction after paracentesis has been performed on the dead eye, struck me forcibly. To verify it I introduced the point of a *Pravaz* syringe into the anterior chamber; by depressing or withdrawing the piston we can make the pupil contract or dilate at will. By removing nearly all the contents of the anterior chamber I was able to reduce the diameter of the pupil to 1 or 2 mm. On the contrary, by forcing the injection as far as possible, the dilatation may extend so far as to make the iris disappear. (1) It is true that one part of the change is only apparent, as *Stadfeldt* has shown: the more the pupil recedes,

(1) When we increase the pressure much, the cornea becomes opaque; we can make it almost as white as the sclera; as soon as the pressure ceases, it again becomes transparent.

the more enlarged it is seen through the cornea; but on examining the eye under water, we find a very noticeable change. The phenomenon is difficult to explain; it is not due to the mere effect of pressure, for we may compress the eye all we want to without observing any change in the diameter of the pupil; nor is it due to a difference of pressure between the chamber and the posterior part of the globe, for, by injecting liquid into the vitreous body or by removing it, we no longer produce any change of the pupil.

I also injected a solution of gelatine into the anterior chamber, and then, by hardening the eyes slightly, I obtained pretty fair casts. Under these circumstances the posterior chamber is also always injected; the cast forms a prismatic ring, with an anterior surface corresponding to the iris, a posterior surface corresponding to the anterior surface of the crystalline lens and an external surface corresponding to the ciliary body. But, between the crystalline lens and the part of the iris next to the pupil, we never find any gelatine, or if there is any, it is so thin a layer that it is destroyed in the work of preparation.

4° During sleep the pupil is greatly contracted, even in amaurotic persons, whose pupil generally is large and motionless. The pupil is also contracted during narcosis, and generally when a person is in agony: at the moment of death it is generally greatly dilated; this dilatation disappears immediately. In spite of the pupillary contraction during sleep the reaction to light persists.

5° On examining the pupil with a magnifying glass we observe rhythmic contractions, which, at least in part, correspond to the systole, and which are due to the fact that the vessels are filling with blood. The contraction is greater when the systole coincides with an expiration. We cannot explain in this way all the slight contractions of the pupil which are observed with a magnifying glass.

6° We observe a dilatation of the pupil following fright; it also accompanies dyspnea, vigorous muscular action or a sharp irritation of any sensitive nerve.

97. Advantage of the Position of the Pupil near the Nodal Point.— *Young* remarked that if the pupil had been situated farther forward in the eye the apparent size of objects would have changed every time we made an effort of accommodation. We have seen that the image of a point for which the eye is not accommodated, forms a circle of diffusion, the center of which, corresponding to the middle of the pupil, is frequently

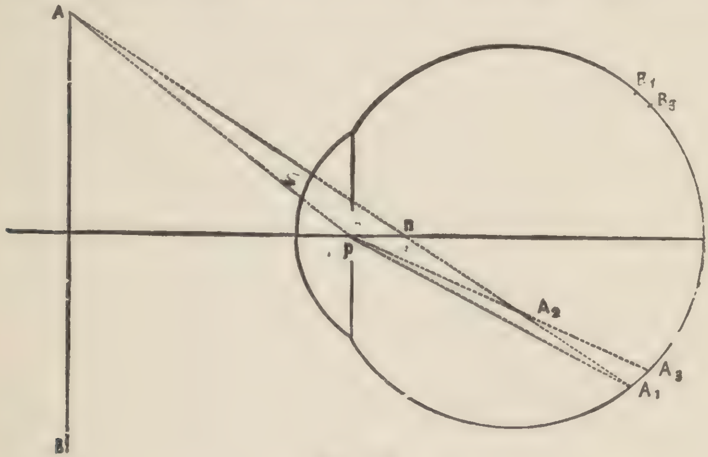


Fig. 142.

brighter on account of spherical aberration; if the pupil is not too large we may consider this center as a vague image of the point. Suppose that, in a state of repose, the eye is focused for the object AB (fig. 142). The image of the point A is formed at A_1 on the line A*n* passing through the nodal point. During accommodation the image is moved forward to A_2. To find the place where the diffuse image is formed on the retina we draw the ray A*p* passing through the middle of the pupil of entrance after refraction, this ray must pass through p_1 (1), the middle of the pupil of exit, and through A_2; the diffuse image is therefore formed at A_3 and the image of the entire object A_3 B_3 is smaller than the distinct image A_1 B_1. In the human eye we may observe a slight effect of this kind by using

(1) On the figure we suppose that p and p_1 coincide; really they are about 0.7 millimeters apart.

our accommodation while observing distant objects; it is more pronounced when we replace the pupil by a stenopaic opening, at some distance from the eye.

The position of the pupil near the nodal point has probably still another advantage. One of the first qualities that we require in a photographic objective is that it be *rectilinear,* that is to say, that the images of the straight lines placed peripherally in the field be straight, and not curved. We usually obtain this effect by placing the diaphragm in the nodal plane, and the position of the pupil near the nodal point of the eye seems to play a part for the correct vision of objects seen indirectly.

Nevertheless, the eye is not rectilinear. It follows from a series of experiments described by *Helmholtz* that, in indirect vision, the straight lines appear in the form of curves, the concavity of which is turned towards the point fixed. If we desire to repeat these experiments, we must place ourselves so that no other line, which we know to be straight, is in the field, for example by stooping over a large table.

1° We place on the table a small piece of paper A (fig. 143), which serves as a point of fixation, and two others, B and C, as far as possible from A, without ceasing to see them distinctly in indirect vision. While fixing A, we try to place a fourth piece, D, on the straight line which joins B and C. We shall nearly always place it too far inwards.

2° If we place on the table a strip of paper with parallel borders, 8 to 10 centimeters in width, and fix the center of it, the borders appear concave towards the point of fixation. The strip, therefore, appears larger at the middle than towards the ends.

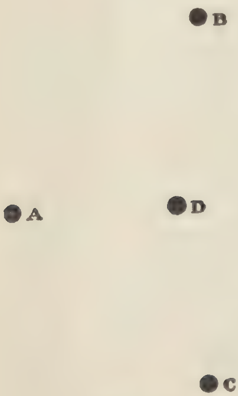


Fig. 143.

3° Guided by theoretical consideration, the value of which may appear doubtful, *Helmholtz* designed the hyperbolic chess-board, of which figure 144 is an illustration diminished in the proportion of 3/16. In

accordance with his theory, he found that, placed at a distance of 20 centimeters, for which the chess-board was calculated, he saw the curves assume the appearance of straight lines when he fixed the middle. When he stood at a greater distance, the lines appeared to have the curvature which they really had; moving nearer and nearer, he saw the curvature diminish and finally completely disappear. The distance at which the curvature disappeared was each time almost exactly 20 centimeters. If he approached nearer still, the lines presented the reverse curvature, appearing concave towards the middle.

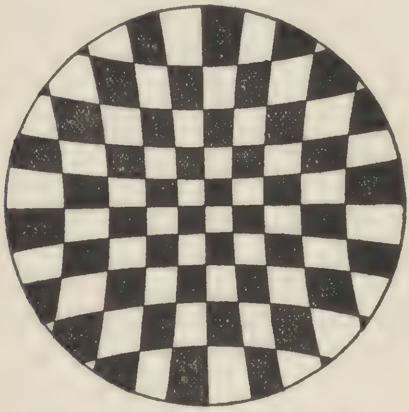


Fig. 144.—Hyperbolic chess-board of *Helmholtz*.

4° Another experiment of the same kind consists in placing a circular piece of cardboard in the periphery of the visual field; above or below we see it elongated in the horizontal direction, while on the two sides it appears elongated in the vertical direction.

We can express all these phenomena by saying that the visual field is seen narrowed towards the periphery. Let us suppose the plane visual field divided into equidistant zones, and suppose that we gave an illustration of it by making the zones diminish towards the periphery. We would thus obtain analogous deformities; the straight lines would be represented by curves concave towards the middle (see page 118). A circle placed

peripherally in the field would become narrower in the radial direction, and so forth.

To explain these observations, *Helmholtz* called attention to another observation which he made, and which is itself a consequence of the law of *Listing* (see chapter XIX).

Standing in front of a wall we look at a point A situated on a level with the eyes; we then raise the look, without changing the position of the head, towards the horizontal line which forms the upper edge of the wall. Moving the look rapidly along this line, we see it concave, with the concavity turned downwards exactly as we would see it in indirect vision by fixing the point A, if it was sufficiently distinct.

Faithful to the *empiric* theories by which he tried to explain most observations on physiologic optics, *Helmholtz* supposed that this illusion was the cause of the preceding one. Surveying the line with the look it appears curved on account of the law of *Listing,* and it is because we have thus learned that it appears curved that it does usually appear so in indirect vision also.—We must note that this way of observing the line, namely by surveying it with the raised look, appears altogether unusual. I do not think that before making this experiment I ever looked at a line in this way, as it would be more natural for me to raise my head to look at it, and in this case the illusion disappears. It is, therefore, not easy to understand how I would have known that the line ought to appear curved.

Fig. 145.
Artificial eye.

But the following experiment is still more at variance with the explanation in question. I had constructed a small artificial eye (fig. 145), all the dimensions of which approached as nearly as possible those of the human eye. The cornea and the crystalline lens are of glass, and have the same curvature as in the human eye; in order to

remedy somewhat the excessive refraction of the crystalline lens, I filled the eye with a mixture of glycerine and water, the index of which is a little higher than that of the vitreous body. The retina is replaced by a hollow hemisphere of ground glass, having nearly the curvature of the retina of the human eye. Although the refraction may not be absolutely identical with that of the human eye, the difference, however, cannot be very great.

With this eye I repeated and succeeded in all the experiments cited above (fig. 146). The image of the black strip has the borders convex towards the periphery; in order that the borders of the image appear straight those of the object must be concave. The image of a circle appeared shrunken in the radial direction, etc. The experiment with the chess-board of *Helmholtz* is still more conclusive. As long as the eye is at a great distance, the image is like the object; but, according as we move the eye nearer, the curvature of the line becomes obliterated, and very close to the drawing the lines of the image appear concave on the inside. I tried to determine the place where the direction of the curvature changes, or in other words the place where the figure appears most rectilinear, and each time I found a distance of 20 centimeters, at least as exactly as when making the experiment with my own eye.

Fig. 146.—Image of a window in the artificial eye.

According to this experiment it seems to me beyond doubt that all these deformities depend primarily upon the form of the retina. Projecting a plane on a hollow sphere, we necessarily obtain towards the periphery a narrowing of the projection analogous to that which we have found for the eye. It is possible, however, that the position of the pupil in front of the nodal point may play a certain part, for the illusion appears to me rather more pronounced if I look through a stenopaic opening, which acts as an artificial pupil placed in front of the eye.

This touches one of the fundamental questions of physiologic optics. I wish to speak of the antagonism between the *nativistic*

and the *empiric* ideas. Although this question is beyond the scope of the present work, I shall consider it for a moment.

Looking at a window, the visual sense tells me that it is square. How can the eye give this information? The nativists, among whom we must first mention *Hering,* say that, by an unknown congenital mechanism, the retinal impression gives directly to the mind the idea of the form of the object. We could express this idea by saying that, by an unknown mechanism, the mind sees the retinal image. The empiricists, among whom *Helmholtz* is the most celebrated, say that the retinal image gives us primarily no information on the form of the object, that it is only a "sign" of the object, almost as the letter A is the sign of a certain sound; by the movements of the eyes and by information furnished by the touch, we learn that this sign is to tell us that the object is square; *Helmholtz* expressed his ideas thus: "As for me, I think it probable that the figure, form and position of the true retina, as well as the deformities of the retinal image, are absolutely unconcerned with vision, provided the image be distinct in its whole length, and that the form of the retina and that of the image remain perceptibly invariable from one moment to another. We have absolutely no knowledge of the existence of our retina."

Under the influence of *Darwin,* an effort was made (*Donders*) to reconcile the two schools by saying that the qualities in question are the result of experiences, not of the individual, but of the species. Understood in this sense the empiric ideas scarcely differ from the nativistic ideas, the qualities being then congenital in the same sense as, for example, the actual form of our organs, and we would then have to distinguish sharply between what we may suppose learned by the same individual and what is due to the experience of the *species*.

The empiric theories are more attractive because they make an attempt at explanation, while the nativistic theories exclude all hope. But it would be necessary to apply them only to the phenomena for which they readily adapt themselves, and it seems to me that the great physicist of Berlin has gone too far in being willing to deny the relation between the illusions here

described and the deformities of the retinal image. It seems to me that there must exist a mechanism by which we can account for the existence of these deformities.

Bibliography.—The opposition to the too free application of empiric ideas does not date from yesterday. See *Œuvres de Young*, p. 239. "We are certainly obliged every moment to call experience to our aid in order to correct the errors of one of the senses by comparison with the perceptions of the others. [But] it seems to me that some scientists go too far when they assert that the use of all our senses is derived from experience alone without being willing to admit the existence of an instinct on a par with it," etc.

Arlt (F.). *Zur Anatomie des Auges.* Arch. f. Ophth. III, 2.—Du Bois-Reymond (Cl.). *Ueber Photographien der Augen bei Magnesiumblitz.* Arch. f. Physiologie, 1888, p. 394.—Tscherning (M.). *La contraction de l'iris accompagnant l'écoulement de l'humeur aqueuse.* Bull. de la Soc. franc. d'opht., 1885, p. 305.—Tscherning (M.). *Quelques conséquences de la loi de Listing.* Ann. d'oc., Sept., 1888.—Tscherning (M.). *La déformation des objets vus indirectement.* Bull. de le Soc. franc. d'opht., 1895, p. 403.

BOOK II
FUNCTIONS OF THE RETINA

CHAPTER XV
CHANGES WHICH THE RETINA UNDERGOES UNDER THE INFLUENCE OF LIGHT

98.— The sensitive layer of the retina is, in all probability, that with the cones and rods. Besides the fact that the very structure of the layer makes this hypothesis probable, it is further strengthened by the experiments and measurements of *H. Müller* (on the entoptic vision of the vessels, see page 186) as well as by observations on visual acuity. But we have not succeeded in explaining in a satisfactory manner the mechanism by which light is transformed into nervous action. We have succeeded in proving a certain number of changes which the retina undergoes under the influence of light, and we have studied on the other hand the functions of the retina, which are now very well known, but we have not succeeded in explaining their mutual relations.

Retinal Purple.—If we examine the eye of an animal which has been left in darkness for some time before enucleation, we find that the external segment of the rods has a purple color which disappears very quickly under the influence of daylight, passing through a yellow tint. The cones have not this coloration and the *fovea* of the human eye, which is composed of cones only, is without color. If we expose the eye of a living rabbit to daylight for a quarter of an hour, the purple first changes to a yellow and then completely fades away. Placing it so that the image of a bright object, a window for example, may be formed on the retina, we can thus obtain a permanent image (*optogram*). If, after having caused the purple to fade away, we leave the animal in darkness, the purple color returns gradually, provided that the retina be in contact with the pig-

ment cells. It is not necessary that they be the pigment cells of the same animal: if we place the retina of one eye in the place of that of another eye the reproduction of the purple is also effected in darkness.

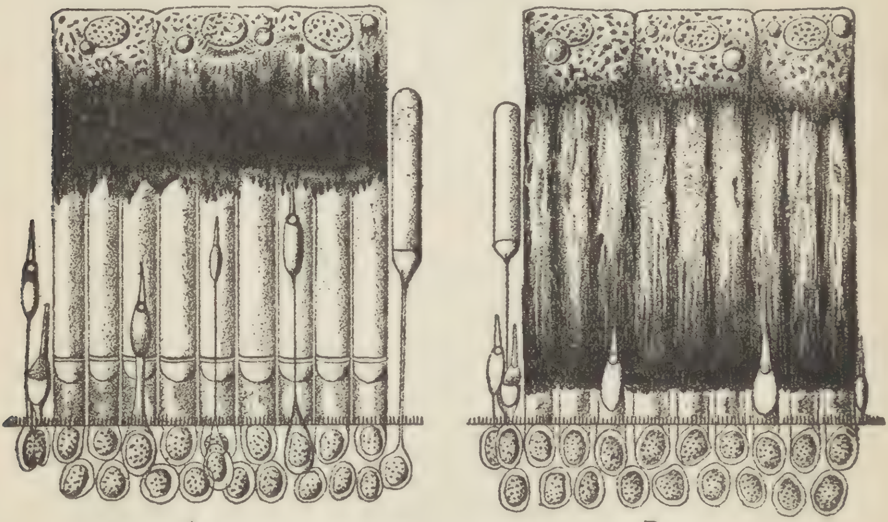
Vision does not depend on the retinal purple, since there is no purple in the fovea, since rabbits whose retinæ we have allowed to fade away completely are not blind, and since there are certain classes of animals, serpents for example, in which the purple is wanting.

The retinal purple was discovered by *Boll* in 1876; subsequently *Kuehne* labored much with the question, studying especially the chemical properties of the retinal purple and yellow. The enthusiasm with which the discovery of Boll was first received quickly grew cold when it was seen that it did not give a direct explanation of the mechanism of vision. Some time ago the question was again taken up and an effort made to put the retinal purple in relation, on the one hand, with the vision of certain colors, on the other with the adaptation of the retina to very feeble light. These efforts, some of which will be mentioned later on, have, up to the present, only a hypothetic character.

99. Movements of the Pigment under the Influence of Light.— By experimenting with frogs, *Boll* observed yet another phenomenon dependent on the influence of light. He observed that it was easy to separate the retina from the epithelium when the animals are left in darkness for an hour or two before death. If the animal has been exposed to light for a certain period before enucleation it is, on the contrary, difficult to separate them, and if we sever the retina we find it covered with black pigment spots which adhere to it. We know that the epithelial cells send prolongations between the rods which they separate from one another. In darkness the pigment is found massed between the exterior segments of the rods, but under the influence of light it is displaced so as to cover the terminal surface of the rod, and is projected among the rods, sometimes even to the external limiting membrane. The external segment of

the rod is swollen at the same time. Analogous phenomena have been described in the eyes of birds, mammals, and also in a human eye.

Van Genderen Stort made a step in advance in the biology of the retina by using a method by which the retina is hardened in a very little while (nitric acid); instead of cutting the retina with a microtome he hacked it with a razor. He showed that there is yet another change which the retina undergoes when exposed to light. In an animal left in darkness some time before death, we find the internal part of the cones long and filiform, and the length differs for different cones so that the latter are arranged in several rows quite a distance from the limitans externa. If, on the contrary, the animal has been exposed to light, the internal part of the cones is shortened and swollen: all the cones are



A B

Fig. 146*a*.—Section of the retina of a frog. After *Van Genderen Stort*. A, in darkness; B, in light.

placed in a row along the limitans externa (fig. 146*a*). According to *Van Genderen Stort* the retinal purple is also in the cells of the pigment epithelium, and it is probably secreted by these cells. He thinks that the pigment displacement has for its object

the protection of the rods against light, and that it is due to this fact that the epithelial cells send, under the influence of light, prolongations between the rods, almost like the cells, called chromatophres, which make the skin of some lower animals change color under the influence of light. *Van Genderen Stort* was kind enough to make a present of some of his beautiful preparations to our laboratory. The phenomena are so distinct that the first glance at the preparation enables one to tell whether the animal was exposed to light or not.

We must note further that *Kuehne* observed certain galvanic phenomena dependent on the action of light on the retina.

Bibliography.—Boll (F.). *Du Bois-Reymonds. Archiv. f. Anat. u. Physiol.*, 1877, p. 4.—Boll (F.). *Monatsber. de. Akad.* Berlin, 1877, Jan. 11.—Kuehne (W.), in Hermann (L.) *Handbuch der Physiologie.* Leipzig, 1879.—Van Genderen Stort. *Acad. d'Amsterdam*, June 28, 1884.

CHAPTER XVI

THE LIGHT SENSE

The functions of the retina are divided into three classes: the *light sense,* the *color sense,* and the *form sense.*

The light sense is the faculty of recognizing the different luminous intensities.

100. Psychophysical Law of Fechner.—According to this law *the smallest difference of perceptible illumination is a constant fraction* (about 1 per cent.) *of the total illumination.*

Fechner came to formulate his law by the following observations. One day he found a scarcely perceptible difference of brightness between two clouds, and was much surprised to see this difference persist on looking through a quite dark smoked glass. He calls this law *psychophysical* because, finding it also for other senses, he was led to consider it as a general law of perception. If, for example, a line must have a length of 105 millimeters in order that we can tell with certainty that it is longer than another of 100 millimeters, we will also find that a line must be at least 210 millimeters for us to be able to tell with certainty that it is longer than another of 200 millimeters. In both cases the relation between the smallest perceptible difference and the total length is the same, one-twentieth. It is so also if we examine the smallest perceptible difference between two weights, and so with the other senses.

We notice that our senses differ in this respect from most of our instruments. With an ordinary double decimeter, the shortest distance that we can *measure* (I do not say estimate) is a half-millimeter; the smallest measurable difference between two lines would be, therefore, a half-millimeter, and this whatever may be the length of the lines to be measured.

To determine the ratio between the smallest difference of perceptible illumination and the toal illumination, *Fechner* used the

following experiment which had already been described in the middle of the last century by *Bouguer* and by *Lambert*. The former had also observed the fact on which *Fechner* later based his law.

1° Let us place at some distance from a screen two candles, A and B (fig. 147), of equal intensity I, and place between the candles and the screen a stick so that it forms two shadows *a* and *b* on the screen. The shadow *a* is formed by A, and consequently illuminated only by B; the shadow *b* receives light only

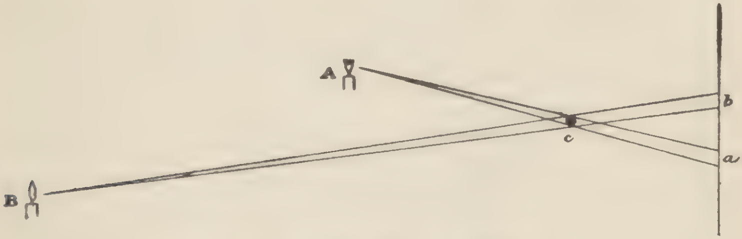


Fig. 147.—Experiment of *Bouguer*.

from A, and the remainder of the screen receives light simultaneously from B and A. By moving B away from the screen, the shadow *b* becomes weaker and weaker, and when the distance of B from the screen is nearly ten times that of A it ceases to be visible.

2° We replace the candles by others of one-half less intensity, and repeat the experiment: we find, as in the preceding case, that the shadow ceases to be visible at the moment when the distance of B from the screen is about ten times that of A.—And we shall find the same result, whatever may be the intensity of the candles.—The law of *Fechner* is thus verified.

Suppose that, in case of 1°, at the moment when the shadow disappears, B is at 500 centimeters from the screen, A at 50 centimeters. We know that the illumination is proportional to the intensity of the luminous source, and inversely proportional to the square of the distance. A gives, therefore, to the screen an illumination of $\frac{1}{50^2}$, B an illumination of $\frac{1}{500^2}$, while the shadow

b receives an illumination of $\frac{I}{50^2}$ only. The difference between the illumination of the screen and that of the shadow is therefore:

$$\left(\frac{I}{50^2} + \frac{I}{500^2}\right) - \frac{I}{50^2} = \frac{I}{500^2}$$

and the ratio between this difference and the illumination of the screen is

$$\frac{\frac{I}{500^2}}{\frac{I}{50^2} + \frac{I}{500^2}} = \frac{1}{10^2 + 1} = \frac{1}{101}$$

or $\frac{1}{100}$, since the measurement is not very exact.

In case 2° the relation is

$$\frac{\frac{1/2\ I}{500^2}}{\frac{1/2\ I}{50^2} + \frac{1/2\ I}{500^2}} = \frac{1}{101}$$

It is consequently the same in both cases.

The law of *Fechner* explains many of the phenomena daily observed.—If, after having performed with the candles the experiment cited above, we open the shutters so that the daylight strikes the screen, the shadows disappear. The difference between the illumination of the shadow and that of the screen remains the same, but the ratio between this difference and the total illumination of the screen is much below the fraction of *Fechner*.—We read as well in the evening, with a gas light, as in day time, although the illumination in day time is enormously more powerful, because the ratio between the light reflected by the black letters and that reflected by the white paper remains the same.—In a space illuminated by a very powerful lamp, the flame of a candle held at some distance from the screen produces a shadow of it, because it absorbs a part of the light of the lamp. If we move the candle nearer the screen, the illumination increases and the shadow disappears, although the difference of brightness between it and the background remains the same.

The law of *Fechner* is true only for medium degrees of illumination. If the illumination becomes very feeble, the difference must be relatively much more considerable. We read very well with a gas light; but if we lower the flame much we cannot read any longer, although the ratio between the light reflected by the letters and that reflected by the paper remains the same.—It is possible that this difference may be due to what is called the *retina's own light,* an expression by which we designate the feeble glow which may still be perceived in a completely dark room, and which is due to internal causes (friction of the blood in the vessels of the retina against the sensitive layer, perhaps also processes in certain parts of the brain, etc.). We can conceive that, if this light is added to that reflected by the printed sheet, the difference of brightness between the letters and the white sheet may fall below the limit of *Fechner.*—The law of Fechner also ceases to be applicable when the light is very strong. This is why we cannot see the spots on the sun with the naked eye, on account of the dazzling, but very well with a smoked glass.

But, within the very extended limits which correspond almost to the limits of illumination which we use, the law of *Fechner* is verified with very great exactness. It is not absolute, however: in order to distinguish very fine shades, it seems that there is a certain illumination which is most favorable, viz., that which approaches the light of a clear day.

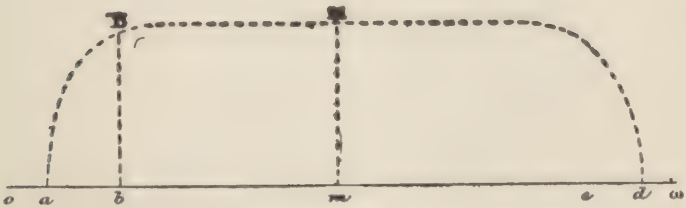


Fig. 148.

The acuity of the light sense may be expressed by the inverse of the fraction of *Fechner.* If the latter be $\frac{1}{100}$, we say that the acuity of the luminous sense is equal to 100; if, by greatly

diminishing the illumination, the fraction rises to $\frac{1}{50}$ we say that the acuity is only 50, and so forth.

We could illustrate the relation between the light sense and the illumination by a curve which would have a form like that of figure 148. The division of the horizontal line would indicate the degree of illumination, beginning on the left by complete darkness, and terminating on the right by the light of the sun. The ordinate of each point of the curve would measure the acuity of the light sense. As long as the illumination is very weak, the eye sees nothing: when it reaches a certain degree which, in the figure, is marked by the letter *a*, the eye begins to be able to distinguish white objects. This degree of illumination, which forms the lowest limit of visibility, is called *threshold* ("Reizschwelle"). As long as the illumination remains so feeble, the light sense is not very acute; the perceptible differences are considerable. But the acuity increases quickly, and when the illumination has reached a certain degree, *b*, the acuity reaches the degree which it holds for a long time, until the illumination has attained the power *c*. It is for the part *bc* that the law of *Fechner* is true, but not exactly, for this part of the curve is not altogether straight. It reaches its highest point at M.

If we increase the light still more, the luminous sense falls quickly; there is again need of very considerable differences of light in order that the differences may be distinguished.

Let us designate by *a* the smallest difference of appreciable sensation. If a light of a certain intensity I produces a certain sensation S, there is need of an intensity $I+\frac{1}{100}I=\frac{101}{100}I$ to produce the sensation $S+a$, an intensity of $\frac{101}{100}I+\frac{101}{100}\times\frac{1}{100}=I\left(\frac{101}{100}\right)^2$ to produce the sensation $S+2a$, an intensity of $I\left(\frac{101}{100}\right)_3$ to produce the sensation $S+3a$, and so forth. It is under this form that the law was promulgated by *Fechner*; for the fact itself was known since the works of *Bouguer* at the commencement of the eighteenth century. The right by which we make the differences designated by *a* equal to one another may be disputed.

101. Measurement of the Light Sense.—We usually limit ourselves to determining:

1° The *threshold,* the lowest limit at which the eye begins to distinguish anything (corresponding to the point *a* of the curve);

2° The least difference of brightness which we can distinguish by ordinary illumination, corresponding to B*b* or to M*m* (fig. 148). It is this determination which we have just made with the candles.

We determine the *threshold* (1) with the *photoptometer* of *Foerster* (fig. 149). It is a box painted black inside. The patient looks through two apertures, corresponding to his eyes *a* and a_1, towards a white surface, placed at the far end of the box, on which are traced large black marks T. The only light which can penetrate into the box comes from a square window F, the aperture of which we can change and which is placed beside the apertures through which the patient looks. Behind the window, which is covered with oil paper, burns a standard candle L. The minimum aperture of the window permitting the patient to see the black marks gives the threshold. The test is not very exact; it is difficult to obtain very uniform answers, and adaptation enormously influences the result.

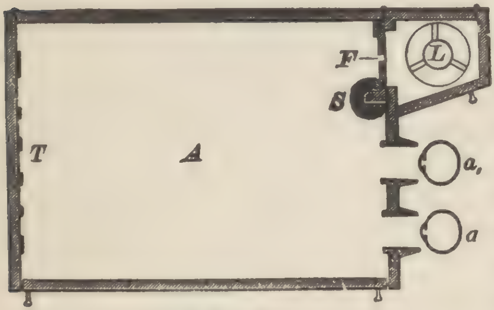


Fig. 149.

The *photoptometer* of *Charpentier,* also intended to determine the threshold, consists of a tube, 22 cm. long and 5 cm. wide, the extremities of which are closed by plates of ground glass A and B. At the middle of the tube are placed two lenses of 11 cm. focal distance, and between them a diaphragm with changeable aperture. On illuminating the plate A the lenses project an image of it on the plate B, the brightness of which

(1) It is doubtful whether the determination of the threshold is really anything else than the determination of the fraction of Fechner for a very weak illumination.—Theoretically, for the determination of the threshold, it ought to be required that the eye can compare a very weak light with absolute black; but we cannot produce absolute black on account of the retina's own light.

image we may cause to change by changing the aperture of the diaphragm. It is the plate B which serves for the observation; for the protection of the eye of the observer we may add to it a second tube blackened internally, the length of which corresponds to the distance for work of the observer. An eye-shade which permits of exact adaptation to the borders of the orbit excludes all extraneous light. The minimum aperture of the diaphragm which permits the observer to distinguish the plate B, determines the threshold.—In every instrument of this kind the difficulty consists especially in finding a luminous source which can give a constant and uniform illumination.

In order to determine the smallest perceptible difference we can use the method with the candles, described above. Another

Fig. 150.—Disc of *Masson*.

method consists in the use of the disc of *Masson*, a white disc of which sectors of different sizes have been blackened (fig. 150). By subjecting this disc to a sufficiently rapid rotation, we see three gray rings separated by white intervals. Supposing that the sector *a* is 20°, the sector *b* 10° and the sector *c* 5°, and supposing, which is not strictly true, that the black does not reflect any light at all, the brightness of the three gray rings would be 340, 350 and 355, if we place the light of the white

rings at 360. The difference between the exterior gray rings and the white will be 5, and the relation between this difference and the white will be $\frac{5}{360}=\frac{1}{72}$, which represents the value of the fraction of *Fechner* of the examined subject, if he can distinguish the three images. If he can distinguish only two, the fraction of Fechner is $\frac{360-350}{360}=\frac{1}{36}$, and so forth. A great number of rings must be used; the illumination must be good, and the patient must not be too far away, in order to eliminate the influence of a diminished visual acuity. It is evident, however, that we cannot completely eliminate it; the acuity may be so poor as to prevent the patient from distinguishing anything.

To obtain an impression of a uniform gray with the disc of *Masson,* it is necessary that it rotate with a certain speed, about 20 to 30 times per second. If the disc carries several black and white sectors, alternating, the speed may be less. In case the speed is not sufficient, the disc gives a scintillating impression

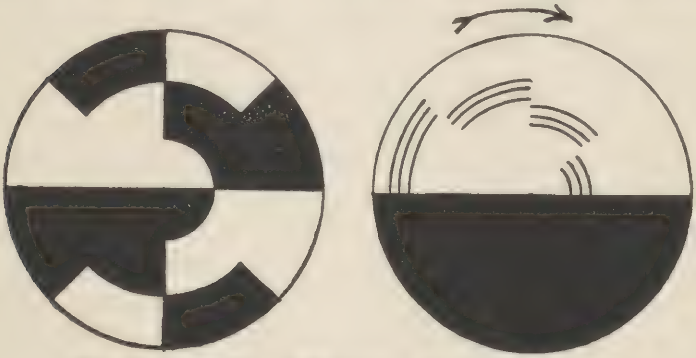


Fig. 150*a*.—A, Disc of *Helmholtz;* B. Disc of *Benham.*

and we often observe on it very beautiful colors. The disc A (fig. 150*a*) has been described by *Helmholtz:* with a certain speed the external ring shows very vivid colors, among which the red and green predominate; they are often arranged in a manner which recalls a series of short spectra, as we observe them with gratings. But the phenomena are very changeable; in the second ring, which has only four sectors, the yellow and blue predominate with this speed, but only to a slight extent. If we

increase the speed the external ring gives a uniform gray, while the second ring assumes the appearance which the external ring had previously. In figure 150*a*, B represents the disc of *Benham*. If we make it rotate in the direction of the arrow, the arcs form concentric circles which present quite vivid colors in the following order, starting from the middle: red, brown, olive-green, blue. Making the disc rotate in the opposite direction, the order of the colors is reversed. The most beautiful of the colors is the red; the circles seem traced in blood.

The nature of these phenomena is not yet elucidated. We must not think that it is due to a decomposition of the white light, for the experiment succeeds perfectly when illuminating the disc with homogenous light, providing it is sufficiently strong We even see colors of this kind when looking towards the homogenous sodium flame.

Another method of studying the power of distinguishing differences of brightness consists in examining the visual acuity for pale letters, the brightness of which we can determine by comparing them with the rings on the disc of *Masson*. This method, which was described by *Javal*, was later developed by *Bjerrum*. It would be better to have a series of tables of visual acuity with paler and paler letters, but generally one suffices; *Bjerrum* recommended the use of letters, the brightness of which is one-twelfth weaker than that of the background. For these letters, a normal individual has an acuity of about one-third the acuity which he has for black letters on a white ground. It is evident that this method cannot be considered as an exact measure of the light sense, since the visual acuity plays a great part in the response of the patient. In order to eliminate to a certain extent this influence, one can use one's own eye as a control, by lowering his visual acuity by means of a convex glass, until it is equal to that of the patient.

102. Results.— The *threshold* of the normal eye was determined by *Aubert*. He found that the weakest light that we can distinguish is that of a sheet of white paper illuminated by a candle placed at a distance of from 200 to 250 meters. The

threshold varies much with the state of *adaptation* of the eye; placed in a dark room, we do not at first distinguish objects which we see very distinctly later on when accustomed to the darkness. For the determination of the threshold it is, therefore, necessary to leave the patient some time (as much as 20 minutes) in the darkness, with eye bandaged, before beginning the examination. It seems that, by this stay in the darkness, the entire curve (fig. 148) is displaced towards the left, and also to its extreme limit, for on leaving the darkness the eye is dazzled by an illumination which it usually bears very well.

The fraction of *Fechner* varies in normal persons between $\frac{1}{100}$ and $\frac{1}{180}$ (0.55 to 1 per cent.).

For a very weak illumination, the light sense of the macula is less acute than that of the surrounding parts; by fixing a point a little to one side of it, we better distinguish objects the brightness of which differs only slightly from that of the background, for example, when we try to distinguish very dim stars. According to certain authors, *Parinaud* for instance, this phenomenon must be attributed to the fact that the fovea does not possess the faculty of being able to adapt itself to very weak illuminations like the rest of the retina, and this difference is explained, because the fovea, composed of cones, has no retinal purple, which is considered as the organ of adaptation. This hypothesis is confirmed by another fact, namely, the knowledge that the time of repose which the eye requires to reach complete adaptation is nearly the same (about 20 minutes) as that which is necessary for the reproduction of the purple. It is possible, however, that the inferiority of the macula may be partly due to its yellow pigmentation. The pigment absorbs a part of the blue rays, which, as we shall see, play a dominant part in vision by weak illuminations.

The threshold is displaced upwards in patients suffering from hemeralopia. It seems, however, that, in many cases, there is question rather of an anomaly of the adaptation, which requires much more time to take place than in the normal eye. Leaving a person affected with hemeralopia in darkness, he continues to improve for some time. We can prove the existence of hemera-

lopia with the photoptometer of Foerster, or by examining the visual acuity while we lessen the illumination. Hemeralopia is a constant symptom of pigmentary retinitis; we meet it as often in cases of syphilitic retino-choroiditis, sometimes in cases of detachment of the retina or in glaucoma. It is extremely rare in cases of pure atrophy of the optic nerve. In cases of idiopathic hemeralopia, we find nothing in the fundus of the eye; this disease is often congenital and hereditary, and therefore incurable; if, on the contrary, the disease has existed only for a short time, its prognosis is favorable; it sometimes has an endemic character. It may happen that the peripheral part of the visual field only is affected; we then establish the existence of the disease by examining the visual field with a weak illumination.

We sometimes meet cases in which the fraction of Fechner is increased; in which, consequently, the patients cannot distinguish gray from white. This affection is met with especially in cases of atrophy of the optic nerve and in central scotoma.—One of the first cases of this kind was observed at the clinic of *Hansen Grut,* at Copenhagen, and described by *Krenchel.* It was a patient who presented himself, saying that he did not see well enough to find his way. Examined with the ophthalmoscope, the papillæ were whitish, the visual acuity was normal, and the visual field was only slightly contracted. It was puzzling, therefore, to explain the complaints of the patient until the idea of examining him with the disc of *Masson* presented itself: the fraction of *Fechner* had increased to $\frac{1}{10}$. The patient distinguished perfectly black on white, but was unable to distinguish between gray shades, as they present themselves, for example, in street paving; whence the difficulty which he experienced finding his way.

We sometimes meet patients who claim that they see better when the illumination is low (*nyctalopia*). Examining their visual acuity, we find, however, that it does not increase when we lessen the illumination (at least in cases in which we have not to do with a purely optic phenomenon: this is why a central leucoma becomes less annoying when the pupil is dilated).—But,

on comparing these persons with a normal person, we note that by lessening the illumination the acuity of the normal person diminishes more quickly than that of the patient. If the normal person has an acuity three times that of the patient by ordinary illumination, it may happen that on diminishing the illumination both would have the same visual acuity. Persons suffering from a central scotoma sometimes complain of nyctalopia for a like reason. We have seen, indeed, that the superiority of the *macula* over the rest of the retina diminishes with the illumination, so that with a very weak illumination the *fovea* does not see so well as the rest of the retina. We can understand, therefore, that a central scotoma may cause relatively less annoyance when the illumination is weak.

We must recall, too, the quantitative measurement of the light sense in persons affected with cataract. The patient ought to be able to recognize the illumination of an ordinary lamp at a distance of 4 to 5 meters, or that of a candle at 2 meters, and its projection must be good, that is to say, the patient must be able to tell the direction in which the luminous source is located. If the patient does not satisfy these conditions, we may conclude that there exists an affection of the fundus of the eye, which compels us to make an unfavorable prognosis.

Bibliography.—Bouguer (P.). *Essai d'optique.* Paris, 1729.—Bouguer (P.). *Traité d'optique sur la gradation de la lumière.* Paris, 1760.—Lambert (J. H.). *Photometria.* Augustæ Vindelic, 1760.—Masson. *Etudes de photométrie électrique.* Ann. de physique et chimie, 1845, t. XIV, p. 129. —Fœrster. *Ueber Hemeralopie und die Anwendung eines Photometers im Gebiete der Ophthalmologie.* Breslau, 1857.—Fechner. *Elemente der Psychophysik.* Leipzig, 1860, 2 vol.—Klein. *De l'influence de l'éclairage sur l'acuité.* Paris, 1873.—Krenchel (V.) in *Klin. Monatsbl. fur Augenheilk.* February, 1880.—Bjerrum (J.). *Undersoegelsen of Synet.* (Danish). Copenhagen, 1894. Charpentier (A.). *La lumière et les couleurs.* Paris, Baillere, 1888.

The work of *Lambert* is first in importance. A German translation with notes by *Anding* has just appeared at *W. Ostwald. Die Klassiker der exakten Wissenschaften.* Leipzig, 1892.

CHAPTER XVII

THE COLOR SENSE

103. General Remarks.—On analyzing any color with the spectroscope, we find no other tints than those which compose the solar spectrum, mixed in different proportions. The only colors which would seem to form an exception, the brownish colors, are really red and yellow colors of slight intensity, more or less mixed with white. To examine the color sense, therefore, we may limit ourselves to the study of spectral colors and their mixtures. We have thus the advantage of experimenting with pure colors, which are easily definable by the wave length of the rays. The use of colored papers, although very convenient, has many drawbacks, in consequence of the impossibility of defining exactly the color of the paper used, so that another experimenter may be able to procure a similar tint. On the contrary, if we obtain a result with spectral light of a certain wave length, the experiment may be described in a very exact manner, the only condition which may be left uncertain being the intensity of the light used. On analyzing blue spectral light with the spectroscope we find only blue, while the light reflected by a paper of this color contains, besides blue, most of the other colors of the spectrum. There is another way of procuring pure colors, for the incandescent vapors give monochromatic light, at least approximately. Thus the sodium flame gives yellow light of a wave length of 0.59μ, the lithium flames red light (0.67μ), the thallium flame green light (0.54μ), and the strontium flame blue light (0.46μ). But, as a rule, these flames are in less common use than spectral light. The light which passes through colored glasses is generally far from being monochromatic; we must, however, except red glasses, colored with oxide of copper, which, when they are a little dark, allow scarcely any but red rays to pass. Among liquids we sometimes use the solution of

bichromate of potash, which absorbs the blue extremity of the spectrum, and the solution of sulphate of copper-ammoniac, which absorbs the red, the yellow and part of the green. A mixture of both allows a quite pure green light to pass.

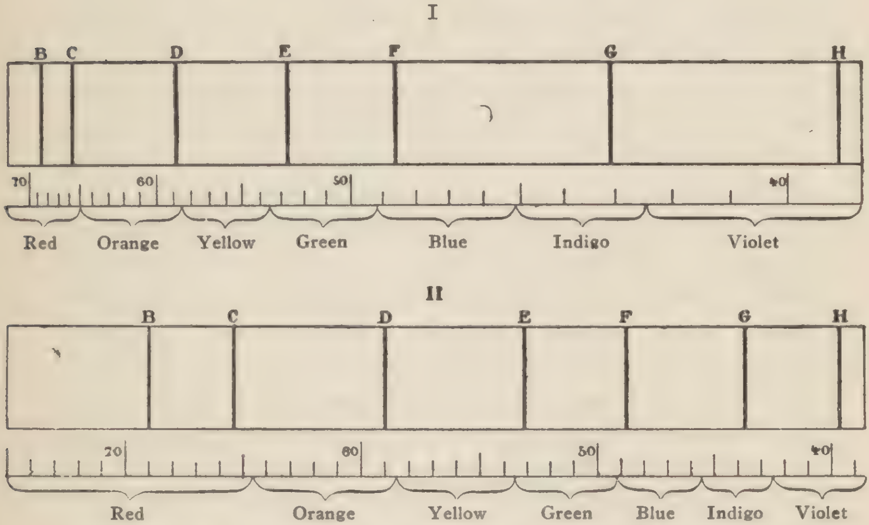


Fig. 151.—I. Spectrum of refraction.—II. Spectrum of diffraction. The numbers indicate the wave length in hundredths of μ.

We distinguish between the spectra of refraction, formed by means of prisms, and the spectra of diffraction, which are obtained by allowing light to pass through a *grating,* that is to say, a glass plate on which a great number of very fine parallel lines have been traced.

The spectra of refraction are preferable because they are, generally, purer than the spectra of diffraction. They have this inconvenience that the relative width of the different colors varies with the prism used. The red and orange colors are reduced to a relatively small space, while the blue and violet colors are stretched out over a large surface. In the spectrum of diffraction, the distance between the different colors is, on the contrary, proportional to the difference of the wave length (fig. 151), so that all the spectra of diffraction are alike and

form, so to speak, the normal spectrum. The yellow is at the middle of the spectrum; the red and orange occupy half, the green, blue, indigo and violet the other half.

As landmarks in the spectrum, we frequently use the lines of *Fraunhofer*, the wave lengths of which have been very exactly determined. Say, for example, that the rays, which we use, are situated at half the distance between E and F; on the scale of figure 151 we see that the light used must have had a wave length of 0.50 to 0.51μ.—It is better, however, to determine the wave length directly, which is easily done by means of a grating.

I have already observed that there are in the spectrum rays beyond the red which are not visible. The extreme visible red corresponds nearly to a wave length of 0.8μ. The colors then follow in the well-known order: red, orange, yellow, green, blue, indigo, violet. Beyond the violet come ultra-violet rays, which are not visible under ordinary conditions, but which can be observed by means of a photographic plate, or by receiving them on a fluorescent screen, or simply by eliminating all other light according to the method given on page 131. They are then seen with a certain grayish color, which is, perhaps, partly due to the fact that the retina is fluorescent.

We distinguish colors according to their *hue* (*tone*), their *purity* or *tint* (*saturation*) and their *brightness* or *shade* (*intensité*). The *tone* or *hue* depends on the wave length alone, or, in other words, on the position of the color in the spectrum: the red has a different hue from the green, etc. The *saturation* or *purity* depends on the white which is found added to nearly all existing colors, except those of the spectrum: the less white there is, the greater the purity of the color. The *intensity* or *brightness* depends on the quantity of light. If we have formed a spectrum by means of a certain luminous source, and then increase the intensity of this source, the intensity of all the colors of the spectrum increases at the same time.

The hue changes constantly in the spectrum: that is to say, if we take light from two different parts of the spectrum, we cannot make them alike by changing their brightness. The change reaches its greatest rapidity in the green-blue part of

the spectrum, where even a variation in the wave length of 0.001μ produces a change of *hue;* the rapidity diminishes towards the extremity, and in the extreme parts of the red and violet the hue remains the same (*Kœnig* and *Dieterici*).—According to *Kœnig* we can distinguish about 160 different hues in the spectrum. On the other hand, according to the same author, the eye can distinguish about 600 different degrees of brightness between the *threshold* and dazzling light. This is true for white and probably also for the different hues of the spectrum, but the total number of different impressions between which the eye can make a distinction is, however, less than one would think in view of these indications, for when the brightness becomes very great or very feeble, the color disappears as we shall see forthwith.

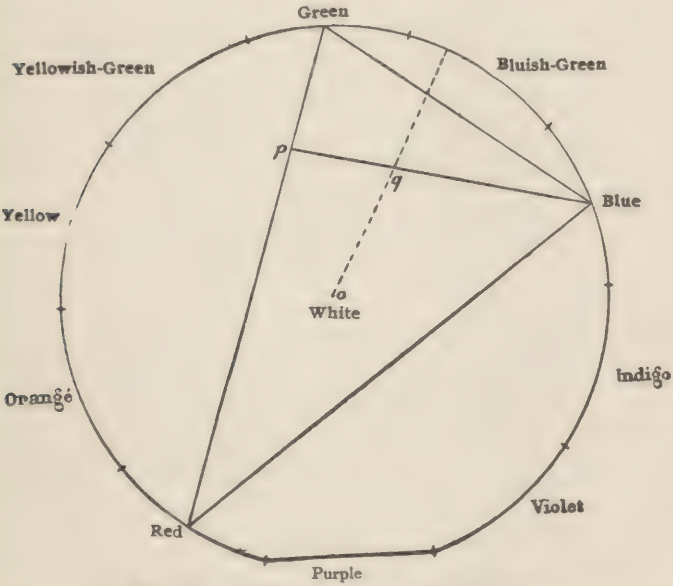


Fig. 152.—Table of colors after *Newton*.

On examining the spectrum it is easy to see that our *sensations or colors* form a continuous series. We begin with the red, which passes from orange to yellow, etc., and end with the violet, the tint of which presents an analogy to the red. The

intermediary color between the red and violet, *purple,* is not found in the spectrum, but it would be possible that this color would be produced by ultra-violet rays if the retina were not fluorescent.

We can, therefore, represent the gamut of the colors by a closed curve. The simplest form we can give to this curve is that of a circle (fig. 152), replacing, however, the part corresponding to the purple by a straight line; we shall soon see why. We suppose all the colors of the spectrum placed on this circle in their natural order. At the center is the white, and on the right, going from the white to one of the spectral colors, are the different tints, the purity being greater as we approach the spectral color. If we mix two colors, the mixture will have one of the intermediary hues often bleached with white, and if we mix, in suitable proportions, two colors situated opposite to each other on the table, we obtain pure white. Two colors which, when mixed, give white, are called *complementary*. For this reason red is complementary to green-blue, green to purple, yellow to indigo and orange to blue.

It was *Newton* who first arranged the colors as in this table. We find in it all hues and all degrees of purity.

I must add a few words on the sensation of black. First, it must be noted that black produces a real sensation: to see black is not the same thing as to see nothing at all. The most striking example is that of the spot of *Mariotte,* which corresponds to the papilla. In this spot we see nothing, but we do not see it black. By looking directly in front, one sees a part of the space in which one is; in regard to that which is beyond the limits of the visual field, one does not see it, but it does not appear black. The impression of black is, therefore, a true sensation, which corresponds to the state of repose of the visual organ.

There exists no completely black object in nature: even black velvets still reflect a comparatively considerable quantity of light. A black object placed in the direct light of the sun may appear clearer than a white object placed in the shadow.

According to some measurements which I have made, the whitest paper which I could find (visiting cards) returns only about a third of the incident light (37 per cent.). The *normal white of Kœnig,* which is obtained by burning a thread of magnesium and allowing the vapor to be deposited on a sheet of paper, sends back about two-thirds of the light; its whiteness is nearly that of snow. Ordinary black paper (bristol black) returns nearly 5 per cent. of the incident light (1.5 per cent. of the quantity reflected by the white paper); black velvety paper sends back about 5 to 1000 of the incident light (1.5 per 1000 the quantity reflected by white paper). The most absolute black that we can produce is that of an aperture made in the side of a closed box, blackened internally. Compared with this black even the velvety paper appears slightly grayish.

Black does not figure on the table of *Newton.* If we desire to include it in the illustration, we must suppose the colors placed on a body of three dimensions, a pyramid or a cone (*Lambert*). The table of *Newton* would form the base of the cone, while the black would form its apex: on the conical surface we would place the colors of little intensity. Thus the brown would be placed between the yellow and the black, etc.

104. Phenomena of Contrast (Simultaneous).—Our judgment of colors is always influenced by the colors of surrounding objects. This fact is well known to painters, whose color sense is generally highly developed, so that they often see colors that inexperienced persons would not perceive. But, in special circumstances, this influence makes itself felt in a very striking manner.

1° Experiment of H. Meyer.—Placing a small piece of gray paper on a sheet of colored paper and covering the whole with a sheet of tissue paper, the small piece is seen to be of the complementary color. *Pflüger* had letters, thus arranged, printed for the examination of color-blindness.

2° Experiment of Ragona Scina.—Two sheets of white cardboard (BC and BD, fig. 153) are placed so as to form between them a right angle; on each is a black spot, *a*, *b*, and a red glass BE is placed so as to form an angle of 45 degrees with the cardboard. The eye A receives from BC the rays which have passed through the red glass and from BD the rays reflected by this glass. The former are red, the latter white, so that the background BC would appear whitish-red. The spot *a* is seen at *a′* of a deep red color, because the eye receives at this place only red rays, the white rays which should come from BD being wanting. Corresponding to *b* the eye receives only white rays coming from BD, and nevertheless, *b* appears of an intense green by contrast. The experiment, which is very pretty, may be performed with other colored glasses. We always see *a′* and *b* in complementary colors.

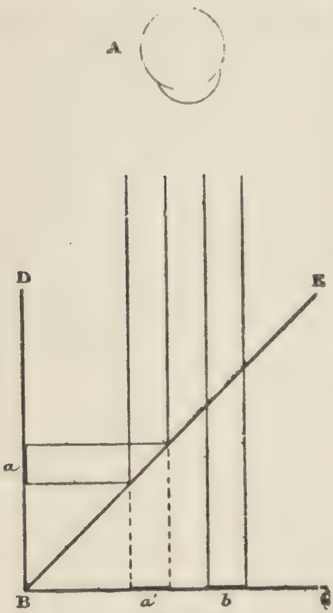


Fig. 153.
Experiment of *Ragona Scina.*

3° Colored Shadows.—Let A and B (fig. 154) be two candles, of which A may be the brighter; in front of A we place a red glass; *a* and *b* are the shadows which the stick *c* forms on a white screen. The screen illuminated by the white light from B and the red light from A, should appear whitish-red, but the red is scarcely perceptible; *b*, which is illuminated only by the red light from A, appears red, and *a*, which should appear white, appears green, by contrast. We can also make the experiment

with daylight and that of a candle, in which case there is no need of the colored glass, since the colors of the two lights already differ. We begin by illuminating the screen with daylight; we see the screen white and the shadow black (gray). On lighting the candle the screen still appears white, although it would seem that it ought to appear yellow, since it is partly illuminated by the yellow light of the candle; the shadow, which just now appeared gray, has become yellow by the illumination of the candle, and the other shadow, which receives the daylight, appears blue "by contrast."

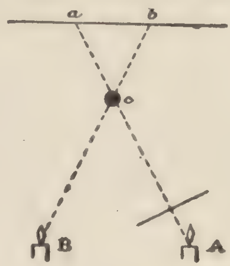


Fig. 154.
Experiment with colored shadows.

4° Experiment of Dove.—Analogous phenomena with colored shadows are observed when we place a colored glass opposite a mirror. We then see two images of a white object, one by reflection on the anterior surface of the glass, the other by reflection on the mirror; this latter has the color of the glass, since the rays have passed through the glass twice. The first, which ought to be white, shows by contrast the complementary color. With a black object on a white ground, the sash of a window for example, we have the phenomena reversed.

We observe that the expression "by contrast" scarcely explains these singular phenomena. In most of these cases it seems that the fundamental phenomenon lies in the defectiveness of our judgment of white. *Thomas Young* already directed attention to the fact that a sheet of white paper appears white to us, as well when illuminated by the yellow light of a candle as by the red light of a coal fire. We may say that we consider always as white the bodies which return the greatest quantity of light, whatever may be the light used (*Javal*). This is primarily independent of the illumination, and this is why a sheet of white paper appears to us white with different illuminations. But the recollection of the illumination by daylight plays, nevertheless, a part, so that, if the real color differs much from it, the paper seems white with a slight colored tone: thus when

we look at it through a red glass, in which case the paper returns red rays only, it appears a reddish-white.

In the experiment with colored shadows the screen appears to us white when it is illuminated by daylight only, and also when it is illuminated by a mixture of daylight and candle light at the same time. But if, under these circumstances, the whitish-yellow light which illuminates the screen appears white to us, it is not strange that the white light which illuminates one of the shadows appears blue, that is to say, less yellow than the screen. We may regard, so to speak, the zero of the scale of our color sensations (the white) displaced, and with it the entire scale.

True Simultaneous Contrast.—While the phenomena of which we have just spoken are due to a false judgment of the

Fig. 155.—Disc of *Masson*.

color white, there are others which are due to a true contrast. By making a disc like that of figure 155, but having a greater number of sectors, rotate we obtain gray rings, and we observe that we cannot see the outer rings which are very pale; we see only the borders of each ring: the external border, which appears deeper than the rest of the ring, by contrast with the following ring which is paler, and the internal border which

appears paler than the rest, by contrast with the neighboring darker ring. By replacing the white and black by yellow and blue, we obtain rings which present different shades of gray; the internal rings are bluish, the external rings yellowish. But each ring has an internal border which is yellow, by contrast with the preceding ring which is bluer, and an external border which is blue, by contrast with the following ring which is yellower. The phenomenon is very pronounced, but disappears, at least in a great part, if we separate the rings by very fine black circles. The diffuse borders favor considerably the effect of the contrast.

105. After-Images (*Successive Contrast*).—When we look at a small colored surface, placed on a white ground, by fixing exactly the same point for a short time, we observe that the color diminishes gradually in brightness: the red becomes brown, etc. We observe at the same time that the object is surrounded by a narrow border of the complementary color, due to the fact that, in spite of himself, the observer makes slight changes in the direction of the look. We explain the phenomenon by saying that the part of the retina where the image is formed is fatigued for the color in question. If we then transfer the look to a sheet of white paper, we see an image tinted with the complementary color. If the surface be red, the image appears bluish-green. We may suppose the white color as composed of two complementary colors, red and green; the retina being fatigued for the red color, it is the green color which predominates. If the object we look at is white, the after-image is black; but if we look at a flame or other very bright object, we obtain a colored after-image, the color of which changes before its disappearance.

The after-images of the complementary color are called *negative:* we can also obtain *positive* images, each part of which has the same color as the original. We close the eyes and cover them with the hand for some minutes, so that no light can enter the eye. We keep in this position for some time until all prior impressions on the retina have disappeared. This done, we remove the hand and open the eyes for an instant, without, how-

ever, changing the direction of the look, shut them immediately and cover them again. If the experiment is very successful, we then see a positive image of exterior objects, of a surprising distinctness. We can scarcely believe that we have really closed our eyes; the hand seems transparent. If we continue to keep the eyes closed, we see the less illuminated parts of the image disappear, while the more illuminated parts change color, becoming bluish, violet, orange, etc.; the image disappears and returns again, and so forth.

A clear after-image of a chess-board, or other analogous figure, shows phenomena exactly like those which I shall describe later under the heading "Phenomenon of *Troxler*." It now becomes probable that the disappearance and reappearance of the after-images are due to the same causes, likewise unknown, as this phenomenon. The after-images, of which I have just spoken, last for a relatively long time, but there are others which last so short a time that they escape observation in the ordinary distances of life. The simplest way of making them appear consists in moving the object which is intended to produce them. The secondary image then seems to follow the object because it is former at the place where the object was a moment before, and because it lasts only an instant. Ordinary after-images form, in these circumstances, a long luminous series. The most striking of these phenomena was described by *Purkinje* and later, under the name of "recurrent vision," by *Davis*. The experiment is very easy to perform: we light a match in darkness, blow out the flame and move the burning wood around. We shall then see the blue after-image, feebly luminous but bright nevertheless, follow the match at some distance, reproducing its form exactly. There are two conditions necessary to the success of the experiment: one is that we do not follow the match with the look, for the phenomenon is visible only in indirect vision; the other is that we use the proper speed, neither too fast nor too slow. With a certain rate of speed the image (called "ghost" by English writers) seems double. According to *Bidwell* the interval between the match and the after-image corresponds to almost one-fifth of a second. This author sees

the space between the match and the remainder of the field blacker, an observation which was confirmed by *Agabobon,* who repeated the experiment at the Sorbonne, but I have not been able to verify it.

By making a black disc with a white sector rotate in full sunlight *Charpentier* observed a black sector which formed in the white sector not far from its anterior border, and which was sometimes followed by several others less pronounced. At times the interval between the anterior border of the white sector and that of the black sector corresponded to about $\frac{1}{60}$ of a second. The observation indicates that when we allow an illumination to act for a very short period on the retina the latter becomes insensible to it after a sixtieth of a second to reacquire its sensibility after the lapse of the same period; sometimes the phenomenon is repeated several times (*retinal oscillations*). The phenomenon must not be confounded with "recurrent vision" for which the interval is much longer.

106. Phenomena Dependent on the Variation of the Brightness of the Colors.— The brightnesses of two sources of light of the same color are compared as easily as if there was a question of white light, and we find almost the same value for the fraction of *Fechner.* If we attempt to compare lights of different color the eye manifests, on the contrary, a very great uncertainty, and besides we encounter a difficulty caused by what is called the phenomenon of *Purkinje.* Suppose that we have two sources of white light, which we have found of equal brightness. If then we diminish the intensity of both one-half we shall find them again equal. But if we equalize two sources, one of which is blue and the other red, and that then we diminish their brightness one-half, the blue light will appear much brighter than the red light.—Let us select two papers, one red and one blue, which by daylight illumination appear to have the same brightness; by diminishing the illumination the blue paper will appear brighter than the red paper. With a very feeble illumination the red paper will appear black, the blue paper a pale gray. In order that the experiment may succeed well the papers

must be seen under an angle which is not too small, for the phenomenon is but slightly pronounced for the macula. In accordance with these observations *Macé de Lépinay* and *Nicati* have shown that the visual acuity falls much more quickly on diminishing the illumination when we use red light than when we use blue light: we select a red glass and a blue glass so that we may have, by daylight illumination, the same acuity on looking at the chart through either. If then we close the shutters almost completely so as to greatly diminish the illumination, we observe that the blue glass enables us to still read half of the chart, while with the red glass we cannot, at the first moment, distinguish even the chart; after a little while we can read the large letters, but the acuity for the red always remains lower than the acuity for the blue which is stationary. *Kœnig* and *Brodhun* also have shown that the increase of the fraction of *Fechner,* at the lower limit, begins sooner for the red than for the blue.

The following experiment shows in a very striking manner the difference which exists in this regard between the two extremities of the spectrum. We project the spectrum on a screen A, pierced by two apertures, allowing the red rays and the blue and violet rays to pass. Behind the screen A we place a lens which reunites these rays on a second screen B, forming on it an image of the surface of the prism which is turned towards A. This image then shows a pretty, purple color. In front of the screen B we place a stick which forms thereon two shadows, one red, the other blue, and it is easy to so regulate the apertures of the screen A that both shadows may have the same brightness. If we now diminish the width of the slit through which light reaches the prism the purple is diluted more and more with white. The blue shadow becomes grayish, and brighter and brighter compared with the background, while the red shadow retains its color, but becomes darker and darker. Finally it is nearly black and alone visible, the other shadow being gray and having nearly the same brightness as the background.

In the spectrum it is the yellow and green rays which have most brightness. The brightness diminishes towards the two extremities of the spectrum, but more towards the blue extremity than towards the red extremity. We must note, however, that if the blue and violet colors seem relatively feeble in the prismatic spectrum, this is partly due to the fact that these colors are spread over a much greater space than the others. In the spectrum of diffraction the intensity is greatest in the middle of the spectrum, and diminishes almost alike towards the two extremities.

If we lessen the intensity of the luminous source the colors of the spectrum change hue. We first see the yellow and blue colors disappear; there remain only the red, green and violet, which take the place of the colors which have disappeared. On still further diminishing the brightness, the blue changes into a blue-gray, the green into a green-gray, the red becomes brownish and finally all the colors disappear, and we see only gray. The red alone forms an exception; it does not seem to change into gray before disappearing.

There exists a very pretty method of showing the change of appearance of the spectrum by the diminution of the brightness. It consists in gluing a board of velvety black paper on a white screen so that by projecting on it a horizontal spectrum the upper half is formed on the black paper and the lower half on the white screen. This latter half shows the spectrum as it ordinarily appears, while the upper half has the form of a gray band, with the exception of the part corresponding to the red which appears brown.

The colors disappear, therefore, when the brightness of the rays becomes very feeble. Also when the brightness becomes very strong the impression approaches white. The sun, seen through a red glass, appears a whitish-yellow, although the glass allows only red rays to pass. Concentrating the light of the sun on a sheet of white paper with a lens, after having made it pass through a blue glass, the image of the sun appears white. When we look at the sun through a prism the spectrum presents itself as a colorless strip of a dazzling brightness. Here also it is the

red which best maintains its color; in most cases it appears a whitish-yellow.

According to *Parinaud,* these phenomena depend on the adaptation of the eye. The spectrum of feeble brightness, which appears gray to the adapted eye, is invisible to the eye not

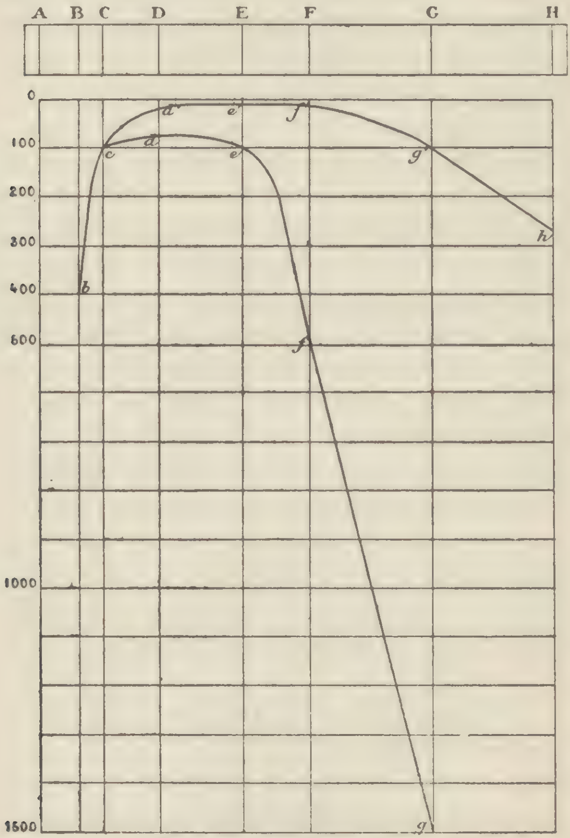


Fig. 156.—After *Parinaud.*

adapted, and when, the intensity increasing, it becomes visible to the non-adapted eye it in turn appears colored. *Parinaud* determined the threshold for different rays of the spectrum, and found the curves represented by figure 156. The upper curve is that of the adapted eye, the lower curve that of the eye not

adapted. The different parts of the spectrum are indicated by the vertical lines, prolongations of the lines of *Fraunhofer* in the diagram of the spectrum which is above the figure. The numbers on the left indicate the quantities of light necessary in order that these different parts of the spectrum may be perceived. Thus the adapted eye requires a quantity of light equal to 1 (this quantity being taken as the unit) in order to perceive the green rays near E, while the non-adapted eye requires a quantity equal to 100 in order to perceive the same rays, and a quantity equal to 1500 to perceive the blue rays near G. We see that the eye, by adaptation, gains nothing for the perception of red rays, whilst it gains enormously for the more refrangible rays. But it gains only in luminous sensibility: except the part *bc*, which is common to the two curves, the whole upper curve corresponds to colorless sensations only. According to *Parinaud*, the fovea gains nothing by adaptation; the rays also appear colored as soon as, with increasing brightness, they become visible to the fovea.

The results of *Parinaud* have been disputed by *Charpentier*, and they no longer harmonize well with the experiments mentioned on page 294. According to *Charpentier*, it is wrong to attribute the colorless sensation which the rays of very feeble brightness call forth to the adaptation of the eye, and, on the other hand, it is certain that if, from full daylight, we enter a relatively dark space, we cannot distinguish right away colors which we observe very well later.

Nevertheless, adaptation plays a considerable part in relation to these phenomena as the following observation of Charpentier shows. He covered the plate B of his photoptometer (see page 275) with a black paper, pierced with seven small openings grouped in a space of nine millimeters square. The plate A was illuminated by spectral light of different colors. On opening gradually the diaphragm of the instrument, he proved that the first impression which is obtained is that of a diffuse luminous spot, without color; let us designate the aperture of the diaphragm for the moment by *a*. To distinguish the color it was necessary to give the diaphragm a larger aperture *b*, and

it is only by making the aperture still greater *c* that we come to distinguish the points. For the eye, adapted to darkness, the apertures *b* and *c* remain almost the same as for the non-adapted eye, while the aperture *a* diminishes enormously especially for the more refrangible colors.

It is not strange that there exist differences of opinion on these questions, for there is very little certainty in the determination of the lower limits of the sensations. It must also be noted that the expressions "adapted" and "non-adapted" applied to the eye are vague. If every one is in accord in considering an eye adapted when it remains for half an hour in darkness, or non-adapted when it remains as long in full daylight, the authors do not agree so well in designating the state of the eye when exposed to an intermediary illumination, such as that of the interior of our houses.

107. Methods of Mixing Colors.— The fundamental examination of the color sense is made by means of what is called *equations of colors:* we mix two or three colors in different proportions until the observer declares the mixture similar to a fourth given color, most frequently white. We then examine whether an eye, of which the color sense is normal, recognizes the equation, that is to say, whether the mixture appears likewise similar to white for this eye.—We can mix the two colors in different ways.

1° *Mixtures of Spectral Colors.* We form two spectra by means of two prisms, and by allowing these spectra to slide over one another we can mix any two hues from them. *Helmholtz* accomplished the same end with a single prism, by using a slit in the form of V; each of the branches formed an oblique spectrum, and the two spectra would overlap to a great extent so that we could obtain all possible mixtures.

The apparatus of *Maxwell* was very ingenious. It consisted of a box, a section of which is shown in figure 157. At E is a narrow slit through which passes light, which is reflected by the mirror *e* towards the prisms P and P_1, through which it passes to meet the concave mirror S. This mirror reflects the

light which passes again through the prisms to go to form a spectrum on the far end of the box, AB. At this place are three movable slits *x*, *y* and *z*, which permit spectral light of any hue to leave the box through each of the slits by displacing them.—Suppose *x* corresponds with the red, *y* with the green and *z* with the violet. It must be noted, in consequence of the reversibility of optic processes, that if we illuminate the slit *x* from the outside by red light, this light will reach an eye placed at E; but if we illuminate the same slit with green light, this

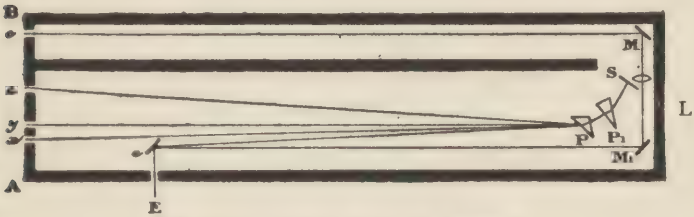


Fig. 157.—"Color box" of *Maxwell*.

light will not reach an eye at E, but will be projected to one side of E. In order that the green light reach E, it must pass through the slit *y*. Consequently the three slits *x*, *y* and *z* by a white luminous source, an eye placed at E sees the surface of the prism P colored by the mixture of the three colors, which a flame placed at E would project on the slits *x*, *y* and *z*.—At the far end of the box is yet another aperture *c* through which enters white light, which, after having been reflected by the mirror M and concentrated by the lens L, meets a plate of ground glass blackened on the back M_1. The eye placed at E sees this plate at the side of the prism, and can thus compare the brightness and color of the mixture with that of the white light, admitted through *c*. By properly placing and opening the slits, we can thus obtain a mixture which is not distinguishable from the white light reflected by M_1, either as to color or brightness.

The latest researches on the mixtures of colors (*Kœnig* and his pupils) have been made with a large spectral instrument,

which was constructed for the laboratory of Berlin, and a description of which is found in the second edition of Helmholtz's work on *Physiologic Optics* (page 355).

2° *Maxwell* also studied the mixtures of colors by placing, on the disc of *Masson,* sectors of different colors (see page 313).

3° We can mix colors by means of a plate of glass *ab* (fig. 158), which is held so that it may reflect rays of one color at the same time that it allows rays of another color to pass (*Lambert*).

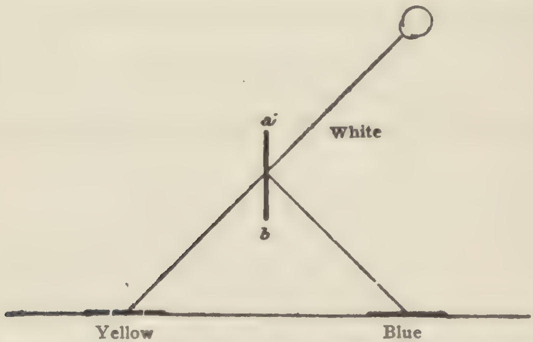


Fig. 158.—Mixture of colors by means of a glass plate.

4° Looking at two colors placed side by side through a double refracting prism, we see them separated by a strip the coloration of which is that of the mixture.

5° Placing two glasses of different colors before the two openings in the experiment of *Scheiner* and looking at the sky, we see the common part of the circles of diffusion in the color of the mixture.

6° Painters frequently use mixtures of coloring matter, but the results which are thus obtained are frequently not in accord with those which are obtained by the other methods. The best known example is the mixture of yellow and blue. Painters thus obtain green, while with a revolving disc we obtain a gray-

white (*Lambert*). *Helmholtz* gave the following explanation of this difference: mixing the colors of yellow and blue pigment the superficial molecules send back yellow light and blue light. Together these rays produce the impression of white, as on the revolving disc. The blue molecules situated deeper also send back blue light, but it must be noted that this blue light, as also that of the superficial molecules, is not pure: by the spectroscope we find that it contains green, blue and violet rays. The yellow molecules send back red, yellow and green rays. Generally the molecules allow to pass rays of the same color as those which they send back. Among the rays reflected by the deep yellow molecules, only green rays, therefore, can pass through the superficial blue molecules, and, among those reflected by the deep blue molecules, likewise only the green rays can pass through the superficial yellow molecules. The result, therefore, becomes a green color, mixed with the white reflected by the surface.

108. Results of the Mixtures of Colors.— *Newton* devised his table to give a graphic illustration of the results which are obtained by mixing colors. The principle of this table is that all the colors we can produce by mixing two given colors are placed on the straight line which joins these two colors, and so much nearer to that one of the two colors which enters most into the mixture. The quantity of the color of the mixture is expressed by the sum of the quantities of the component colors. Suppose, for example, that we want the result of the mixture of three parts of green with one part of red and two parts of blue. We begin by joining the green and red by a straight line which is divided into two by the point p (fig. 159), so that the distance of p from the green may be a third of its distance from the red; p is then the place of the mixture of the green and red, the mixture being represented by the number 4, the sum of the two component colors. We then join the point p with the blue by a second straight line which is divided into two by the point q, so that the distance pq is to the distance of

q from the blue, in the proportion of 2 to 4; q is the place of the mixture of the three colors, and the quantity of this mixture is expressed by the number 6. Drawing the line oq and prolongating it until it cuts the spectral curve, we see that the color of the mixture is a bluish-green strongly diluted with white.

There enters into this illustration of *Newton* an expression which is not defined, that of the *quantity* of the colors. While it is easy to tell what must be expected from equal quantities of the same color, it is not easy to define the expression of equal quantities of two different colors, the result of which is that the form of the curve becomes, up to a certain point, arbitrary. With *Newton,* we must consider as equal the quantities of two complementary colors, which, when mixed, give white, since the white, on his table, is situated at an equal distance from both. If we take two other complementary colors, we must also consider as equal the quantities of these colors which, mixed, give white, but on condition that this white be of the same brightness as the former. As we shall see, *Maxwell* and *Helmholtz* used other definitions.

The table of *Newton* shows that, with the exception of purple, we cannot produce new colors by mixing spectral colors, for we can always, after having found the position of the mixture, draw a straight line passing through the center and this point. Prolonged, this straight line will meet a spectral color, and the mixture is equal to this color diluted with white.

The table of *Newton* indicates also another peculiarity of the normal color sense, namely the fact that *we can reproduce all existing hues by mixing, two by two, three colors properly chosen.* Let us select, for example, *red, green* and *blue,* and draw on the table (fig. 159) straight lines which join these colors. If, then, we select any spectral color, we can always join it to the center of the table by a straight line; this straight line must necessarily cut one of the sides of the red-green-blue triangle and at the place of intersection is found the mixture which is similar, in hue, to the spectral color. On account of this peculiarity the normal eye is called *trichromatic*. Observe particu-

larly that I have said that the two colors are alike as to hue. Generally they are not alike as to purity, the color of the mixture being diluted with white. The table of *Newton* also requires that the spectral color must always have greater purity, for, if we could, by mixing two spectral colors, reproduce a third color exactly, these three colors should be placed on a straight line, and the spectral curve could not be circular. But this last condition of the table is not fulfilled.

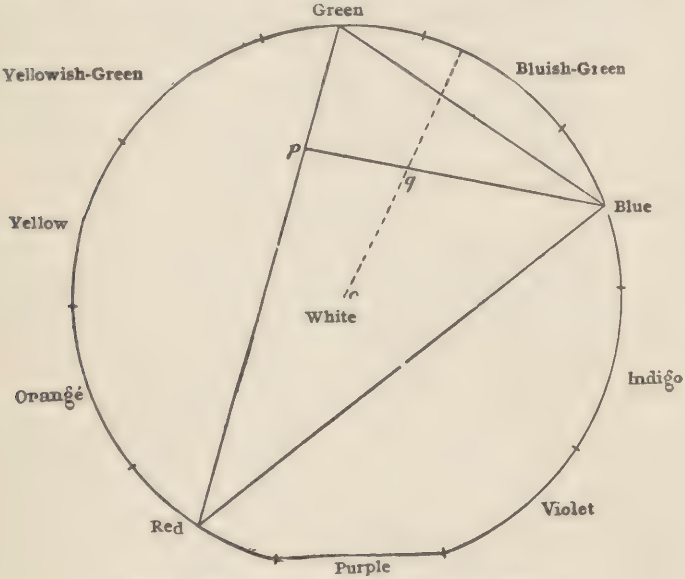


Fig. 159.—Table of colors after *Newton*.

The accuracy of the illustration of *Newton* has, indeed, been verified by the admirable works of *Maxwell*. This author found that *Newton's* table gives a very exact illustration of the results of the mixtures of colors, but that the spectral colors cannot be arranged in a circle, because there are quite extended parts on the spectrum, the colors of which can be reproduced exactly, or nearly exactly, by the mixture of two given colors, and which, consequently, must be placed on straight lines.

Figure 160 shows the spectral curve of *Maxwell*. While the curve of *Newton* must be considered merely as a conception of

the mind, *Maxwell* determined his experimentally with the instrument described in the preceding chapter (fig. 161). To use it he placed it in such a position that the slits x, y z and c were turned towards a sheet of white paper illuminated by the sun. As a starting point he selected the three following colors (*standard colors*):

	Red (R)	Green (G)	Blue (Bl)
Wave length:	0.630 μ	0.528 μ	0.457 μ

Fig. 160.—Color table of *Maxwell.*

He placed the slits x, y and z so as to give access to these colors, and, by regulating the width of the slits, he produced a mixture which differed neither in tint nor brightness from white introduced through the slit c.

By measuring the slits he found for x a width of 2.36 mm., for y 3.99 mm. and for z 3.87 mm., and by designating the white,

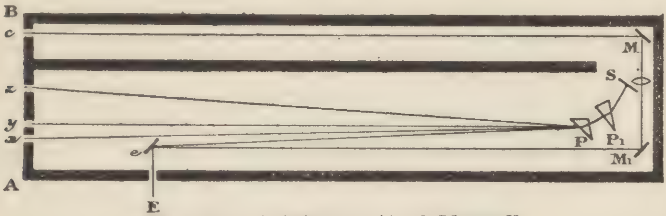


Fig. 161.—"Color box" of *Maxwell.*

which remained constant through all the experiments, by W, he had thus the equation

$$2.36\ \mathrm{R} + 3.99\ \mathrm{G} + 3.87\ \mathrm{Bl} = \mathrm{W}$$

He then displaced the slit x so as to give access to orange light; by regulating the slits he again produced a mixture similar to white which gave him the equation.

$$2.04\ \mathrm{Or} + 3.25\ \mathrm{G} + 3.88\ \mathrm{Bl} = \mathrm{W}$$

As white was the same in both cases, we can combine the two equations, which gives

$$2.04\ \mathrm{Or} + 3.25\ \mathrm{G} + 3.88\ \mathrm{Bl} = 2.36\ \mathrm{R} + 3.99\ \mathrm{G} + 3.87\ \mathrm{Bl}$$

or

$$2.04\ \mathrm{Or} = 2.36\ \mathrm{R} + 0.74\ \mathrm{G} - 0.01\ \mathrm{Bl}$$

or

$$1\ \mathrm{Or} = 1.155\ \mathrm{R} + 0.362\ \mathrm{G} - 0.006\ \mathrm{Bl}$$

He then repeated the measurement for the other colors, by always combining two of the *standard colors* with the color in question to produce white. He thus succeeded in expressing all the colors of the spectrum by three colors. The following table shows the results of these measurements (see next page).

By dividing each equation by the coefficient on the left, we obtain the expression corresponding to the width of the slit of 1 millimeter.

Under this form the result is found expressed on figure 162. The three curves, designated by R, G, B, correspond to the three *standard colors;* the numbers underneath are the wave lengths

COLOR	QUANTITY	WAVE LENGTH		RED	GREEN	BLUE	SUM	UNITY
Red.......	5.63	(663)	=	2.36	+ 0.05	+ 0.36	2.77	2.032
	2.36	(630)	=	2.36	+ 0.00	+ 0.00	2.36	1
Orange....	2.04	(606)	=	2.36	+ 0.74	— 0.01	3.09	0.662
Yellow....	2.79	(583)	=	2.36	+ 2.45	— 0.01	4.80	0.582
	3.20	(562)	=	1.55	+ 3.99	— 0.10	5.43	0.589
	3.30	(544)	=	0.42	+ 3.99	— 0.03	4.38	0.754
Green.....	3.99	(528)	=	0.00	+ 3.99	+ 0.00	3.99	1
	5.26	(513)	=	— 0.33	+ 3.99	+ 0.44	4.10	1.282
	7.87	(500)	=	— 0.43	+ 3.99	+ 2.22	5.77	1.363
Blue......	7.83	(488)	=	— 0.39	+ 2.67	+ 3.87	6.15	1.275
	5.14	(477)	=	— 0.24	+ 0.98	+ 3.87	4.61	1.116
	4.28	(467)	=	— 0.14	+ 0.14	+ 3.87	3.87	1.105
Indigo....	3.87	(457)	=	0.00	+ 0.00	+ 3.87	3.87	1
	4.10	(449)	=	0.08	+ 0.03	+ 3.87	3.98	1.032
	5.59	(441)	=	0.14	+ 0.09	+ 3.87	4.10	1.362
Violet.....	8.09	(434)	=	0.04	— 0.23	+ 3.87	3.68	2.197

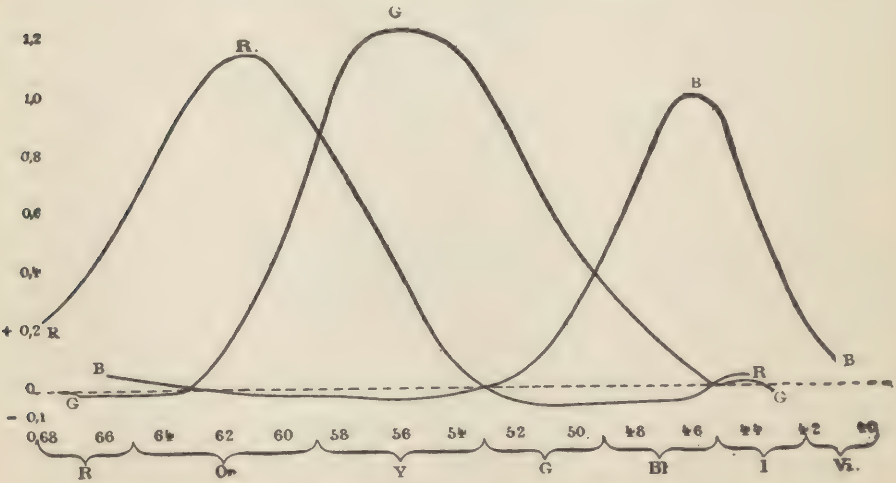


Fig. 162.—Color-curves of *Maxwell.*

of the different colors of the spectrum, and the position of the three points in which the curves cut the vertical line corresponding to each of the colors, indicates the quantities of the three *standard colors* needed to produce the mixture.

The negative sign of the blue, in the equation of the orange, is found again for the greater number of the colors added to one or other of the *standard colors.* Its significance is easy to grasp. In fact, if we write the equation of the orange thus:

$$2.04 \text{ Or} + 0.01 \text{ Bl} = 2.36 \text{ R} + 0.74 \text{ G}$$

it indicates that we cannot, with the three *standard colors,* produce a mixture exactly like orange, but must, on the contrary, add a little blue to the orange so that it may be like the mixture of red and green.

It should be noted that, up to the present, I have simply expressed the quantity of a color by the width in millimeters of the slit giving access to this color. To construct the table of colors we do the same for the three *standard colors;* but for other colors we will be obliged to select the units in another manner. I have said, indeed, that with *Newton* the quantity of a mixture is considered as equal to the sum of the quantities of the component colors. The sum of the three component colors of the orange was

$$2.36 + 0.74 - 0.01 = 3.09$$

while the width of the slit was 2.04 mm. According to *Newton,* the quantity of orange passing through the slit of 2.04 mm. is, therefore, 3.09, that is to say, the unit of the orange corresponds to a width of the slit of $\frac{2.04}{3.09} = 0.662$ mm.

If we wish to use the table to solve questions of mixtures of colors we must, therefore, multiply the quantities found by the table by the figures indicating the units, in order to obtain a result expressed by the width of the slit in millimeters. The units are in the last column of the table. They are obtained by dividing the coefficients on the left by the figures in the column before the last, which indicate the sum of the component colors.

To construct the spectral curve, we begin by drawing the dotted equilateral triangle of figure 163. We suppose the three *standard colors* placed at the three angles, an arrangement which was proposed by *Young*. To find the position of the orange, we begin by dividing the red-green side into two parts, in the proportion of 0.74 : 2.36. Let P be the point of division: join

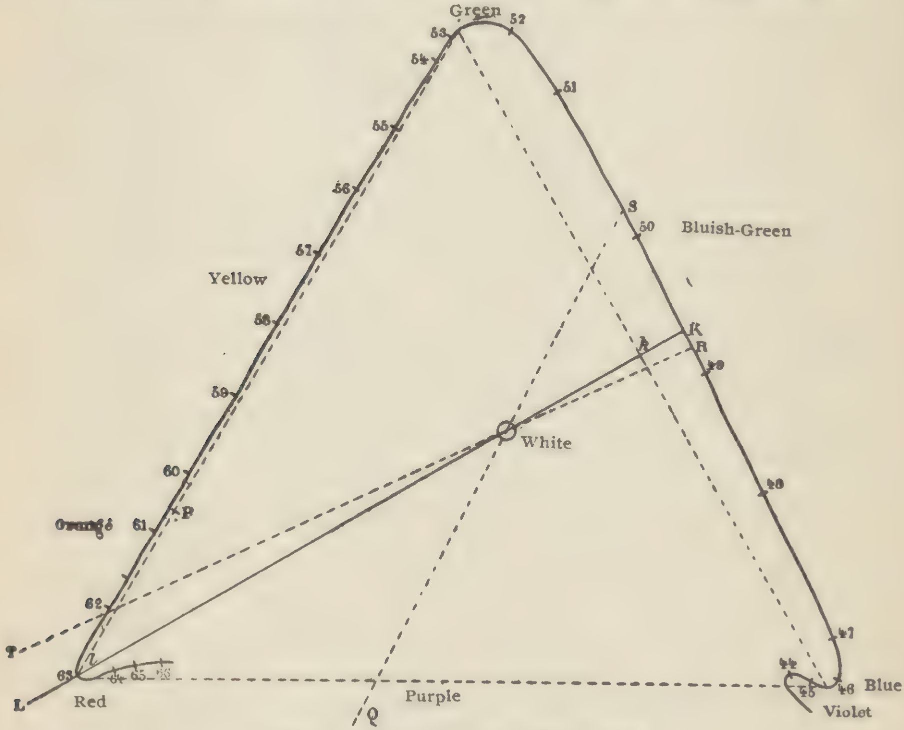


Fig. 163.—Color table of *Maxwell.*

this point to the blue angle by a straight line, of which we measure the length a. The color at P can be considered either as a mixture of 2.36 R with 0.74 G, or as a mixture of 3.09, or with 0.01 Bl. It follows that the orange must be placed on the prolongation of a, beyond the point P, and by designating its distance from P by x we should have $x = \frac{0.01}{3.09}a$. This distance is, for the orange, so small that it is scarcely visible on the figure,

the curve coinciding at this position almost with the dotted line. —We observe that, on account of the presence of the negative coefficient, the color in question must be placed outside of the triangle. A color which is situated in the interior of the triangle may be reproduced exactly by a mixture of the three *standard colors;* this is not possible for a color situated outside of the triangle: it is necessary, on the contrary, to mix it with one of the *standard colors,* in order that it may seem equal to the mixture of the two others.

On the table of *Maxwell* the greater part of the spectrum (from 0.63 μ in the orange-red to 0.53 μ in the green, and from 0.51 μ in the green to 0.47 μ in the blue) is arranged on the two sides of a triangle of which the green, between 0.53 μ and 0.51 μ, forms a rounded angle, while the extremities of the spectrum form two other somewhat irregular angles. We must imagine the third side of the triangle occupied by the purple colors, which are obtained by mixing red with blue. As nearly all the spectral colors have one of the coefficients negative, almost the entire curve is situated outside of the triangle of the *standard colors,* which indicates that the mixture colors have nearly all a little less purity than the spectral colors. The part situated between the red and the green coincides, however, very nearly with the corresponding side. By selecting another *standard color,* green, we could make the part of the curve situated between 0.51 μ and 0.47 μ coincide with the other side of the triangle, but it is easy to see that we cannot select the green color so as to make the two sides coincide with the curve at once. *We cannot, therefore, select three spectral colors* such that we can *reproduce all the other spectral colors exactly by their mixtures;* we can reproduce all the hues, but some of the mixture colors always continue to have less purity than the corresponding spectral colors, whatever may be the *standard colors* we have chosen.

By means of the table of Maxwell we can construct the result of mixtures of any colors. If we mix two colors placed on the same side of the approximately triangular curve, we obtain a mixture color which has as much purity as the spectral

color, while if we mix two colors situated each on a different side, we obtain a mixture strongly diluted with white. The three colors which *Maxwell* selected as standard colors, the red, green and blue, have, therefore, this peculiarity that they *cannot be reproduced by mixing other spectral colors,* the mixture being always strongly diluted with white.—The approximately triangular form of the curve, with the three colors, red, green and blue, placed at the angles, does not depend on the choice of the *standard colors.* By means of the equations of *Maxwell,* we can, by a simple calculation, express all the spectral colors by three colors other than his *standard colors,* for example by orange, blue-green and blue. The curve even then retains its approximately triangular form, having the red, green and blue at the angles, but it differs considerably from the equilateral triangle formed by the straight lines joining the three new *standard colors,* which indicates that the mixture colors have, in this case, very little purity. *Maxwell* selected red, green and blue, so that the curve would come as near the triangle in form as possible.

Contrary to what has taken place in the case of these three colors, those which are placed on each of the two sides of the triangular curve, may be reproduced exactly by mixing other spectral colors. They are, in this regard, analogous to the purple colors which are obtained by mixing the red and spectral blue, and which appear to the eye as pure as the pure spectral colors.

The most interesting phenomenon among the great number of facts which are expressed by the table of Maxwell, is certainly this, that we can produce a perfect sensation of yellow by mixing red and green. The fact was already known to *Young,* and formed the principal basis of his theory of colors, which I shall mention later on. *Lord Raleigh* had constructed a special instrument for determining the quantities of spectral red and spectral green necessary to produce a complete equality with spectral yellow. In his numerous examinations he could always obtain a perfect equality, but in the matter of the quantities required of the component colors, he found quite unexpected individual differences (see page 315). We can also mix the

light of the lithium and thallium flames so as to obtain a light which cannot be distinguished from that of the sodium flame. Another method, also pointed out by *Lord Raleigh,* consists in looking through a liquid which allows only red and green rays to pass (a mixture of bichromate of potash and blue aniline dissolved in water). By observing through this liquid an object of a bright white, a cloud illuminated by the sun for example, it appears of a pure yellow, although all the yellow rays are completely absorbed.—The liquid is, besides, very sensitive to tints of white light; the light of the blue sky, which contains too little red, appears greenish, while the light of an arc lamp appears reddish.

The yellow occupies a special position among the colors. An observer completely ignorant of the results of the mixtures, as well as those of the physicists who obtain yellow by mixing spectral red and green, as those of the painters who, with their pigments, obtain green by mixing yellow with blue, would probably be temped to class the yellow among the three *standard colors* of Maxwell, so as to reckon four principal colors in the spectrum: red, yellow, green and blue. As we have seen, the yellow is distinguished from the three others in that it can be reproduced by a mixture of other colors. In this respect it is analogous to the colors which are placed on the other sides of the triangle, the purple and the blue-green, and it is distinguished from the latter in this that the eye may not perceive any trace of red or green in the yellow, while no one would hesitate to declare that he saw blue and red in the purple, or green and blue in the blue-green. The yellow, in this regard, resembles white in which the eye no longer distinguishes any trace of the component colors. The yellow is also that one of the spectral colors, which, to the eye, seems to offer most resemblance to white.—Another peculiarity of the yellow, on which Herschel laid stress, is the considerable change which this color undergoes when its brightness diminishes. A dark blue still seems blue, while a dark yellow appears brown, a color which the observer not prejudiced would consider rather as a special color.

We can obtain the impression of *white* in many different ways. The celebrated experiment by which *Newton* combined by means of a lens all the colored rays of the spectrum in a white image shows, in the first place, that all the colors of the spectrum, when mixed, give white. The equations of *Maxwell* furnish a long series of examples of the possibility of forming white by mixing three colors. Lastly the table indicates a great number of pairs of complementary colors, that is to say, colors which, mixed two by two in the proper proportions, give white. To find the color complementary to a given color, we have only to prolong the line which joins it to the white, until it meets the curve again. The point of intersection is the place of the complementary color, and the quantities to take of both colors are inversely proportional to their distances from the white. We must recollect, however, that if we wish to express the quantity by the width of the slit in millimeters, we must reduce the numbers, as already pointed out.

A glance at the table shows that the green colors (greenish) from 57 to 49.5 have no complementary colors in the spectrum. Their complementaries are the purple colors. The complementaries of the red extremity, up to 61, are situated very near one another (from 49.5 to 49.2), those of the blue extremity are condensed near 57. The hue varies, therefore, very slowly towards the extremities of the spectrum, while the variation reaches its greatest rapidity in the blue-green, where the divisions are separated by very marked intervals.

Maxwell did not determine the extreme parts of the spectrum; one might think, therefore, that the curve ought to be really more extended; but, according to the researches of *Kœnig* and *Dieterici*, this is not the case. These authors made a long series of very minute researches, like those of *Maxwell*, with their large spectral instrument. Their results seemed to agree well with those of the latter author; however, they could not verify the bend which the curve of *Maxwell* makes in the red. According to these authors, the hue does not vary in the spectrum beyond 67 and 43, so that the divisions beyond these limits must on the table coincide with these limits. *Maxwell*, indeed,

himself calls the form of the extremities of the curve somewhat doubtful.

If we compare the complementary quantities of red and blue-green, we notice that the red appears darker than the green. To illustrate facts of this kind on the table, *Helmholtz* supposed as equal quantities of two different colors quantities appearing to have the same brilliancy. He thus obtained the spectral curve illustrated in figure 164. The small circle indicates the position of the white. Since the red complementary to the blue-green

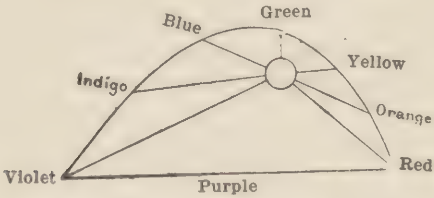


Fig. 164.—Color table of *Helmholtz.*

appears darker than the latter, we consider its quantity as smaller and place it consequently farther from the white. Indeed, such a comparison of the brightness of two different colors is not easy, as *Helmholtz* himself remarked, and the result depends besides on the phenomenon of *Purkinje.* If, for example, a certain quantity A of yellow light appears to have the same brightness as the quantity B of blue light, we find that the quantity $\frac{A}{2}$ of yellow light will appear darker than the quantity $\frac{B}{2}$ of blue light. The form of the curve would vary, therefore, according to the brightness used.

Maxwell showed how, without the help of a spectral instrument, we can make determinations analogous to his own by means of the revolving disc of *Masson.* It is necessary to have paper discs (colored, whites and blacks) of two different sizes, so as to be able to make two mixtures at once, by covering the central part of the large disc with the small ones.

We cut the discs along a radius, in order to be able to combine them so as to obtain colored sectors of any angle. We select three *standard colors,* the red, green and blue, and we

combine three large discs so as to have a sector of each color. In the middle we place two small discs combined so as to have a black and a white sector. Making the whole rotate, we obtain in the middle a gray circle, surrounded with a ring tinted with the mixture of three *standard colors*. By regulating the angles of the sectors we make the two tints alike, and write the equation as thus:

$$165\ R + 122\ G + 73\ Bl = 100\ W + 260\ B \quad (\textit{Aubert})$$

W denotes the white, B the black, and the numbers indicate the angles of the sectors. Neglecting the little light reflected by the black, we may write:

$$165\ R + 122\ G + 73\ Bl = 100\ W$$

To express any other color, the yellow for example, by the *standard colors* we replace the red sector by a sector of this color. Regulating the size of the sectors, we find for example:

$$146\ Y + 17\ G + 197\ Bl = 159\ W + 201\ B$$

or, by dividing by 1.59,

$$92\ Y + 11\ G + 124\ Bl = 100\ W$$

We then combine this equation with that of the *standard colors,* which gives

$$92\ Y + 11\ G + 124\ Bl = 165\ R + 122\ G + 73\ Bl$$

or

$$1\ Y = 1.97\ R + 1.21\ G - 0.55\ Bl$$

With these equations we can construct graphic illustrations of the same kind as figures 160 and 162, and, by always working with the same kind of papers, we may thus study and compare the color sense of different eyes; but the spectral method always remains superior.

109. Abnormal Trichromasia.— If we examine a certain number of persons by the method of *Maxwell,* on constructing the color table of each person, we often find small differences: a mixture which one observer declares like white, seems to another colored. It is probable that these differences are due, at least in part, to the fact that a portion of the rays is absorbed by the media of the eye, and that this absorption is more pronounced in some persons than in others. Thus the yellowish color of the crystalline lens of old persons indicates that it must absorb a part of the blue rays. A mixture of yellow and blue, which, to a normal person, appears equal to the white, must appear yellowish to the old person, whose crystalline lens absorbs relatively more of the light of the mixture than of the white light. After extraction of a cataract, the patient often, at the first moment, affects to see all blue, almost as everything appears tinted with the complementary color when we have looked for a little while through a colored glass and then remove it suddenly. *Maxwell* attributed some of the phenomena in question to the absorption of the green-blue rays by the yellow pigment of the macula. Looking at a bright line through a prism, he observed a dark spot corresponding to the fovea, which moved up and down with the look, as long as the latter remained in the blue part of the spectrum, but which disappeared as soon as the look left the blue. He recommended also, in order to observe the phenomenon, fixing a yellow paper for a little while, and then transferring the look to a blue paper. The spot then appears for some moments. Taking two equal whites, one made of ordinary white light and the other of a mixture composed in great part of green-blue rays, the latter, seen in indirect vision, seemed greenish and more luminous than the former.

We have seen (page 238) that the existence of the yellow pigment of the macula may appear doubtful, but the fact that the macula is less sensitive to blue than the remainder of the retina is unquestionable. I do not see the scotoma in the blue part of the spectrum, but another observation which I have made is equally convincing. There exist in commerce transparent sheets of colored gelatine which may often with advantage

replace the colored glasses in many experiments. I have such a sheet, tinted probably with an aniline color, which allows the red and blue rays to pass. When, looking at the sky, I put this sheet before my eye, I see at the point fixed a somewhat diffuse red spot, almost the size of the moon or a little larger. After an instant it disappears; if then I remove the sheet without changing the direction of the look, I see the after-image of the spot, very slightly greenish and clearer than the surrounding parts.—The color table of Maxwell himself differs somewhat from that of Mrs. Maxwell, illustrated in figure 160, differences which could very well be due to the fact that inferiority of the macula for the blue was more pronounced in him than in her.

Neglecting these slight differences, *an equation of color which is true for a normal eye, remains true for all eyes as well for normal eyes as for dichromatic eyes.*

This latter assertion was considered entirely general, until *Lord Rayleigh,* in 1880, discovered a class of eyes for which it is not true. After having produced a mixture of spectral red and spectral green which appeared to him identical with spectral yellow, he asked a certain number of people to compare the two hues. Most of them found the hues identical, but some, amongst whom were his three brothers-in-law, declared that they saw scarcely any resemblance; the pure color appeared yellow to them, while the compound color seemed to them nearly as red as sealing wax. To see the hues alike, these persons had to add so much green to the mixture that it appeared nearly pure green to a normal eye. The mixture of *Lord Rayleigh* was 3.13 R+1.00 G; that of his brother-in-law 1.5 R+1.0 G. (1)

The persons in question presented no other anomalies of the chromatic system; they were by no means dichromatics (*daltonists*). Later researches (*Donders, Kœnig* and *Dieterici*) confirmed the opinion of *Lord Rayleigh* that these people formed a

(1) The numbers are not comparable with those of *Maxwell, Lord Rayleigh* having probably used colors different from the *standard colors.* Otherwise *Maxwell* and *Mrs. Maxwell* would both have belonged to the category of abnormal trichromasia, which is not at all probable.

group by themselves: no intermediary forms have been found between their anomaly, which *Kœnig* called *abnormal trichromasia,* and the normal chromatic system. The anomaly seems almost as frequent as dichromatism; *Kœnig* and *Dieterici* found three cases of it among seventy persons examined, but no case is known in which the anomaly was discovered by the person himself who was affected.

110. Color-Blindness or Dichromasia (Daltonism).— The most prevalent form of dyschromatopsia is called *daltonism* after the celebrated English chemist, *Dalton,* who was affected with it, and who gave the first fairly exact description of it. It is calculated that about 4 per cent. of men are affected with this anomaly; it is much rarer in women, especially in its complete form.

For the daltonists, there is in the spectrum a place, in the green-blue, the color of which resembles white (gray). We call this place the *neutral point.* Instead of the great variation which the normal eye perceives in the spectrum, the daltonists see only two colors: one which they most frequently call yellow, and which fills the entire part situated between the neutral point and the red extremity, and the other which they call blue, and which extends from the neutral point to the violet extremity. In no part belonging to either of the colors does the hue change; there are differences of purity and brightness only. The color called yellow seems to them pure in the red, orange, yellow and green, until about 0.54μ or 0.53μ near the line E. In all this part there are differences of brightness only; we can make one of these colors like any other color by changing the brightness. The red and orange of the spectrum are often so feeble that they are not perceived unless the spectrum is very clear. Starting from the line E, the color becomes more and more grayish, and at the neutral point in the neighborhood of 0.50μ (see fig. 165) the color is like gray. The brightness diminishes at the same time; generally, the daltonists tell you that the parts situated near the neutral point are darker than those situated at some distance away from it. It is possible that this diminu-

tion of brightness is due to the fact that the neutral point is situated in the green-blue part of the spectrum, the rays of which are most affected by the influence of absorption in the yellow pigment of the macula, a phenomenon which often seems very pronounced in the dichromatics. Starting from the neutral point the other color called blue begins to make itself felt: gaining in purity, it becomes pure at about 0.46μ, and, starting from this point, presents differences of brightness only; the maximum is near the place where the color becomes pure.

The dichromatics see, therefore, in the spectrum only two colors, but it is difficult to tell which. If we designate the colors as yellow and blue, it is not a sure sign that the spectral colors give them the same impressions as those which we obtain by yellow and blue. Generally speaking, it is impossible to communicate to any one the nature of a sensation which we experience otherwise than by a comparison. If, for example, one man told another that an object had a sugary taste, he only means to convey that the object gives him a sensation similar to that which sugar would give him. The other can then verify this if he also finds that the taste of the object is similar to that of sugar, and if he finds it so he will say that the former has a normal taste; but it is impossible to tell whether the object has the same taste for both.—As we cannot know how the daltonists see colors, *Donders* proposed to replace in their case the expressions of yellow and blue colors by those of warm and cold colors, terms which are in use among painters.

We must observe, however, that while in all other known cases the daltonism was bilateral, there exists in literature a unique case of uniocular daltonism; it is clear that such a patient would be well qualified to give information on the question of knowing how the daltonists see the colors. The case was very well investigated by *Hippel.* The left eye was normal, while the right eye, which squinted, but which had been operated on and presented no ophthalmoscopic lesion, showed an anomaly wholly analogous to ordinary daltonism. The neutral point (situated at 0.512μ) divided the spectrum into a yellow part and a blue part. The red and green of the spectrum were, in

hue, similar to the yellow, but appeared a little less bright. Now, looking at the yellow sodium line, first with one eye and then with the other, the subject declared that the appearance was the same for both eyes, apart from a slight diminution of brightness for the dichromatic eye. It was the same for the blue indium ray as for the white. If, therefore, we can consider the case of *Hippel* as a case of true daltonism the difficulty

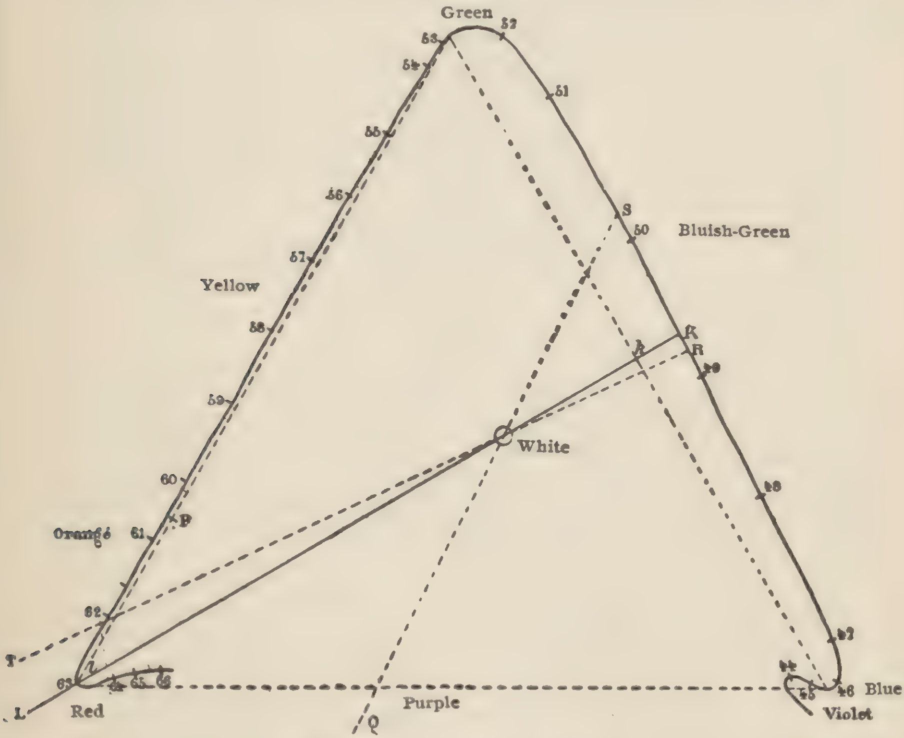


Fig. 165.—Color table of *Maxwell.*

seems solved. The sensations which the daltonists designate as yellow and blue would be identical with those of normal persons.

As color-blind persons recognize the equation of the normal eyes, the colors which are complementary for normal eyes are also complementary for them. It follows that the color comple-

mentary to the neutral point must also appear gray to them (or be invisible), as well as all the colors situated on the diameter of the table which joins them. As the colors next to the neutral point appear strongly mixed with white, their complementaries, as long as they are in the spectrum, must appear of very little brightness, since they must neutralize only the little chromatic value which is in these grayish colors.

While an equation of colors, which is true for a normal eye, is so also for the color-blind, the reverse is not true: color-blind persons recognize as similar, mixtures which are by no means so for a normal eye. For a daltonist, we can reproduce the impression of any color of the spectrum, as well as that of white, by mixtures of *two* colors. On account of this peculiarity, the anomaly in question is also termed *dichromasia*.

Maxwell used two of his *standard colors,* green and blue. He thus found, for a dichromatic student, the equation

$$4.28\ \mathrm{G} + 4.20\ \mathrm{Bl} = \mathrm{W}.$$

The position of this mixture color is marked on the table (fig. 165) by the letter *k;* the letter K indicates the corresponding spectral color, which is the neutral point. As the daltonists recognize the equations of the normal eyes, we can combine this equation with that of the normal eye (page 305)

$$2.36\ \mathrm{R} + 3.99\ \mathrm{G} + 3.87\ \mathrm{Bl} = \mathrm{W}.$$

We have, therefore, for the daltonist

$$2.36\ \mathrm{R} + 3.99\ \mathrm{G} + 3.87\ \mathrm{Bl} = 4.28\ \mathrm{G} + 4.20\ \mathrm{Bl}$$

an equation which we can also write

$$\mathrm{L} = 2.36\ \mathrm{R} - 0.29\ \mathrm{G} - 0.33\ \mathrm{Bl} = 0.$$

This latter color would not, therefore, produce any impression on the dichromatic eye and would represent, up to a certain point, the element which is wanting in it. Its place is marked by the letter L on the table (fig. 165). As L is situated outside

the spectral curve, it is a fictitious color which really does not exist, but which we must suppose still purer than the corresponding spectral color which is marked *l*, since it is situated farther from the white than the latter. Compared with L, *l* is to be considered as a mixture of white. Nor is it wholly invisible, but very feeble.

For his daltonist, *Maxwell* succeeded in reproducing all the colors of the spectrum by mixtures of his two *standard colors*. The results are represented by the curves in figure 166. More-

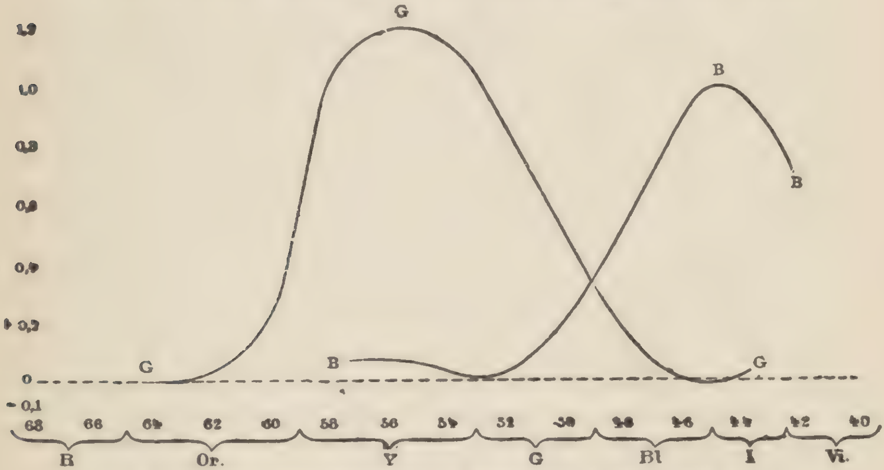


Fig. 166.—Color curves of a dichromatic, after Maxwell.

over, it would be simpler to select two colors which appear pure to the daltonists, as *van der Weyde* and latterly *Kœnig* and *Dieterici* have done. The green color of *Maxwell* seemed to the daltonists slightly mixed with gray, as the curves show.

Fig. 167.—Color table of a dichromatic, after the measurements of *Koenig* and *Dieterici*.

On the table of colors the whole chromatic system of the daltonists is reduced to a straight line (fig. 167), since all the colors which we can produce by mixing two given colors must be placed on the straight line which joins them. The line, too,

corresponds only to the part of the spectrum in which the colors are seen mixed with white, because all the parts where the colors seem pure, must come together in the two points which form the extremities of the line.

Examining a series of daltonists, we observe that the position of the neutral point is not exactly the same in all. It varies in different persons between 0.492μ and 0.502μ. In figure 165 these two points are marked R and S; it is, therefore, between R and S that the position of the neutral point may vary, and consequently, the direction of the neutral diameter would vary between RT and SQ. There results a certain difference between daltonists whose neutral point is situated nearer R, and those in whom it is situated nearer S. In the former, the neutral diameter passes through the green-blue and the red (1), and the spectrum seems shortened, because the red extremity contains the colors complementary to the grayish colors and must, consequently, as we have seen, appear very dark. In the others, the neutral point corresponds to a color situated nearer the green, the complementary of which is purple, and not found in the spectrum. As the colors complementary to the gray parts of the spectrum do not correspond to the red extremity, the latter preserves its ordinary intensity and the spectrum is not seen shortened.

Guided especially by theoretical considerations (see page 329), it has been proposed to distinguish between these two forms by designating the former as *anerythropsia* (*Rothblindheit*), the latter as *achloropsia* (*Grünblindheit*). It was *Seebeck* who first distinguished between these two forms; but although he has been followed by a great number of scientists, among others by *Helmholtz, Holmgrén, Leber* and *Kœnig,* this distinction does not yet seem completely justified. If the neutral diameter had always either the direction SQ or the direction RT, it would be

---

(1) In order not to depart from the terminology which is generally used, I have designated the colors from 0.62 to 0.63μ as reds, but it must be noted that with the division of the spectrum which I have adopted in figure 151, and which was proposed by *Listing,* these colors are already in the orange. On the other hand, *Chibret* found with his instrument that the colors which the daltonists confound most frequently are the orange and blue.

reasonable to distinguish between the two forms, but there seem to exist intermediary forms. The position of the neutral point is, moreover, not constant, even for the same individual: it is displaced a little towards the blue when we increase the brightness of the spectrum (*Preyer*).

There have been described some very rare cases of anomalies of color vision, which are usually classified under the name of *akyanopsia* (*Blaublindheit*). In these cases the neutral point would be found in the yellow-green, and the spectrum would be seen shortened at its blue extremity. But the existence of this form is far from being established. In cases of poisoning with santonine, we meet anomalies of color vision which are somewhat in accord with these observations, but these phenomena seem rather to be attributed to a slight transient coloration of the vitreous body.

In consequence of the deficiency of their chromatic system, the daltonists are often exposed to errors, which are especially striking when they confound red with green. This is why *Dalton* used to walk in the street with the scarlet cloak of the *Oxford* doctors, thinking that it was black or gray. Cherries seem to them of the same color as the leaves of the cherry tree, etc. To understand these errors we must recollect that the colors of objects are never pure; they always contain white, and this is why red objects appear gray and not almost black like the red of the spectrum. In spite of these errors it is often astonishing to see how the daltonists know how to overcome their defect by making use of the differences which the colors present to them. Comparing, for example, red with yellow, they can frequently give their true names to these colors. The hue for both is the same, but the red appears to them less pure than the yellow, and they know that this less pure yellow is what is generally called red. They generally seem more sensitive to differences of brightness than normal persons do, and they can sometimes see traces of color which the normal eye does not discover. Thus *Mauthner* relates a case, in which the daltonist claimed that he saw yellow on a sheet of black paper. On examining the paper it was found that it really did reflect

a little of the yellow light, which had escaped the normal observer.

111. Monochromasia.— There exists yet another anomaly of the color sense, which is very rare, but seemingly well-established, name *monochromasia*. While color-blindness implies no other abnormality, mono-chromatic eyes manifest all other signs of weakness: photophobia, albinism, diminution of the visual acuity, etc. For these people differences of color do not exist; the only differences they perceive are differences of brightness, almost as on an engraving. The whole color table is narrowed to a point. The spectrum seems to them simply a luminous band, the brightness of which reaches its maximum, not in the yellow as is the case with the normal eye, but in the green (at about 0.52μ). *Hering* emphasized the analogy which exists between the manner in which monochromatics see the spectrum, and the appearance which it presents to the normal eye when its brightness is very feeble.

112. Clinical Examination of the Color Sense.— The method of mixing colors forms the fundamental examination of the color sense, and we can scarcely pass it over if we desire to form an exact idea of the chromatic system of the person whom we observe; but the method is too complicated for clinical use, and it is, besides, completely dependent on the good faith of the person whom we examine. For the clinician it is important to be able to decide quickly and surely whether his client is a dichromatic or not. With this object in view different methods have been invented.

It must first be noted that we obtain only little useful information by asking a color-blind person how he would term the color of such and such an object. If we present red to him, for example, it may not unlikely happen that he will designate this color as red, although he does not see it different from certain greens.

The method most used is the test with colored yarns (*Holmgrén*). We present to the subject the green shade of least purity

and we request him to find the shades which resemble the latter, adding that they may be a little more or a little less pronounced. Besides the green shades, the daltonist matches yellow grays, brown grays, red grays and pure grays. We then present to him *pure purple.* It is here that the alleged difference between the two kinds of daltonists becomes apparent. A person affected with *anerythropsia* would find that the blue and violet hues resemble pure purple, while a person affected with *achloropsia* would select the green and gray shades. Individuals who have only an incomplete color-blindness would stand the latter test, but not the former. *Krenchel, Daae* and others arranged colored yarns in the form of charts; *Cohn* used colored powders: *Seebeck,* who invented the method, used colored papers.

On the tables of *Stilling* are arranged a great number of spots of two colors, selected so as to be seen alike by the daltonist. There are, for example, on one sheet complementary spots, red and green; the reds are arranged between the greens so as to form numbers visible to the normal eye, but invisible to the dichromatic eye, which sees all the spots of the same color. The tables of *Stilling* do not seem very good; it appears that there are daltonists who read them, and normal eyes which do not read them. The tables of *Pflüger,* which I have already mentioned, are preferable; they are based on a phenomenon of contrast. The patient looks at a purple sheet on which are printed gray letters; the whole is covered with tissue paper. A normal eye sees the purple ground through the tissue paper, and easily reads the letters which appear by contrast in the complementary color. The daltonist sees the ground gray like the letters, so that he cannot distinguish the latter.

We can prove that the anomaly is not feigned by making the patient look through a colored glass. If the patient confounds green and red he should no longer confound them when looking through a red glass, for, as the green rays do not pass through this glass, the green must appear to him much darker than the red. Daltonists who need to be able to distinguish colors, chemists for example, may sometimes use with advantage

a colored glass, which puts them in a position to distinguish between two colors which they otherwise confound.

Polarization instruments have been used to discover color-blindness; *Rose* constructed the first instrument of this character; the leucoscope of *Kœnig* is founded on the same principle. The best of these instruments is the chromatoptometer of *Chibret.* If we place a plate of quartz cut parallel to the axis between two Nicols, parallel to each other and forming an angle of 45° with the axis of the quartz, we see the plate tinted a certain color which depends on the thickness of the quartz. Making the Nicol nearest the eye (the analyzer) rotate around the axis of the tube, the color becomes less and less pure. At 45° the field is white, and if we continue to rotate the Nicol we obtain the complementary color, which increases the purity, up to 90°, when it attains its highest point. Replacing the analyzer by a double refracting crystal, a plate of spar, for example,

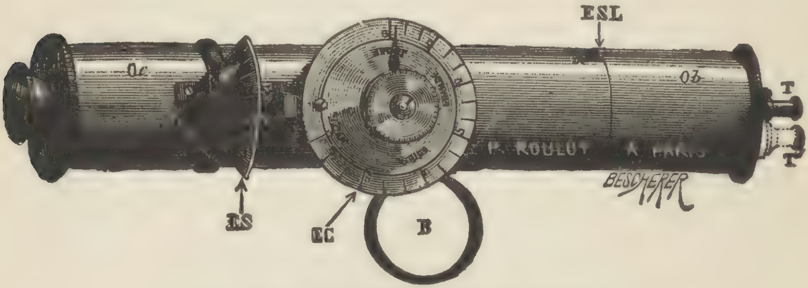


Fig. 168.—Chromatoptometer of *Chibret*.

which acts like two Nicols, perpendicular to each other, the field is seen double and one of the images of the field has the color complementary to that of the other. Rotating the spar, the colors become less and less pure, and at 45° the two fields are white. The hues of the two complementary colors depend on the thickness of the plate of quartz. In the instrument of *Chibret,* by placing the plate more or less obliquely, we can use a greater or less thickness, and thus obtain the whole gamut of colors. The instrument thus presents a very great number of hues and degrees of purity.

The patient looks towards a window through the instrument. We place the index of purity ES (fig. 168), which regulates the position of the doubly refracting crystal, at 5°, which gives colors strongly mixed with white, and after having put the index of the hues E G, which regulates the inclination of the quartz on the orange, at zero, we ask the patient if the fields are alike. If they are not, we rotate the index of the *hues* slowly towards the red, yellow and violet. If the patient always sees the two fields different we repeat the experiment after having placed the index of purity at zero, which makes the two fields white. He ought now to see them alike. If the patient stands these tests, he is not color-blind. If, on the contrary, in the first experiment he sees the two fields alike for a certain hue, he is color-blind. We then increase more and more the purity of these hues. If we thus succeed in producing a difference between the two fields the daltonism is incomplete; in the contrary case, it is complete.

If there is question of persons who desire a certificate to be employed on railroads, or as sailors, etc., it may, in addition, be useful to examine whether they can distinguish signals. An aperture of 3 millimeters diameter in a screen, covered with white paper, and illuminated from behind by a lamp, suffices for this examination. We place the person to be examined at 5 or 6 meters distance, and we see whether he commits errors when we place glasses of different colors before the aperture.

113. Hypotheses on the Mechanism of Color Vision. — To explain the mechanism of color vision different hypotheses have been tried: the old ones were without any anatomical basis; the more recent have been more or less inspired by the discovery of the retinal purple. None of these hypotheses are satisfactory in character, and the facts known up to the present do not seem yet sufficient to explain the mechanism of color vision. Let us mention briefly these hypotheses.

Theory of Young.—The following is how *Young* explained his hypothesis: "It is certain that we can produce a perfect sensation of yellow and blue by a mixture of green and red

light and of green and violet light. There are reasons for supposing that these sensations are always composed of a combination of separate sensations. This supposition at least simplifies the theory of colors; we may, therefore, accept it with advantage until such time as we shall find it incompatible with some phenomenon. We shall proceed, therefore, to consider white light as composed of a mixture of three colors only, *red, green* and *violet*."

According to this hypothesis, we suppose each nervous fibre of the retina composed of three fibres of the second order; each of these three fibres would be provided with a special terminal organ (a photo-chemical substance) and also with a special central organ. An irritation of the first fibre would produce a red sensation, an irritation of the second fibre a green sensation and an irritation of the third a violet sensation. These three colors are termed *principal colors*. An irritation of the first two fibres would produce yellow, etc. An irritation at once of the three fibres produces white, and if none of the fibres is irritated, we have the sensation of black. The red rays irritate the first fibre, the green rays the second, the violet rays the third; the yellow rays irritate the first and second, and so forth. *Young* explained color-blindness by supposing that one of the fibres was wanting.—One of the advantages of this hypothesis is that we can suppose the action identical in the three fibres. The action in the terminal organs must necessarily be different, but the one in which the impression is conducted to the brain may be the same in the three cases. The difference between the three sensations would be produced by the different reaction of the central organs.

In this form the theory is very attractive, but does not accord with observations on color vision. It requires, indeed, that we can select three spectral colors so as to be able to reproduce all existing hues and degrees of purity by mixing them. But we have seen that this is not possible; there always remain some of the spectral colors which are purer than the mixtures. According to *Young* the color table must have an exactly triangular form, but the observations of *Maxwell* have shown that

this is not the case. We cannot use, for example, the *standard colors* of *Maxwell* as principal colors, because we cannot reproduce with them the colors situated outside of the triangle.

Modification of the Theory of Young by Helmholtz.—We must, therefore, suppose that the sensations corresponding to the principal colors are still purer than the spectral colors, for then their mixtures could have the same purity as the latter. On the table the principal colors would then be placed farther from the center than the spectral colors, so that the triangle, which we would obtain by joining them, would complete the entire curve.

Helmholtz supposed that each spectral color irritated the three fibres at once, but in a different degree. Thus the red rays would irritate the first fibre strongly, the other two feebly. The impression produced by the spectral red would already contain white. *Helmholtz* remarked, in this regard, that this impression is not the purest sensation of red that we can have. If we first produce an after-image of an object of the complementary color, before looking at the spectral red, the impression becomes much more vivid, because we would thus have fatigued the two other fibres.

Helmholtz at first tried to explain color-blindness, as Young did, by the absence of one of the fibres. He supposed, therefore, three kinds of color-blindness: *anerythropsia, achloropsia* and *akyanopsia.* As we have seen, the last form is very doubtful, and the first two seem to become blended into one. But, there are yet other difficulties. Persons who are color-blind declare that they see yellow or blue in the spectrum, while, according to *Helmholtz,* they should see green and violet or red and violet. The hypothesis was saved by saying that it was not possible to know what they meant to convey by blue and yellow, but as this explanation became very doubtful, after the observation of *Hippel,* the hypothesis was modified once more by supposing that color-blind persons possess three fibres, but that in them the colors act equally on two of the fibres. If, for example, the red rays act as much on the first as on the second fibre, they must produce a yellow sensation. It is the same for

green rays. Taking the blue as the third principal color, we could thus explain the manner in which color-blind people see the colors; but all these modifications do not add to the plausibility of the hypothesis.

THEORY OF HERING.—This scientist assumes a "visual substance" which is a mixture of three others: one, which determines the sensation of black and white, another, which determines that of red and green, and a third, which determines that of yellow and blue. The red light acts on the red-green substance, causing a katobolic change (disassimilation) which produces the sensation of red. The green light, on the contrary, would cause an anabolic change in this substance by its action (assimilation) which would produce the sensation of green. The same takes place in the case of the yellow and blue rays in relation to the yellow-blue substance. The intermediary rays act on the two substances alike. But all the rays acts on the whitish-black substance, which *Hering* expresses by saying that these rays have besides their color value (*Valenz*), a white value (*Valenz*) also. It is not only the white light, but also the colored rays, which disassimilate this substance. If the two other substances did not exist, all the rays would produce a white sensation, but of different brightness. This is what takes place in the case of monochromatics (achromatics). If only one of the two substances is wanting we have the dichromatic system.

Hering supposes, therefore, four principal colors: red and green, yellow and blue, and he thinks that we have a direct impression of the fact that these four colors are pure, and that the others, perceived by an action on the two substances together, are compound.

The rivalry between these two theories, the first of which was inspired by observations on mixtures of colors, whilst the second seems to be derived especially from the study of after images, has formed the subject of a great number of works; the pupils of *Helmholtz* tried to prove that the hypothesis of *Hering* was false, and *vice versa.* It seems to me that both theories have suffered by it. The theory of *Hering* seems rather to give a statement of known facts, than to explain them. It

is based on the fact, which it seems to me difficult to deny, that the human eye does not see any resemblance between the four principal colors of the spectrum, red, yellow, green and blue, while each of the intermediary colors resembles two of the principal colors. But it must be noted that the red of *Hering* ought to be complementary to the green; it does not correspond, therefore, to the spectral red, which, according to *Hering*, already contains yellow, but to a purple color which we cannot readily claim to give the direct impression of a pure color. (1) It seems to me also that a theory which renders no account of the special situation of the yellow among the colors, is necessarily insufficient.

Other Theories.—Among the more recent theories, we may cite that of *Ebbinghaus*, who supposes the existence, in the cones, of a green substance, the decomposition of which would produce the sensation of red and green, while the purple, by its decomposition, would produce the sensation of yellow and blue. *Parinaud* supposes that stimulation of the rods produces a sensation of non-colored light, while stimulation of the cones may produce all possible sensations, the sensation of colors and the sensation of white. The retina would have two systems sensitive to light, one monochromatic, the other trichromatic. The ideas of *v. Kries* almost agree with those of *Parinaud*.

Arthur Kœnig exploited a theory which may be considered as a development of the theory of *Young-Helmholtz*. He supposes the *red, green* and *blue* as principal colors. According to *Kœnig*, the decomposition of the retinal purple into yellow produces the weak sensation of gray, which causes any color when it is sufficiently weak. Further decomposition produces the sensation of blue. Perception of the two other principal colors, green and red, is effected by the agency of the pigment cells, while the cones must be considered as dioptric instruments in-

(1) Towards the periphery of the visual field there exists a dichromatic zone, in which we see only yellow and blue colors. A red object seems yellow at this place, while a purple color appears blue; it is the intermediary tint which corresponds to the red of *Hering*.

tended to concentrate the light on the epithelial layer.—I have already mentioned that *H. Müller* measured the distance of the retinal vessels from the sensitive layer by means of the parallax of the vessels, seen entoptically (see page 183). In collaboration with *Zumft, Kœnig* repeated these experiments with spectral light. He found that the distance increases according as we approach the red end of the spectrum. The layer sensitive to green light, and especially that sensitive to red light, would, therefore, be situated behind the layer sensitive to blue. The distance of these two layers exceeded even the retinal thickness, which led *Kœnig* to suppose that the perception of these two colors takes place in the epithelial layer.—These experiments still need to be verified; *Koster* repeated them without success.

Bibliography.—In spite of the great number of works on color vision, this question still seems imperfectly elucidated. In the preface to his treatise on light which appeared a few years before *Newton's* works on optics, *Huyghens* said he would not speak of colors, "a question in which, up to the present, no one can pride himself on his success." It seems to me that this phrase, which was true at the time of *Huyghens* as to the physics of colors, may be applied to-day to their physiology. This subject has not yet found its *Newton.*

Newton (I.). *Optics.* London, 1704.—Lambert. *Farbenpyramide.* Augsburg, 1772.—Dalton, *Edinburgh. Philos. Journal.* Vol. VI.—*Œuvres de* Young, edited by Tscherning, p. 217-232.—Purkinje. *Zur Physiologie der Sinne.* II, p. 109, 1825.—Seebeck. *Ueber den bei manchen Personen vorkommenden Mangel an Farbensinn.* Pogg. Ann., 1837, p. 177.—Helmholtz (H.). *Ueber die Theorie der zusammengesetzten Farben.* Pogg. Ann., 1852, p. 45.—Helmholtz (H.). *Ueber die Zusammensetzung der Spectralfarben.* Pogg. Ann., 1855, p. 1.—Helmholtz (H.). *Ueber die Empfindlichkeit der menschlichen Netzhaut für die brechbarsten Strahlen des Sonnenlichts.* Pogg. Ann., 1855, p. 205.—Maxwell (C.). *Experiments on Colors as Perceived by the Eye with Remarks on Color Blindness. Transact. of the Roy. Soc. of Edinb.,* XXI, 1855.—Maxwell (C.). *On the Theorie of Compound Colors and the Relations of the Colors of the Spectrum. Phil. trans.,* 1860.—Maxwell (C.). *On the Unequal Sensibility of the Foramen Centrale to Light of Different Colors.* Edinb. Journ., 1856, IV, p. 337.—Hering (E.) in *Lotos* Prag., 1880-82-85-87.—Rayleigh. *Nature.* Vol. XXV, p. 64, 1881.—Macé de Lepinay and Nicati. *Ann. de chimie et de physique.* Ser. 5, t. 24, p. 289, 1881 et t. 30, p. 145, 1883.—Uhthoff (W.). *Ueber das Abhängigkeitsverhältniss der Sehschärfe von der Beleuchtungsintensität.* Gräfes Arch. XXXII, 1886.—Uhthoff (W.). *Weitere Untersuchungen über die Abhängigkeit der Sehschärfe von der Intensität sowiè von der Wellen-*

länge im Spektrum. Gräfes Arch. XXXVI, 1890.—Kriess (I. v.). *Die Gesichtsempfindungen und ihre Analyse.* Leipzig, 1882.—*v. Hippel.* Gräfes Archiv. XXVII, 3, p. 47, 1881.—Krenchel (W.). *Ueber die Hypothesen von Grundfarben.* Gräfes Arch. XXVI, p. 91, 1880.—Kœnig u. Brodhun. *Experimentelle Untersuchungen über die psychophysische Fundamentalformel in Bezug auf den Gesichtssinn.* Acad. of Berlin, July 26, 1888, and June 27, 1889.—Kœnig (A.) and Dieterici (C.). *Die Grundempfindungen in normalen und anomalen Farbensystemen und ihre Intensitätsverthelung im Spectrum.* Zeitschrift für Psychol., IV, p. 241, 1892.—Kœnig (A.) et Zumft (I.). *Ueber die lichtempfindliche Schlicht in der Netzhaut des menschlichen Auges.* Acad. of Berlin, 1894, May 24.—Kœnig (A.). *Ueber den menschlichen Sehpurpur und seine Bedeutung für das Sehen.* Acad. of Berlin, 1894, June 21.—Chibret. *Chromatoptometre.* Bulletin de la Soc. fr. d'opht., 1886, p. 336.—Ebbinghaus (H.). *Theorie des Farbensehens.* Hamburg, 1893.—Parinaud (H.). *La sensibilité de l'oeil aux couleurs spectrales; fonctions des éléments rétiniens et du pourpre visuel.* Ann. d'oc. t. CXII, p. 228, 1894.—Koster (W.). *Ueber die percipirende Schicht der Netzhaut beim Menschen.* Gräfes Arch., LXI, 1, p. 1, 1895.

CHAPTER XVIII

THE FORM SENSE

114. Central Visual Acuity.— The power of distinguishing forms is a very complex faculty, which is in great part connected with the ocular movements. To judge of the form of objects we grope for them, so to speak, with the look. Nevertheless, indirect vision furnishes an idea of the form of objects. According to empiric ideas (page 263) it would be the observations made during the displacements of the look that would have taught us the meaning of the impressions obtained in indirect vision.

The lowest angle under which two points can be distinguished from each other has been taken as the measure of the form sense. Astronomers for a long time devoted attention to this question. *Hooke,* for instance, said that in order that a double star can be recognized as such by the eye, the interval must correspond to one minute, and moreover, that good eyes would be necessary to see two stars under these conditions. Later, the physiologists took up the question, generally by working with a small grating the bars and intervals of which were of the same size. We place the grating towards the sky and try how far we can move away from it before the bars become confused. Care must be taken that the image formed on the retina is distinct, by correcting defects of refraction, if there are any. In accord with most observers *Helmholtz* found nearly the same angle as *Hooke,* that is to say, one minute, but it must be observed that it is neither the width of a bar nor that of the interval, but the sum of the two, which corresponds to this angle.

Considering the anatomical structure of the retina, we would expect that the angle of least distinction would correspond to the size of a cone. In the experiment of *Hooke* we may suppose, indeed, that we can distinguish two stars if, between the two cones on which their images are formed, there is found a

third, which does not receive any impression (fig. 169). We may, therefore, conclude that the angular size of a cone must be smaller than the angular distance separating the two stars.

Fig. 169.

Experiment of *Hooke*. The images of two stars (e, e) are formed on two cones separated by a third.

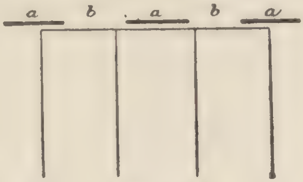


Fig. 170.

Measurement of the visual acuity by a grating.

aa, Images of the bars separated by those of the intervals, *bb*.

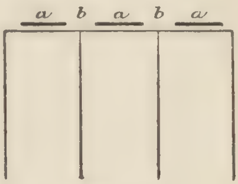


Fig. 171.

Measurement of the visual acuity with a grating.

Limit.—All the cones receive the same impression.

In the experiment of *Helmholtz,* on the contrary, we cannot conclude that the size of the cone must be smaller than the angular size of the black bar; for we can very well imagine a larger cone, the central part of which may be occupied by the image of the black bar, while the lateral parts would be occupied by a part of the images of the intervals, but which would receive, however, less light than the neighboring cones (fig. 170). But we can conclude that the cone must be smaller than the angular distance separating the centers of the two neighboring luminous intervals (or, which amounts to the same thing, smaller than the sum of the black bar and a luminous interval), for if the size of the cones were equal to this distance, all the cones would receive the same quantity of light (fig. 171), and the bars would be confused. Thus the result obtained by *Helmholtz* is in agreement with that of *Hooke.*

Placing the distance of the nodal point of the eye from the retina at 15 mm. the angular size of a minute corresponds to $\frac{2 \times 15\pi}{60 \times 360} = 0.004$ mm. In the *fovea* the size of the cones is about 0.002 mm. The visual acuity does not seem, therefore, to alto-

gether reach the degree which we would expect according to the structure of the retina, probably on account of optic irregularities. It seems rare, indeed, that a luminous point forms its image on a single cone, and if it extends over several cones, it is not strange that the angle of least distinction is larger than the angular size of a cone (fig. 172).

Fig. 172.—Experiment of *Hooke*, the optics of the eye being defective. Instead of distinct images, the stars form diffusion spots, *ee*, *ee*.

One might think that the least angle of *visibility* may serve as a measure of the form sense, that is to say, that we can measure it by determining what is the smallest visual angle under which an object may be seen; but it is evident that this angle depends solely on the luminous intensity of the object, for, in spite of their minimum angular size, we see fixed stars very well when they are sufficiently luminous.

If the eye were optically perfect, so that the image of star could be formed on the surface of a single cone, it is easy to see that the luminous impression which this cone may receive, if it be sufficiently strong, would suffice to make the object visible, even if the image did not occupy the entire surface of the cone. But, as a rule, the optic properties of the eye are not so good. Most people do not see the stars as points, but as small surfaces so much greater in proportion as the star is brighter; the image of the star is, indeed, a circle of diffusion composed of more or less luminous parts: when the light is feeble these latter parts disappear so that the star appears smaller. As long as the star is luminous the image, therefore, generally covers several cones; if the light diminishes the image may be formed on a single cone, but the visibility always depends on the brightness only. A comparison with the preceding

experiment shows also that we cannot use the visibility of a single star as a measure of visual acuity; the experiment would be identical with that of the grating, if we imagine two infinitely large bars separated by an interval corresponding to the star. We have seen that we may conclude that the angular size of the cone is smaller than the angular size of a bar plus an interval; but this, in the present case, has no application.

In clinics we use, for the measurement of visual acuity, the charts of *Snellen* or others constructed on the same principle. The letters are arranged so as to be seen under an angle of 5 minutes; the lines which form the letters, as well as most of the intervals which separate them, are seen under an angle of 1 minute. We see that the normal acuity of *Snellen* corresponds to half of that which *Helmholtz* found, with his grating, in which each bar and each interval corresponded to a half minute. We have found also that the best eyes have a visual acuity which approaches 2 ($\frac{3}{2}$ or $\frac{4}{3}$), and we can be almost certain that if, with a good illumination, the acuity is only equal to 1, the eye presents defects sufficiently pronounced to be easily established.

We have said that the angle under which the letters are seen corresponds to 5 minutes. The angle being equal to the linear size of the letter divided by the distance at which it is seen, it is clear that the letters which are intended to be seen at a distance of 12 meters must have double the linear size of those which are seen at 6 meters. If the former are seen at a distance of 6 meters only, we say that the visual acuity is equal to $\frac{6}{12}=\frac{1}{2}$ Different authors, *Javal* among others, have observed that this way of designating the visual acuity is not very logical, and that we should, in this case, say that the acuity is equal to $\frac{1}{4}$, since the surface of the letter in question is 4 times greater than that which corresponds to the acuity 1.

In spite of the theoretical objections which may be made to it, the chart of *Snellen* is, however, very practical. It is certain indeed, that some of the letters are much more easily read than others on the same line. The legibility of a letter is, indeed, a very complex affair, which is far from depending altogether

on the size of the intervals separating the different lines. Attempts have been made to remedy this, sometimes by making larger the letters which are read with difficulty, sometimes by selecting only letters which are easily legible. These improvements are not widely employed, for they are without much utility; by using the chart we learn, in fact, very quickly the degree of legibility which each letter has for a normal eye. A more serious inconvenience is the small number of large letters, which frequently renders the determination of refraction difficult in cases in which the acuity is not so good, because the patients learn the letters by heart. To have a constant illumination, it is well to place the chart in a dark place and to illuminate it with a gas jet provided with a reflector, which protects the eyes of the patient. The chart of *Javal* is transparent and placed by the side of the patient, who looks at it in a looking-glass. We thus achieve this result, that the letters, being opaque, are always seen perfectly black, and that the distance is double by reflection. The size of the letters increases in geometrical progression, which had already been proposed by *Green*. *Burcardt* had printed series of groups of dots of different sizes arranged after the principle of *Snellen*. The patient must be able to count the number of dots which compose a group. Many oculists followed the example of *Snellen* and constructed charts on the same principle.

We still use the reading test types of *Jaeger,* the first fairly complete collection of characters of different sizes which had been used. The advantage which the chart of *Snellen* presents is that it has written upon it the distance at which the patient ought to be able to see each line, which enables oculists to examine the sight of all patients at a like distance. This principle had already been applied by *Stellwag*.

In 1891, *Guillery* proposed to measure the visual acuity simply by the distance at which we can distinguish a black point on a white ground. By comparisons with the letters of *Snellen,* he found that a black point seen under an angle of 50 seconds

corresponds to the normal acuity; at 5 meters it should have a diameter of 1.2 mm. This point is designated as No. 1. No. 2 has the *surface* twice as large as No. 1, and the patient who sees only No. 2 at 5 meters distance, has an acuity of $\frac{1}{2}$, etc. Each point is on a white square, sometimes in the center, sometimes below, sometimes in an angle, etc., and there are on the same line several tests side by side in which the point has the same size. The patient must tell at what part of the square he sees the point. It seems that we measure the visual acuity quite as well in this way as by the principle of *Snellen,* which is quite interesting, and shows that we cannot identify the examinations with the luminous point on a black ground with that made by means of a black point on a white ground. *Javal* constructed a small portable scale on the same principle: it is composed of small black squares, such that the side of a square is also equal to the diagonal of the preceding one. If the side is equal to 1, the diagonal is $\sqrt{1^2+1^2} = \sqrt{2}$, which is the side of the following square; the diagonal of this latter is then 2, and so forth. In this manner the area of a square is always double that of the preceding square.

Relations Between Visual Acuity and Illumination.—The visual acuity depends directly on the illumination of the chart, but it is quite difficult to determine the relation in a general way, because there are many different factors which affect it. Thus the relation must depend on the pupillary size, on the manner in which the pupil contracts under the influence of light, on the degree of optic perfection and especially on the adaptation of the eye to darkness. *Druault* has made some researches on this question, by moving a candle (of stearine of 22 mm. diameter) towards the visual acuity chart, and noting the distance at which this light would allow each line to be read; the eye was in a degree of medium adaptation. In order to obtain high degrees of illumination, he replaced the candle by a lamp equivalent to fifty-four candles. The following table shows his results, taking as unit the illumination obtained by placing a candle at a distance of one meter.

Illumination.	Acuity.
0.016 meter candles	$\frac{15}{200} = 0.075$
0.020 " "	$\frac{15}{100} = 0.15$
0.028 " "	$\frac{15}{70} = 0.21$
0.047 " "	$\frac{15}{50} = 0.30$
0.12 " "	$\frac{15}{40} = 0.37$
0.25 " "	$\frac{15}{30} = 0.50$
0.67 " "	$\frac{15}{20} = 0.75$
1.50 " "	$\frac{15}{15} = 1.00$
16.7 " "	$\frac{15}{12} = 1.25$
5400 " "	$\frac{15}{10} = 1.50$

We note that the acuity increases rapidly at first, then slowly, with the illumination, and finally there is need of an enormous increase of illumination in order to make the acuity rise from 1.25 to 1.50. Still increasing the illumination, the acuity would probably still increase, but very little, so that the curve indicating the visual acuity for the different illuminations would be a flattened curve much elongated and more or less like the curve of the light sense (fig. 148).

I have already observed that the relation between the visual acuity and the illumination depends, furthermore, on the color of the light used (page 293).

The theory according to which the layer of the cones and rods would be the sensitive layer, explains sufficiently well the acuity which we obtain with a good illumination, but it gives by no means a satisfactory explanation of the manner in which the acuity falls when the illumination diminishes.

115. Peripheral Acuity.— We determine the limits of the visual field with a perimeter or campimeter, by allowing the person examined to fix the center, and finding up to what limit the patient can still see the object in indirect vision. The distance of the eye from the plane of the campimeter, or from the arc of the perimeter, varies slightly for different instruments. The object is generally a white square (or a colored one), the side of which is about 1 centimeter. With the white object we thus find the absolute limits of the field; taking larger or brighter objects we scarcely obtain any more extended limits. It is otherwise for the examination with colors. It seems, indeed, that by taking sufficiently large and bright objects we obtain larger limits than by ordinary examination. In clinics, we examine generally with the *white, blue, red* and *green,* and we find, as a rule, the field less extended in the order in which I have named the colors. If one finds different limits for the red and green, this is probably due to the fact that colors which are not complementary or which have a different brightness are used. Otherwise we ought to find the same limits.

The visual acuity falls greatly as soon as the image is moved away from the fovea. If, for example, we fix the border of the chart of *Snellen* the acuity falls in consequence to $\frac{1}{8}$ or $\frac{1}{10}$. Attempts have been made to determine the peripheral acuity according to the principle of *Snellen,* but the method is very difficult to use clinically, whilst another method introduced by *Bjerrum* seems to give good results. He simply repeats the perimetric examination with smaller and smaller objects. He uses a distance of 2 meters, placing the patient in front of a large black curtain; the objects used are small ivory discs of different sizes, fixed on black rods of 1 meter in length. The

observer must wear black gloves. By thus examining, *Bjerrum* found as the limits of the normal field:

		Outside.	Inside.	Below.	Above.
With a disk of..........	3mm	35°	30°	30°	25°
— —	6mm	50°	40°	40°	35°
Normal limits		90°	60°	70°	60°

By this method we can frequently establish defects which we could not otherwise find. We thus meet cases of atrophy of the optic nerves, in which the field examined in the ordinary manner is normal, whilst the method of *Bjerrum* reveals considerable contractions. In glaucoma *Bjerrum* has, by his method, discovered scotomata scattered in the field, but which are generally connected with a spot of *Mariotte* by a lacuna in the form of a bridge. The paracentral scotoma is thus connected with the papilla by a lacuna which surrounds the upper or lower half of the macula. Its form indicates directly the course of the nerves. Sometimes it may be useful to repeat the examination with diminished illumination.

More recently, *Groenouw* has made analogous measurements with a black point on a white ground. He designates as *isopters* the lines drawn in the visual field through the points where the visual acuity is the same. These methods are founded on the same principle which was used by *Guillery* for the measurement of central acuity. Their theory is still to be formulated.

In the normal field there is only one interruption, namely, the blind spot which corresponds to the papilla. It was discovered by *Mariotte,* whose name it bears, and created at the time a very great sensation. From his discovery Mariotte drew this conclusion, that it is the choroid which is the sensitive layer of the eye, since it was absent in this place, and this idea was for a long time accepted. We can determine the form of the blind spot by the ordinary methods with the perimeter, and still better by placing ourselves at a distance of one or two meters. The spot has an elliptical form; generally we succeed, on examining with a very small object, in following the big vessels a little

outside of the papilla (fig. 173). If we do not succeed in following them farther, it is due to the lack of stability of the fixation. According to the researches of *Dr. Holth,* who drew figure 173, it is almost impossible to maintain an almost exact fixation for more than 5 or 6 seconds; after this time the look makes in-

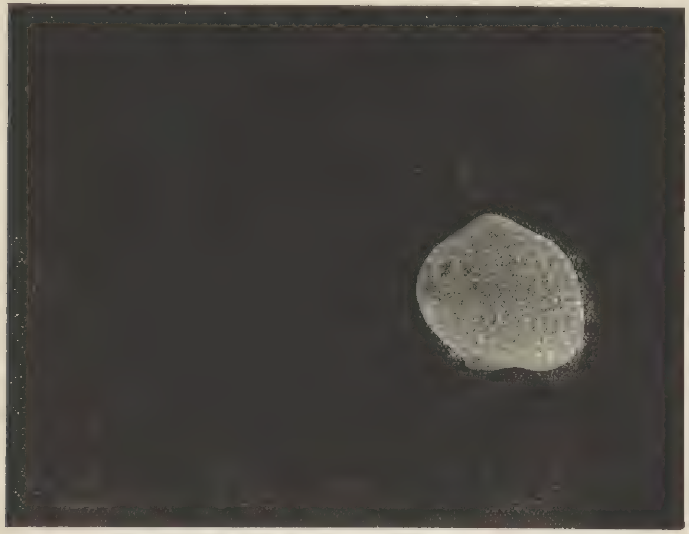


Fig. 173.—Mariotte's blind spot in my right eye, drawn by Holth.

voluntary deviations which may reach a third or half a degree, and after 20 or 30 seconds we frequently observe deviations which often exceed one degree. We can control fixation by using as the object of fixation a point marked on a small colored surface on a white ground. After a very short time we see the surface surrounded with a border of the complementary color.—The internal border of the spot of Mariotte is about 12 degrees from the point of fixation, and the diameter corresponds to about 6 degrees, or 12 times the diameter of the moon.

Phenomenon of Troxler.—If we draw several black spots on a sheet of paper and fix one of them for some time, we see sometimes one, sometimes another of the surrounding spots disappear, to reappear a little while after, generally at the moment of winking or of making a slight movement of the eye. This

singular phenomenon which was described at the beginning of this century by *Troxler,* has recently been studied by *Dr. Holth.* The color of the background, as well as that of the spots, plays no part; during the disappearance of these latter we see in their place the background only; the scotoma is, therefore, filled almost like the spot of *Mariotte.* Even the spot fixed may disappear after a long period of fixation. In order to study the phenomenon we can observe a regular diagram as in figure 174.

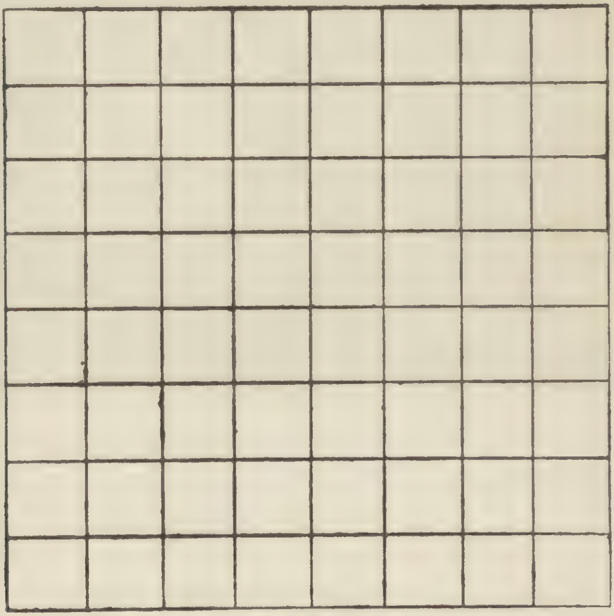


Fig. 174.

For my eye the phenomenon begins after having fixed the middle for 8 or 9 seconds, that is to say, at the moment when the fixation begins to be less steady. From this moment the figure shows continuous changes: sometimes one part of the figure disappears, sometimes another. An interesting fact is that most frequently the scotomata are not absolute: sometimes it is the horizontal lines which disappear at one place, while the vertical lines persist, sometimes the contrary takes place. These phe-

nomena recall forcibly that which has been described under the name of antagonism of the visual fields and which we observe, for example, when presenting in a stereoscope horizontal lines to one eye and vertical lines to the other.—If we fix the center of a figure composed of concentric circles and radii, we see sometimes the latter, sometimes the circles. On a chess-board we see sometimes one, sometimes another of the squares disappear, and so forth. *Holth* even caused luminous objects to disappear, the moon for example; according to him small objects disappear even if we give them a slow motion. There is reason, therefore, to be on the guard against this source of error, if we wish to perform perimetry with precision.

Bibliography.—Hooke v. Smith, Robert. *Cours complet d'optique,* translated by Pezenas. Paris, 1767, p. 44.—Troxler. *Ueber das Verschwinden gesehner Gegenstände innerhalb unseres Gesichtskreises. Himly v. Schmidt. Ophthalm. Bibliothek.,* 1802, II, p. 1.—*Œuvres de Young,* edited by Tscherning, p. 78.—Stellwag v. Carion. *Die Accommodationsfehler des Auges.* Wien, 1855.—Guillery. *Ein Vorschlag zur Vereinfachung der Sehproben.* Arch. f. Augenheilk., XXIII, p. 323, 1891.—Grœnouw. *Ueber die Sehschäfe der Netzhautperipherie und eine neue Untersuchungsmethode derselben.* Arch. f. Augenheilk., XXVI, p. 85, 1893.—Bjerrum. *Undersoegelsen af Synet.* Copenhagen, 1894.—S. Holth. *Om det normale Synsorgans Stirreblindhed. Norsk Magazin for Laegevidenkaben.* August, 1895.

BOOK III

THE OCULAR MOVEMENTS

AND

BINOCULAR VISION

CHAPTER XIX

THE LAW OF LISTING

116. Center and Axes of Rotation of the Eye.—The movements of the eye are made freely in all directions; the extent of the field of fixation is about 55° in all directions.—It is easy to prove that the soft parts which fill the orbit are incompressible: if we try to push the eye backwards, we meet with considerable resistance; the movements of the eye are limited, therefore, to its rotations.

These rotations are made, at least approximately, around a center which, according to the determinations of *Donders*, is situated about 10 mm. in front of the posterior surface of the sclera, or 14 mm. behind the summit of the cornea. It coincides with the center of the posterior surface of the globe, supposed to be spherical. It is not certain that the center of rotation is exactly the same for movements in different directions.

Donders, in collaboration with *Dojer*, determined the position of the center of rotation of the eye in the following manner. He first measured the diameter of the cornea with the ophthalmometer of *Helmholtz*, and then placed a hair (*a*, fig. 175) stretched vertically in a ring, in front of the middle of the cornea. He then examined the angular size of the lateral movements of the look, which the observed person had to make, in order that the hair would be seen successively in coincidence

with the left and right borders of the cornea. Let ACD (fig. 175) be one of these movements, p half the diameter of the cornea, and x the distance CE. Then we have $p = x \operatorname{tg}$ ACD,

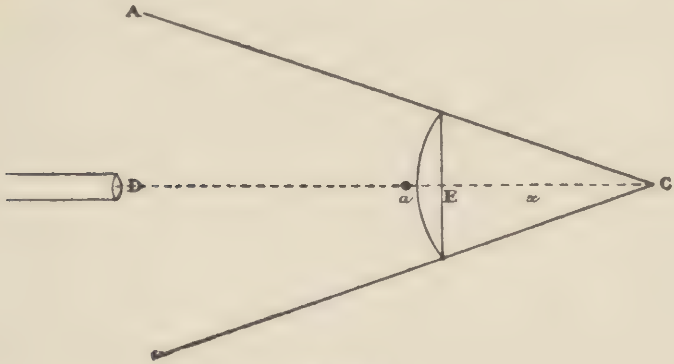


Fig. 175.

from which we can calculate x. Adding to this distance the height of the cornea, we find the distance of the center of rotation from the cornea.

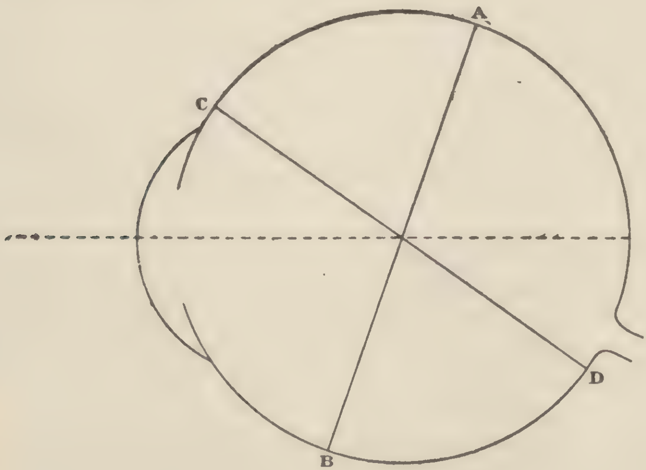


Fig. 176.

The six motor muscles form, as we know, three pairs, (1) which cause the eye to turn around three axes passing through the center of rotation of the eye. The axis of the external and internal recti is vertical. The axes of the two other pairs are situated in the horizontal plane. The nasal extremity of the axis of the superior and inferior recti, BA (fig. 176) is situated a little in front, so as to form an angle of about 70° with the visual line. The temporal extremity of the axis of the oblique muscles CD (fig. 176) is directed very much forwards; it forms an angle of about 35° with the visual line.

The internal and external recti turn the eye, therefore, directly inwards and outwards. The superior and inferior recti direct the look upwards and downards, but at the same time a little inwards. The inferior and superior oblique direct the look either downwards or upwards but at the same time outwards. The look is directed straight upwards by the combined action of the superior rectus and the inferior oblique, and the direction downwards is obtained by the combined action of the inferior rectus and superior oblique.

The muscles make possible the rotation of the globe around any axis. This is all that it is of importance to know for the physiology of the eye. We must not think that the eye turns oftener around the axes which we have just described, than around the intermediary axes. It seems, indeed, that all six muscles are concerned each time the eye makes any motion; the axis around which the eye turns is, therefore, always different from the three which we have just mentioned.

117. The Law of Listing.—Supposing the head to be motionless, the position of the eye is determined for a given point of fixation. This is far from being evident *a priori,* for the eye could still perform rotations around the visual line. Each time that the look returns to the same point, no matter in what

(1) [This statement is only approximately true, as according to the careful measurements of *Volkmann,* each of the six muscles of the eye seems to rotate the latter around its own axis. See paper by the translator in the Archives of Ophthalmology, Vol. XXVII, No. 1, 1898: Are our present ideas about the mechanism of the eye-movements correct?]—*W.*

way, the eye always reassumes the same position (*Donders*). If, by fixing a colored ribbon stretched horizontally, we produce an after image, and then project the latter on a wall, keeping the head motionless, the image assumes a position which is not always horizontal, but which is always the same every time that the look returns to a given point. This position is determined by the law of *Listing*.

There exists a certain direction of the visual line in relation to the head, which we call *primary direction;* the corresponding position of the eye is named *primary position,* and every other position (direction) is called *secondary*. The primary direction generally corresponds to the direction which the visual line assumes when we look at the horizon, giving to the head the position which seems most natural; but it happens quite frequently, however, that one is, under these circumstances, obliged to lower the look slightly, in order to put the eye in the primary position. In this case, one is obliged to lean the head slightly backwards in order to make the primary direction horizontal. We must suppose this direction invariably connected with the head, in all the movements of which it partakes.

According to the law of *Listing, the eye may be brought from the primary position to any secondary position by a rotation around an axis perpendicular to the two successive directions of the visual line.* This defines for us at the same time the primary position.—The axes of *Listing* are all contained in a plane perpendicular to the primary direction and pass through the center of rotation of the eye. This plane is, therefore, as invariably, connected with a head.

To demonstrate the law of *Listing,* we place ourselves at a distance of one or two meters from a wall on which is placed a fixation mark A (fig. 177), on a level with the eyes. It is necessary to make the position of the head secure. If we do not wish to make very exact measurements, a head-rest, like that of the ophthalmometer of *Javal* and *Schioetz,* suffices. If, on the contrary, we desire a very great exactness, we use the little mouth-board (*planchette*) of *Helmholtz,* the border of which is covered with sealing wax. We squeeze the planchette

between the teeth while the sealing wax is still warm, so that the latter may receive the imprint of the teeth. We then fix the planchette on a stand, so as to be able to turn it to the right or to the left or to incline it any number of degrees fixed upon (*Hering*).

We place on the wall, at A, a rectangular cross so that its arms may be horizontal and vertical. The cross ought to contrast boldly with the background, so as to permit us to obtain a very pronounced after image by fixing it for a little while. We take the planchette between the teeth and, inclining the head

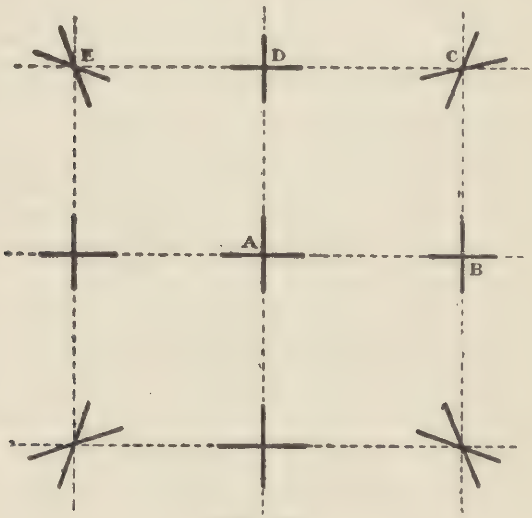


Fig. 177.

(with the planchette) a little forward or backward, or inclining it a little to the right or to the left, we find a position such that on moving the look along the prolongation of each of the arms of the cross, the after image of this arm glides all the time on itself (fig. 177). We then observe that there exists only one position of the head for which this is possible; for every other position of the head the after image of the cross turns around during the displacement of the look. When we have found this position of the head, we fix the planchette, so as to be able to

again find the position every time that we take the planchette between the teeth. Then, when we fix the point A, the eye is in the primary position. Suppose, indeed, that we fix a second

Fig. 178.

point B, situated on a prolongation of the horizontal arm: since the meridian which was horizontal when fixing A, is also horizontal when fixing B, it is clear that the look may be brought from A to B by a motion around a vertical axis, that is to say, around an axis perpendicular to the two directions of the visual line. It is the same for displacement in the vertical direction. In order to demonstrate that this is also the case for the oblique displacements, we tilt the cross (fig. 178). It is then easy to prove that the after image of one of the arms of the cross glides all the time on its prolongation, when the look follows this prolongation, and that, consequently, the eye turns around an axis perpendicular to this meridian. The law of *Listing* is thus verified.

If, in these experiments, the look does not follow the prolongation of one of the arms of the cross, we observe phenomena which might seem in contradiction with the law of *Listing*. Thus fixing the point C (fig. 177) we observe that

the after image of the vertical arm of the cross is no longer vertical; it has undergone a rotation, and the upper extremity is carried to the right. A little reflection shows that this is simply a consequence of the law of *Listing,* and that the meridian which was vertical when fixing A', cannot remain vertical when the eye turns around an axis perpendicular to the direction AC. *Donders,* who first described this phenomena, attributed it to a rotary movement (*Raddrehung*) of the eye, that is to

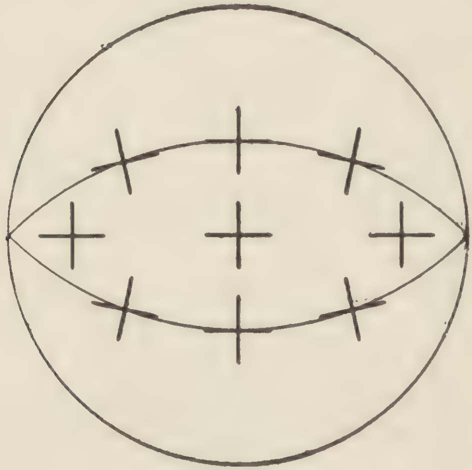


Fig. 179.

say, a rotation around the visual line, but it is clear that such a rotation cannot take place since the axis of Listing is perpendicular to the visual line.—The horizontal arm of the cross seems to have suffered a rotation in a contrary direction, but this is merely the result of the projection of the after image on a plane which is not perpendicular to the visual line. (1) If we project the image on the concave surface of a hollow hemi-

---

(1) [How much these after images ought to be inclined towards the horizontal and vertical lines of the wall has been explained by the translator in a paper entitled "The Law of Listing and Some Disputed Points about Its Proof." Archives of Ophthalmology, Vol. XXVIII, March, 1899. The relation between these angles and the angles of Helmholtz is elucidated in a paper by *Dr. G. Hay* in the Journal of the Boston Society of Medical Sciences, in Oct., 1899, and in a paper by *Professor L. Hermann,* in Pflüger's Archiv. der Physiologie, Nov., 1899.]—*W.*

sphere, in the center of which is the eye, the cross remains rectangular and seems to have suffered a complete rotation to the right (fig. 179).—In these experiments, the position of the two eyes is exactly the same: we can cover sometimes one eye, sometimes the other, and the position of the after image does not change.

It must be noted that the eye *may* be transferred from the primary position to a secondary position, by rotating around the axis of *Listing*. I do not say that it really makes this movement, for the law of *Listing* defines solely the position of the eye in the state of repose.—We know nothing, or almost nothing, of the manner in which the eye makes its movements. There is no reason to assert that it turns around the axes of *Listing*, nor even to suppose that the look always follows the same way to go from one point to another. The best method of studying this question would probably be to bring the look quickly from one point to another, leaving the eye exposed to a pretty intense light. The after image of the luminous source then assumes the form of a line which permits some conclusion as to the nature of the movement.

What we have said suffices to determine any position of the eye. If the look passes from one secondary direction to another, the position of the eye is nevertheless determined by the law of *Listing*, since, having reached its new secondary position, it must have the same position as if it had reached there, starting from the primary position. Note that the look cannot be brought from one secondary position to another by turning around an axis perpendicular to the two directions in the visual line. For, if the look goes from B to C (fig. 177), following the prolongations of the vertical arm, we observe that the after image of this arm starts from the prolongation and rotates more and more so as to attain the position which it should have when the look will have arrived at C. In making this movement of the look, the eye does not rotate, therefore, around an axis perpendicular to the visual line, and we can in this case speak of a true rotary movement. If we displace the look so that the after image moves always on itself, the point of fixation describes a

curve the convexity of which is turned towards the point A. It is the same for the horizontal arm: if we bring the look from C to E, so that its after image moves on itself, we obtain a curve with its convexity downwards. The following illusion, described by *Helmholtz,* results from this fact.

If, after having fixed the point A in the primary position, we raise the eyes and survey quickly with the look a horizontal straight line situated higher up, it appears concave towards the floor (compare page 261). This is due to the fact that oblique directions of the look are very rare. Generally, we take care when we desire to look at any object, to turn the head in such a way that the eyes are nearly in their primary position, and that the horizontal lines are drawn on the *retinal horizon* (the meridian of the retina which is horizontal in the primary position: in the experiment fig. 177, the retinal horizon is marked by the after image of the horizontal arm of the cross). On account of this custom we have a tendency to consider the direction of the retinal horizon as horizontal, even when it is not. Looking upwards and to the left, the retinal horizon inclines its right extremity downwards, and, if we consider this direction as horizontal, it follows that the straight line which we observe must appear inclined to the left; when the look reaches the other extremity, this latter will seem inclined to the right; thus it is that the line assumes its curved aspect, but we must survey it quickly, otherwise it seems rather to lean sometimes to the right, sometimes to the left.

Another Method of Demonstrating the Law of Listing.—As the retinal horizon passes through the papilla, we can use the position of the spot of *Mariotte* to account for its direction. *Fick* drew, on a cardboard movable around a point O, a black spot just large enough to disappear in the spot of Mariotte, when he fixed the point O in the primary position. Turning the head to the right or to the left and inclining it at the same time, while he continued to fix the point O, the spot reappeared and he then measured how much it was necessary to turn the cardboard to make it disappear again.—Proceeding thus, we find, as by the

preceding method, that the eyes follow pretty exactly the law of *Listing,* at least while the visual lines remain parallel.

118. Experiments of Meissner.—Apparently vertical meridian.—There exists another method which has been described by *Meissner,* and which enables us to verify the law of *Listing* in a very exact manner. But before explaining this method, I must mention a singular phenomenon which we meet when we wish to judge whether a line is vertical or not.

We hold a plumb-line in front of a wall painted uniformly and we fix a point situated a little in front of this line (1): we then see the latter in double homonymous images, and we would expect to see two vertical and parallel lines; but the two lines seem to converge upwards: seen with the right eye, the upper extremity of the line seems to lean to the left. If we fix a point situated behind the line, the images are crossed and seem to converge downwards. A vertical line seen with one eye only does not, therefore, appear vertical, but its upper extremity seems to lean to the left or to the right, according as it is the right eye or the left eye which looks at it.—Looking at a rectangular cross, one of the arms of which is horizontal and the other vertical, the two angles, the upper right and lower left, will appear, for the right eye, larger than the other two, while the contrary takes place for the left eye.

Since, for the right eye, a vertical line appears to lean to the left, there must exist a line leaning to the right, which seems vertical. We can determine the direction of this line by observing a white disc movable around its center and on which we draw one diameter. Along the border is a scale graduated in degrees, the zero of which corresponds to the vertical line, and which must be placed so as not to be visible. The observer tries to turn the disc so as to place the diameter vertically. With the right eye he places nearly always the upper extremity some degrees too far to the right, with the left eye some degrees too

(1) We must not place ourselves too near the line, in order that the influence of convergence, of which I shall speak immediately, may not interfere.

far to the left. For the horizontal meridian, the phenomenon is less pronounced.—It is necessary to arrange the experiment in such a manner that the observer cannot be guided by the view of the surrounding objects.

Another method of determining the angle between the apparently vertical meridians of the two eyes has been described by *Volkmann* (fig. 180). He placed two small revolving discs on a vertical wall so that the distance separating their centers would be equal to the distance between the eyes. On each disc was shown one radius. He observed the discs as with the stereoscope, the right eye fixing the disc on the right, the

Fig. 180.—Discs of *Volkmann.*

left eye that on the left. He placed one of the radii vertically, and then tried to place the other so that the two radii would appear to form a single straight line; it was necessary that they should form an angle of about two degrees.—Among the stereoscopic tests which are given in *Javal's* manual on strabismus, several show small discs like those of *Volkmann,* on which the two radii are exactly parallel. On overlapping the two discs they form only one, but the diameter appears broken; the two radii seem to form an obtuse angle. If we present to the right eye the figure which was intended for the left eye, the angle seems turned in the opposite direction.

It is probable that these phenomena are due to the more important part played by the downward look in everyday life: we look downwards when reading, and when walking the look most frequently follows the ground, etc. By repeating the experiment of *Meissner,* we will find that the two images appear

parallel if we bring the lower extremity of the plumb-line towards the observer, until, in relation to the line of the look, it has almost the inclination which a book has when we hold it in the ordinary position of reading. If we draw a straight line on a sheet of paper placed on a table so that this line is in the median plane of the observer, we see, on placing ourselves in the position which we ordinarily assume in order to read or write, making the visual lines parallel, that the two images of the line appear parallel. Glancing at figure 181, in which the eyes are shown projected on the table, it is easy to see that the extremity A of the line which is nearest the observer forms its image on more peripheral parts of the retina than the extremity B. The two meridians of the retinæ which receive the images, converge therefore downwards, since the extremity A forms its image higher and more towards the periphery than the extremity B. We have formed our judgment according to this experiment, and when, under other circumstances, a line comes to form its image upon this meridian, we consider it as situated in the median plane. According to *Javal,* the experiments establishing binocular vision in persons affected with strabismus, confirm absolutely the preceding explanations.

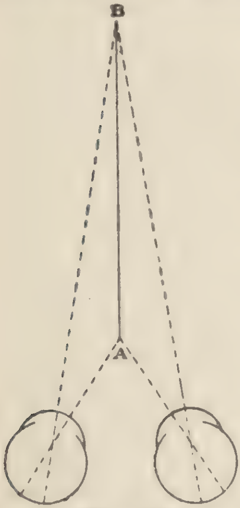


Fig. 181.

One can understand how these methods may be used, if not to directly verify the law of *Listing,* at least to compare the position of the two eyes. Working in the primary position, and with the two visual lines parallel, *Volkmann* found that it was necessary to give to the radii of his discs directions converging about two degrees downwards, in order that they would appear to form an unbroken line. Leaving the visual lines parallel, he found the same angle for all secondary directions, and the law of *Listing* was thus verified. It is otherwise when we converge. After having placed the eyes in the primary position, *Volkmann*

converged for a point situated at 30 cm. in the same horizontal plane. Since, under these circumstances, the eyes pass from the primary position to an internal position, the law of *Listing* would have demanded that the directions of the two radii would continue to form an angle of two degrees; but *Volkmann* found that it was necessary to increase their inclination to four degrees, in order that the resulting line would be seen unbroken. Converging, each eye had, therefore, made a rotary movement of one degree, which it would not have made by taking the same position, if the visual lines were parallel. The eyes do not, therefore, follow exactly the law of *Listing* when the visual lines are not parallel.

The following experiment is very easy to perform. We place two candles one meter from each other, and we observe them at one or two meters distance, taking care to put the eye nearly in the primary position. We then try to converge as if to fuse the two candles. We will then observe that they appear slightly inclined towards each other; the nearer to each other we bring the candles, the greater the inclination; the angle between the two candles may reach 15° or more. The image of the left eye is inclined, the upper extremity to the right and *vice versa*. *Hering,* and later *Landolt,* have made exact measurements of these deviations from the law of *Listing.*

119. Historical.— The question of knowing whether the eye performs rotary movements around the visual line has been much disputed. *Hueck* thought that he observed that the eye undergoes a rotation in a reverse direction when the head is leant towards the shoulder so that the meridian of the retina, which is vertical in the ordinary circumstances of life, remains vertical. He attributed this rotation to the contraction of the oblique muscles, and his ideas were shared by all scientists until *Ruete* demonstrated the error of *Hueck* by means of the examination with the after images, and gave a correct explanation of the action of the oblique muscles. *Donders* took up the question, and enunciated a law which bears his name, according to which the position of the after image is always

the same for the same direction of the eye; but the question was stated clearly only by the enunciation of the law of *Listing*, which is found for the first time in the treatise of *Ruete* of 1853. *Listing* did not publish it himself. *Meissner* was the first who verified this law by experiments.

After the experiments of *Ruete* and *Donders* everybody supposed the rotary movements of *Hueck* did not exist, when *Javal* demonstrated that the eye performs, nevertheless, a very slight rotation in this direction. He had observed, indeed, that when he leant his head to the right or to the left the direction of the axis of his cylindrical glasses no longer coincided with that of his astigmatism. This is, perhaps, the most exact test to see whether glasses are properly placed. *Helmholtz* verified the fact by placing on a level with his eyes a small colored band on a frame fixed on his planchette. By leaning the head with the planchette, the secondary image turned a little in the opposite direction, so as no longer to coincide with the ribbon.

Bibliography.—*Œuvres de Young*, edited by Tscherning, p. 145.—Hueck. *Die Achsendrehung des Auges.* Dorpat, 1838.—Donders (F. C.). *Holländische Beiträge*, 1848.—Ruete. *Lehrbuch der Ophthalmalogie*, 1853.—Fick (A.). *Die Bewegungen des menschlichen Augapfels.* Zeitschrift für rat. Medizin, IV, 1854.—Meissner (G.). *Die Bewegungen des Auges. Arch. f. Ophth.*, II, 1, 1855.—v. Helmholtz. *Ueber die normalen Bewegungen des menschlichen Auges. Arch. f. Ophth.*, IX, 2, 1863.—Volkmann (A. W.). *Physiologische Unersuchungen im Gebiete der Optik.* II. Leipzig, 1864.—Donders and Doyer in Donders. *Anomalies of Refraction of the Eye.* London, 1864, p. 180.—Javal (E.) in de Wecker. *Traité des maladies des yeux.* I, p. 815. Paris, 1866.—Tscherning (M.). *La loi de Listing.* Paris, 1887.

CHAPTER XX

THE OCULAR MOVEMENTS

120. Jerking Movements of the Eyes.—It seems as if the eye should be kept motionless in order to obtain an impression, at least an impression which can be perceived with some distinctness. If, in a railroad train which is going quite fast, we fix a point on the window, the landscape appears confused, the images of its different parts succeeding one another too quickly on the retinæ to be perceived distinctly. Observing the eyes of any one who is looking at the landscape, we see that they move by jerks. The eyes of the person observed make alternately a rapid movement in the direction of the train to catch the object, and a slower movement in the opposite direction to keep the image of the object on the *fovea*. Then they again make a rapid movement with the train to catch a new object, and so forth.

The eye cannot fix the same point for even a little while, without the formation of after images which annoy the vision, and without the phenomenon of *Troxler* interfering. The eyes are, therefore, in perpetual motion which is made by jerks: they fix a point, make a movement, fix another point, and so forth. While reading, the eyes move also by jerks, four or five for each line of an ordinary book. *Lamare* constructed a small instrument, formed by a point which is supported on the eye across the upper eyelid, and which is fastened to the ears of the observer by rubber tubes. With this instrument each movement of the eye causes a sound to be heard. We hear four or five slight sounds during the reading of one line, and a louder sound when we begin to read a new line.

121. Relative Movements of the two Eyes.—The relative movements of the two eyes are governed by the necessity of seeing the object single. It is necessary for this purpose that an image

of the object fixed be formed on each fovea. When, after having looked at an object at a certain distance, we look at another situated at the same distance, the two eyes make *associated* movements: both turn to the right, or both to the left, upwards or downwards, etc., and one as much as the other. If the objects are both in the median plane, but at different distances, it is necessary, in order to bring the look from the more distant to the nearer, that the eyes make a movement of convergence: both turn inwards to the same extent; finally, if the two objects are in different directions, the second nearer than the first, the eyes perform a combination of an associated movement and a movement of convergence.—If the second object is situated farther away than the first, the eyes make a movement of divergence (negative convergence).

It is impossible to cause a movement to be made with one eye without the other moving also, or at least without its having a tendency to move. A very simple experiment would seem to indicate the contrary. Suppose that the two eyes fix a point *a*, and that we place in the visual line of the right eye an object *b*. If we ask the observed person to fix *b*, the left eye is directed towards this point, while the right eye remains motionless. But, if we observe closely, we shall see that this eye makes really two slight changes of position, for instead of receiving no innervation, as one would think, its muscles receive two, one which would cause it to make an associated movement (to the right), and another which would cause it to make a movement of convergence (to the left); the results of these two innervations neutralize so that the eye remains motionless. It was *Hering* who described this experiment, which is of great importance for the understanding of the relation between the movements of the two eyes.

The two kinds of movements of which we have spoken are the only ones which the eyes have usually to make in the interest of fusion, and they are the only ones which they *can* make. It is possible, however, to make them diverge a little.—I mean absolute divergence and not relative divergence, which is only

a less degree of convergence.—We can make this divergence necessary for fusion by placing before one eye a prism with its apex turned outwards; but the angle of the prism which the eyes can thus overcome does not much exceed fixed degrees. We are unable to raise the look of one eye while leaving the other motionless; but by placing before one eye a very weak prism, apex upwards, this eye deviates a little, however, in the interest of fusion. The prism which we may thus overcome generally does not exceed two or three degrees.

These peculiarities of the ocular movements are evidently not due to the muscular apparatus. There is, indeed, nothing to prevent the right eye from making a movement to the right, but it cannot make it while the left eye makes a movement to the left. If we cannot perform two movements at once, this is due to the fact that we cannot give the necessary innervation for this movement. And we cannot give this innervation because we are not accustomed to give it, since, far from being useful, it would be harmful, on account of the diplopia to which it would necessarily give rise.—The impulse which guides the ocular movements is, up to a certain point, analogous to that which makes us keep our eyes open and the head erect, with this difference, however, that the innervation which guides the movement of the eyes is much more rigorous; we can lower the head or close the eyes if we desire to do so, but we cannot put the eyes in divergence. The innervation in question disappears during sleep. When struggling against sleep, we observe diplopia, and the two images affect relative positions which they never have in a state of wakefulness. The homonymous images whch we obtain by squinting voluntarily are always parallel, if I except the phenomena mentioned in the preceding chapter, and they are at the same height (if the head be kept erect). The images which we obtain when sleep comes upon us have, on the contrary, wholly irregular positions: sometimes one is higher than the other, sometimes they undergo rotations, etc. At the same time the eyelids have a tendency to close and the head to fall.

122. Measurement of Convergence.— This measurement is made preferably with the rotary prism of *Crétès*. As we know, this instrument is composed of two superimposed prisms of the same strength. A special mechanism allows them to be turned in opposite directions. When the apices have the same direction, the effect is double that of each of the prisms. If we cause them to rotate the deviation always takes place in the same direction, but it gradually diminishes and disappears when the apices are directed in different directions. The instrument replaces, therefore, a whole series of prisms of different strength.

We place the prism with the apex outwards while the patient looks at a distant flame, and we increase the strength of the prism until the subject sees two images of the flame. We thus find *abduction;* for healthy eyes, it is five to seven degrees of prism. We then turn the prism apex inwards and increase its strength until diplopia is produced. *Adduction* is much stronger than abduction; it may reach 20 or 30 degrees of prism, or more. We can also measure adduction and abduction for a nearer point. Adduction often exceeds the maximum value of the prism of *Crétès,* and on the other hand, it quite frequently happens that it is greater than we find it at that moment, because the observed person does not do his best to fuse the images. It would also be better to measure the adduction simply by trying how near we could approach an object without its appearing double (ophthalmodynamometer of *Landolt*).—We sometimes meet rare cases of *defect of convergence,* where the adduction is greatly diminished, while the abduction is normal.—In other cases both are diminished: the patient can fuse well two images which are formed on the two maculæ, but he experiences no *need of fusion;* even when the double images are very near each other, the eyes do not make the slight motion necessary to fuse them.

We have seen (page 13) that the deviation produced by a prism corresponds nearly to half its angle. If we can overcome a prism of six degrees, apex outwards, it is equivalent to saying that we can make the visual line diverge three degrees. This manner of indicating the degree of deviation is the simplest, and

that which is most frequently used. It has been attempted to introduce another notation first described by *Javal,* and afterwards adopted by *Nagel.* This author names *meter angle* the deviation which one of the visual lines undergoes when, after having fixed a point at infinity, we look at a point situated at one meter distance on the visual line of the other eye. ω (fig. 182) is, therefore, a meter angle, if A is situated at a distance of one meter; two meter angles, if A is at 50 centimeters, and so forth. The system was invented to measure the convergence in a manner analogous to the measurement in dioptries which we use for refraction (accommodation). The meter angle corresponds to about three degrees and a half.—This system seems to offer scarcely any advantages, and it has this quite serious disadvantage, that the value of a meter angle is not the same for different persons. It varies with the *base* line.

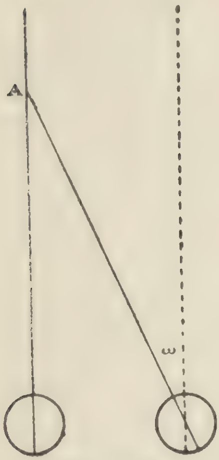


Fig. 182.

We call by this name the distance between the centers of rotation of the two eyes; it varies between 66 mm. and 58 mm., or still less. We can measure it by sighting a distant object, a lightning rod for example, along the surface of a planchette held horizontally. We close one eye and fix a needle in the planchette, so that it may appear to coincide with the lightning rod. The needle must not be placed too near the eye in order that its images may not be too diffuse. Then we repeat the experiment with the other eye without displacing the head; opening the two eyes, we should see the two needles blended into one, which coincides with the lightning rod. The distance between the needles is equal to the base line.—We find also very great variations, especially if we examine children, whose base line is manifestly very short.

Now it is clear that the deviation which the eye must undergo, in order to pass from infinity to one meter distance, is so much

more considerable in proportion as the base line is greater.—A meter angle corresponds to $3°40'$ for a person who has a base line of 64 mm., to $3°20'$ if the base line is 58 mm. To do well, therefore, it would be necessary each time we measure the convergence in meter angles, to tell also the length of the base line.

Prentice proposed to number prisms according to the linear deviation which they produce at a given distance, observing that at a distance of one meter the deviation produced by a prism of one degree is about one centimeter.

123. Relations between Accommodation and Convergence.—In the interest of single and distinct vision, it is necessary that there be formed on each *fovea* a distinct image of the object fixed. In order that the images be formed on the two foveas, it is necessary that the individual make his eyes converge towards the observed object, and in order that the images be distinct, it is necessary that each accommodate exactly for the object. There is thus formed a relation between accommodation and convergence, so that we cannot easily converge towards an object without also accommodating for this object. The rule, however, is not absolute; we can, if it is necessary for distinctness of vision, change within certain limits the degree of accommodation without changing the degree of convergence. This play of the accommodation, which is possible while the convergence remains the same, has the name *relative amplitude of accommodation* (*Donders*). We can measure this amplitude by placing convex and concave glasses before the eyes until the object appears double or diffuse.

Bibliography.—Javal (E.). In de Wecker. *Traité des maladies des yeux.* Paris, 1866.—Donders. *Anomalies of the Refraction of the Eye.* London, 1864.—Nagel (A.). *Ueber die Beziehungen dioptrischer Werthe und der Beträge symmetrischer Convergenzbewegungen nach metrischen Einheiten. Mittheilungen aus der ophtalmiatrischen Klinik in Tübingen.* Tübingen, 1880. Lamare. *Les mouvements des yeux dans la lecture. Bull. de la Soc. fr. d'opht.* 1882, p. 354.—Prentice (Ch. F.). *Ein metrisches System zur Bezeichnung u. Bestimmung v. Prismen. Archiv. f. Augenheilk.* XXII, p. 215.

CHAPTER XXI

THE PROJECTION OF VISUAL IMPRESSIONS

124. Projection Outwards in Uniocular Vision.—In order to be able to form a correct idea of the position of an exterior object, it is necessary to be informed as to the direction and distance of this object. Judgment of the direction is formed as well, or better, with a single eye; the superiority of binocular vision is apparent in the judgment of distance, but at the same time, in the matter of direction, it causes certain illusions from which persons blind of one eye are exempt. We shall first discuss the vision of these latter.

General Law of Projection.—An impression of any point of the retina is projected outwards into the visual field, following the *line of direction;* that is to say, following a straight line passing through the retinal point and the nodal point of the eye. We have seen that inversely, an exterior point for which the eye is focused forms its image at the point of intersection of the line of direction with the retina. As long as there is question only of objects seen distinctly, the law of projection is equivalent to saying that we see exterior objects in the direction in which they really are. The law of projection does not apply merely to the ordinary phenomena of vision: all the retinal impressions, the phosphenes, after images, entoptic phenomena, circles of diffusion, etc., are projected according to this law, which is entirely general. As exceptions we can cite only the deformities of objects seen indirectly, which seem to show that the law is not followed very exactly for very peripheral parts of the retina, and perhaps for some of the illusions which I shall mention later.

125. Projection of the Visual Field.— The law which we have just announced regulates the manner in which we localize objects in the visual field, but it does not regulate the projection of the visual field in its entirety. The latter depends on the manner in which we judge the position of the eye, or rather the direction of the visual line. If in uniocular vision, we judge cor-

rectly the direction of the visual line, the entire visual field is projected in a correct manner. We shall, therefore, proceed to discuss the means by which we form this judgment.

Supposing that we fix a point A, and that we desire to fix another point B. As long as we fix A, B is seen in indirect vision, and the distance between the images enables us to judge of the degree of innervation necessary to bring the look towards B; generally this judgment is quite exact so that we bring the look towards B almost without hesitation. From innervation results the contraction of the muscles, the change of position of the eye and the change of the retinal image until B forms its image on the *fovea*.—One might think that the sensation of the more or less considerable contraction of the muscles, the gliding of the eye between the lids, etc., could furnish us with information on the direction of the visual line, but this is not so; we judge this direction solely by the degree of innervation which we have used to bring the look into this direction. This fact is well established by the observation of patients affected with ocular paralysis. If, for example, we tell a patient affected with paralysis of the right external rectus to close his left eye and look to the right, he furnishes the innervation necessary; the eye remains motionless on account of the paralysis, but the patient thinks he has moved it to the right, so that there results a false projection; if we tell the patient to move his finger rapidly towards an object situated to the right, not having time to guide himself by the sight of the finger, he constantly moves it too far to the right. A healthy person can make the experiment by looking to one side, while he exerts a traction in the opposite direction on a fold of the skin, near the external canthus. The traction is communicated by the conjunctiva to the globe, and on account of the resistance which it exerts, one is obliged to use a stronger innervation to bring the look to the opposite side; we conclude from this that the look is carried farther in this direction than it really is, which causes projection of the visual field in a false manner.

Judgment of the degree of innervation used is very exact, because it is always corrected by the result obtained, as the fol-

lowing experiment shows. One looks directly in front after having put a prism of ten degrees, apex to the left, before each eye. Seen through the prisms, an object situated at ten degrees to the right, appears five degrees from the visual line, and we need only an innervation corresponding to five degrees to fix it; we think, therefore, that it is situated at five degrees to the right, and, if we wish to grasp it, we do not bring the hand far enough to the right. But it suffices to repeat the experiment only a few times in order to be no longer deceived: we learn very quickly to reckon with prisms. If then we repeat the experiment after having removed them, we bring the hand too far to the right.

When we judge correctly the direction of the visual line there is in monocular vision no possible illusion as to the direction in which objects are. In mathematics we often determine the position of a point by means of what are called polar coördinates. Being given a fixed point, named *center of coördinates,* the position of any other point is determined by the direction and length of the *radius vector,* that is to say, of the line which joins the two points. In uniocular vision, the center of coördinates is represented by the eye, or, more exactly, by its nodal point; the law of projection gives the direction of the radius vector. To know the exact position of the exterior point, there is wanting, therefore, only the length of the radius vector, but this the eye does not give, at least not in a direct manner.

It is easy, indeed, to convince oneself that while the eye informs us very exactly on the direction in which the light comes, it gives us no information as to the distance whence it comes. The information which the greater or less degree of accommodation used could furnish is too indefinite.—In the tenth chapter I laid stress on the importance which the study of the form under which a distant luminous point is seen may have in the matter of exact knowledge on the optics of the eye. One might think that one can replace the distant luminous point by a near luminous point placed at the focus of a strong lens. If the eye would inform us on the distance whence the light comes to it, the result of the two experiments ought to be the same,

since the rays reaching the eye are parallel in both cases. But this is not so. Other information tells us, in fact, that, in the latter case, the luminous point is very near, which makes us see the figure of diffusion extremely small, and makes this form of experiment not to be recommended.—We know also that the after images appear to us large or small, according as we project them on a distant or near surface, which shows clearly that the eye does not accord to them a real distance. If we do not present to them a surface on which they can be projected, for example by closing the eyes, they generally seem to have the same apparent size as the object of which they are the image; we accord to them the distance of this object, a distance which is not told by a direct sensation, but which we judge by an unconscious reasoning, as we shall see in the following chapter.

126. Projection in Binocular Vision.—*The impressions of the two maculæ are projected towards the same place.* When the eyes perform their functions correctly, both of them always fix the same object, so that under these circumstances the fact stated is not surprising. But it is the same when they do not fix the same object, as is evident among others from stereoscopic experiments. The following experiment seems to me to demonstrate this fact in a very striking manner, but it is necessary to be able to squint in order to repeat it. It is quite easy to learn to squint inwards; in order to squint outwards we take hold of a fold of the skin near the outer canthus of one eye, while we look towards the opposite side.—To perform the experiment, we close one eye and look at the flame with the other, so as to produce an after image. We then open the closed eye and select a point which we fix *with this eye* while we are endeavoring to squint. We then see the after image place itself on the point of fixation, although the visual line of the eye to which it belongs is not at all directed towards this point. We can squint more or less considerably, placing the visual line in divergence or in convergence: as long as the other eye fixes the point of fixation, the after image is located there also.

Physiologic Binocular Diplopia.—Let A, figure 183, be an object which both eyes fix, B another nearer object. If we close the right eye, the point B is seen five degrees to the right of A; if we close the left eye, it is seen five degrees to the left of A. Opening both eyes, A is seen single at the place where it really is; we see two images of B, one five degrees to the left, the other five degrees to the right of A.—We therefore see B in double crossed images; if we fix B, A is presented in double homonymous images.—We can perform the experiment with two candles, and, if necessary, we can make the diplopia more striking by placing a red glass in front of one eye.

This singular phenomenon, which had already been described by *Alhazen*, is known as *physiologic binocular diplopia.*

Center of Projections.—We observe that the correct information which the eyes furnish to us gives rise to a false interpretation, for it is evident that, when an object is seen double, there is at least one of the images which does not coincide with the object. When we close one eye, the corresponding image disappears, while the other image does not change position. The false judgment must, therefore, persist also in this case, at least for one of the eyes. The sight of normal persons does not, therefore, necessarily become similar to that of a one-eyed person.

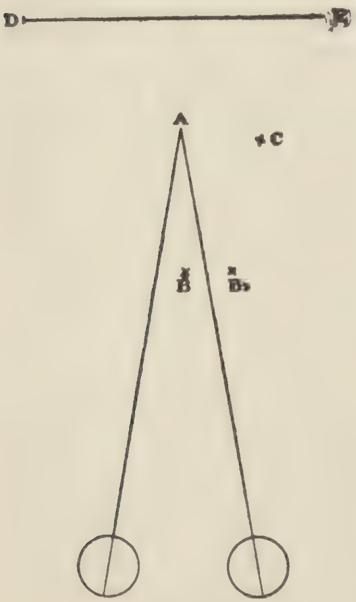


Fig. 183.

The physiologic diplopia is due to the fact that we do not take into consideration the different position of the two eyes; without a special examination we cannot tell whether an image belongs to one eye or the other. We refer every

visual impression, from whatever eye it may come, to a common and single center, which, in my case, coincides pretty exactly with the right eye. Recalling the mathematical terms which we have used in the preceding chapter, we may say that it is the center of the co-ordinates the position of which we judge imperfectly. If we took into account the different position of the two eyes, we would have two centers of co-ordinates, and the idea of the direction of the object would suffice to fully determine its position. In the experiment (fig. 183) we would thus reason as follows: Since we see with the right eye an object five degrees to the left of A, with the left eye the same object five degrees to the right of A, the object must be in the middle plane and nearer than A; we would therefore see B single and in its right place. Instead of this we refer the impressions, as in uniocular vision, to a single center, and we inform ourselves that the object must be double, since it is seen at once to the right and the left.

Directing Eye. (1)—In my case this center of coördinates coincides almost exactly with the right eye, probably because, having used it so much separately, I have acquired the faculty of judging exactly with this eye the position of exterior objects, or, in other words, because there is developed a kind of uniocular vision in addition to binocular vision. I must add, however, that this condition was not developed as a result of my labors on physiologic optics, because the phenomena were the same when, twelve years ago, I began to devote my attention to this subject. According to *Hering* the center is often at an equal distance between the two eyes, and this would, in fact, be the true type of binocular vision, in which neither of the eyes plays a dominant part.—The reasons why I say that in my case the center of projections coincides with the right eye, are as follows:

(1) According to a communication from *Javal,* the binocular vision of *Vallée* was like mine. He described this condition as general (in a communication to the Academy of Sciences, about 1830), and gave the name *directing eye* to the eye which controls projection outwards. *H. Kaiser* has also described the same condiiton for his eyes.

1° When on looking at a distant object I see a near object in double crossed images, and when I try to touch this object by a quick motion, I grasp it correctly if I sight the image with the right eye, while I bring the hand far from the object if I sight the image with the left eye. It is the same if I close one eye. With the right eye I judge accurately the position of objects seen indirectly, as a one-eyed person would do; with the left eye I judge falsely. Thus, in the experiment figure 183, closing the right eye, I see B five degrees to the right of A, as I ought to, but I refer the impression to my right eye, and, thinking that the object B is five degrees to the right of the visual line of my right eye, in order to reach it I bring my hand toward B_1.—I have also noticed, especially when I observe the double images of near objects, accidentally and without trying to, that one of them, that of the right eye, presents a more material appearance, while the other rather resembles a kind of shadow; *Dr. Knapp, Jr.,* made the same remark to me. It must be noted that my eyes are practically equal, as to acuity and refraction.

2° I fix a mark P (fig. 184), not too bright, placed on a dark and uniform background. Interposing a stick between my eyes and the background, on the visual line of the right eye, I see it in double images; the image of the right eye (*d*) coincides with the mark of fixation, while the image of the left eye is seen more to the right (*g*) (fig. 184 A). If now I fix the stick, it is the image *g* of the left eye which is brought towards that of the right eye, *d,* in order to coincide with it, while the latter remains motionless. One might think that this is due to the fact that I placed the stick on the visual line of the right eye, but this is not so; if I place the stick on the visual line of the left eye (fig. 184, B) so that the image of the right eye *d* is seen to the left, it is still the latter which remains motionless, while that of the left eye makes a great movement to join itself to it when I fix the stick.—This apparent movement exists also when I close the right eye, although, under these circumstances, the left eye does not make any movement. Under this latter form the experiment was described by *Hering.*

3° This author furthermore described the following experiment: we fix binocularly an object placed at some distance in the median plane, and we try, by a quick movement, to place a stick quite near the face in the direction in which we see the object; it is better to conceal the movement of the hand with a screen. Making this experiment, I bring the stick pretty exactly on the visual line of the right eye. The experiment is easy to repeat even with persons who are not accustomed to study such

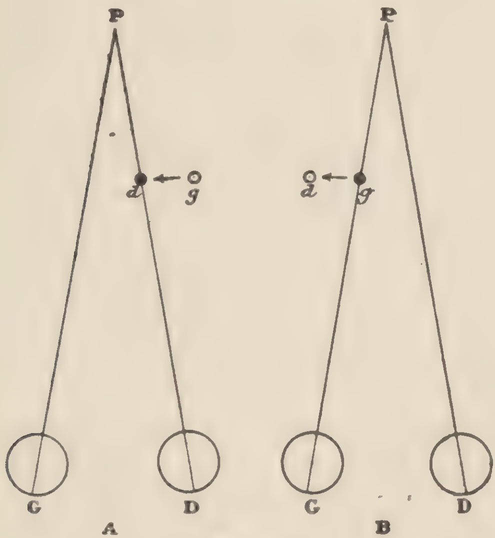


Fig. 184.

questions, and we can control by placing ourselves in front of the observed person and sighting with one eye along the mark of fixation and the space between the eye-brows (*glabella*) of the observed person. I have observed several persons in this way. Most of them show a marked tendency to prefer one or other eye, which seems to indicate a tendency to a development of a uniocular vision in addition to the binocular vision like that which I have described for my eyes. Persons enjoying pure binocular vision must place the stick in the median plane; as the center of projection does not coincide with either

of the eyes, these people cannot project correctly objects seen indirectly. This type of vision, therefore, seems inferior to the other, as far as orientation is concerned.

Horopter.—All the points outside the point fixed are not seen double; the point C (fig. 183), for example, is seen ten degrees to the right of A, as well with the right eye as with the left

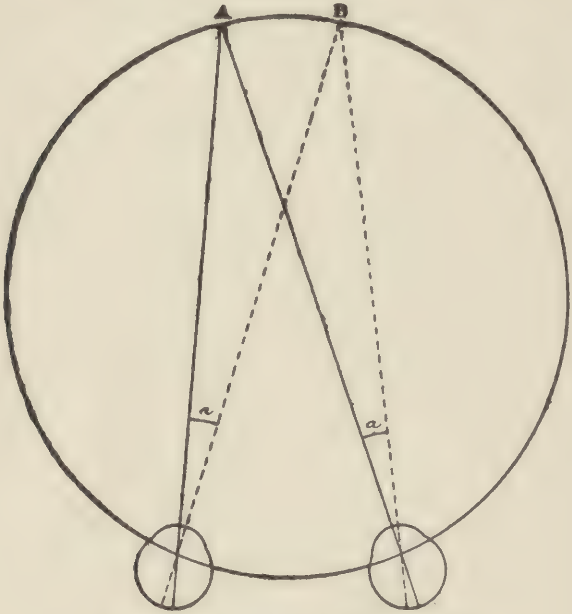


Fig. 185.—Horopter of *Johannes Muller.*

eye; it is therefore seen single.—The entirety of the points seen single while we fix a given point, is called *horopter.*—The study of the horopter is quite a complicated mathematical problem, and without much interest, since the diplopia is only very slightly indicated when the object is a little distant from the point of fixation. It may be solved when we know the position of the corresponding points (see the following chapter) and the law which regulates the position of the eyes (law of *Listing*). When the point of fixation is in the plane which contains the primary position of the visual lines, we see single all the points which are

on a circle passing through the point of fixation and the nodal points (*horopter* of *Johannes Müller*, fig. 185). It is easy to see that on fixing A, B is seen single, because the two angles designated by *a* are equal, since both correspond to the arc AB. —If we fix a point on the floor, situated in the median plane, the horopter corresponds almost to the plane of the floor.

Suppression of Double Images.—As one sees some exterior objects double, and some single, one might think that it would result in great confusion. It does not: most people have never observed double physiologic images before making the experiment described above. Under ordinary circumstances the attention is always brought to bear on the object fixed, and the look never remains for any length of time on the same object, so that we have not much time to perceive double images. It must also be observed that the objects, not fixed, form their images on the peripheral parts of the retina, where the perception is less distinct than at the macula. It is scarcely possible to suppose a serviceable binocular vision if the entire retina had an acuity like that of the *fovea*. But we also make important use of the phenomenon known under the name of neutralization of images, and which has been given special prominence by the works of *Javal* on the vision of persons affected with strabismus (see chapter XXIII).

In addition to the fact that most of the time an object seems to be at two different places, binocular vision gives rise to yet another contradiction. Making the experiment with the two candles before the screen DE (fig. 183), we have seen that the right eye sees the candle B at five degrees to the left of A; in this direction the left eye sees a part of the screen; and as we do not take into consideration the different position of the two eyes, but refer our impressions to a common center, the result is that we seem to see two objects in the same direction. Interposing a stick between the eyes and a book (controlled reading of *Javal*) we can read without interruption only when both eyes are open; if we close one eye, the stick covers some of the characters. We here meet the same contradiction; we see the stick

in the same direction as the characters which it conceals, and as, on the other hand, we know that it is nearer than the book it appears transparent. But, in cases in which such an interpretation is not possible, for example when we present to both eyes wholly different images, in a stereoscope, we observe what is called *antagonism of the visual fields.* It is sometimes the images of one eye that predominate, sometimes those of the other, and as long as we see in a part of the visual field images of one eye, those of the other are completely suppressed.

It seems that this suppression of the images of one eye plays a great part in binocular vision, and that it is this which generally causes us not to observe double physiologic images.—It is not easy to know which of the two images is suppressed, for as soon as we pay attention to this question both appear. Generally it is the more eccentric image, or, in other cases, the image which, on account of the perspective, occupies the smallest retinal surface (*Javal*) which disappears. But, in most persons, there seems, as I have already stated, to be developed a certain superiority of the eye which is most frequently used separately, and then it is always the image of the other eye which is suppressed.

Bibliography.—Muller (Johannes). *Beiträge zur vergleichenden Physiologie des Gesichtssinnes.* Leipzig, 1826.—Hering (E.). *Beiträge zur Physiologie.* Leipzig, 1861.—Kaiser (H.). *Compendium der physiologischen Optik.* Wiesbaden, 1872, p. 298.

CHAPTER XXII

MONOCULAR PERCEPTION OF DEPTH

127. Influence of Accommodation.— I have already said that the eye gives us no direct information as to the distance from which light comes to it. We might think that the degree of accommodation used in order to see the object distinctly would inform us as to its distance. When the eye is accommodated for distant objects, near objects do not appear distinct, and an experienced observer might use this circumstance to judge of the distance of an object. *Young* said that painters must take care to show near objects vaguely under penalty of obtaining a hard and disagreeable effect. But the importance of accommodation for the judgment of distances is but small, because, generally, we are dealing with such long distances that the difference of accommodation is insignificant. For all distances exceeding one meter, the variation of accommodation does not reach one dioptry.

128. Indirect Judgment of Distance.— In the absence of direct information, a whole series of circumstances enable us to judge of the distance of an object, generally by an *unconscious judgment*.

a. The knowledge of the nature of objects often furnishes us with a means of knowing their distances. Thus, if we know the size of an object, we can judge its distance from its angular size. It is the size of man especially which enables us to make this judgment. Generally we judge directly of distance. When we see a man very far off, he does not appear to us small, because we know what size he ought to be, but we conclude that he must be very far away, since the angular size is small, and this, without this latter fact directly striking our consciousness. This observation is quite characteristic of the manner in which

unconscious judgments are formed and it must be noted that this way of judging is something to be learned. I recall very well that the first time I saw a man climb the mast of a ship, he appeared to me like a doll, and *Helmholtz* reports a similar observation.—If we look at distant objects through a telescope they are enlarged; but as long as we have to do only with objects of known size, such as men, houses, etc., they seem to preserve their natural size, but appear near. We must open the other eye to convince ourselves that they are really enlarged.

b. A means which is often used to judge whether one object is nearer than another, is to observe whether it conceals a part of the other. If one hill conceals the lower part of another hill it must be nearer.

c. If we are acquainted with the object at which we are looking, or if there is a certain regularity, we easily come to know what part is nearest. On the photograph of a house, we easily judge the distance at which the different parts ought to be, while photographs of rocks, landscapes, etc., are frequently more difficult to interpret.

d. The shadows thrown are often important for the judgment of distance. If a surface is illuminated, the luminous source must be in front of it, and if an object casts a shadow on this surface, it must be nearer the observer than the surface. It is for this reason that we obtain a much better idea of the reality by adding shading to a drawing.

e. Finally, *aerial perspective* sometimes influences the idea which we form of distance. We comprise under this term the darkening and change of color which distant objects undergo on account of the incomplete transparency of the layers of air which separate them from the observer. The vapors of water which are in the atmosphere reflect the blue rays, and allow the red rays to pass. Comparing the spectra of a blue sky and a cloudy

sky, *Lord Rayleigh* thus found that the brightness of the latter diminishes greatly towards the blue extremity. When the spectra had the same brightness in the red, the green of the cloudy sky was already less strong than that of the blue sky. It is for this reason that the setting sun appears red, and distant mountains blue. When there is much water vapor in the atmosphere, we see distant objects, such as forests and hills, more distant and consequently larger than they really are. In the mountains the air is, as a rule, very pure, which causes us to often judge the distance and height of the summits much smaller than they really are.

We know that the sun and moon appear larger when they are near the horizon, which is merely an illusion. If we measure their angular size, we find it exactly the same in both cases. Likewise, if we try to divide the distance between the zenith and the horizon into two equal parts, we are greatly deceived;

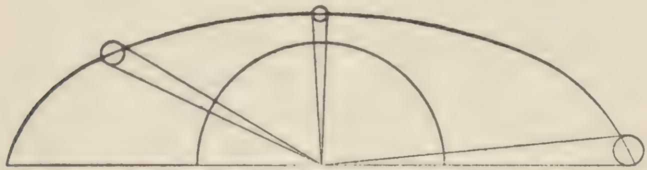


Fig. 186. After Young.—The curve indicates the apparent form of the sky. The sun, although seen under the same angle, seems of variable size.

the lower part is always too small. Since the moon, near the horizon, appears larger than near the zenith, although it has the same angular size, it is equivalent to saying that we judge it to be farther away. The illusion is due to the aerial perspective. The moon is seen through a much thicker layer of the terrestial atmosphere when it is near the horizon than when it is at the zenith. It seems, however, that the comparison with terrestial objects also plays a part in this judgment (fig. 186).

These different means enable us to judge more or less exactly of the distance of an object. They are especially useful to us when we have to do with long distances, on which the parallax, of which I am about to speak, cannot give any information.

129. Influence of the Parallax.— The idea which we obtain of the relief, by displacements of the head, is well known to all who use the ophthalmoscope. We thus obtain a very distinct idea of the depth of an excavation, etc.—We often use this means, without knowing it, to study an object difficult to interpret, and it is the principal means by which one-eyed people account for the relief. The observer often sees thus, without his perceiving that he does so, the relative movements of exterior objects, and he uses them to account for their position. If, for example, while the eye is displaced from *a* to *b* (fig. 187) the observer sees the object A displaced to the right relatively to the object B, A must be nearer than B; to draw this conclusion, we need not look during the displacement. If, after having observed the objects in the position *a*, we close the eye to open it again only in the position *b*, *we* observe, nevertheless, that A has changed place relatively to B, which suffices to judge of its distance.

The judgment is here based on the comparison of the successive retinal images; images change for each new position of the eye. But, as all comparison by memory is defective, we obtain a much more distinct idea of the difference between the images, and consequently of the relief, by comparing the images simultaneously with the two eyes, and it is for this reason that we always judge distances better with two eyes than with one. It is easy to convince ourselves that this is so by trying to reach a stick placed at some distance with the finger coming from the side. Looking with one eye only we are deceived much more frequently than when we open both eyes.

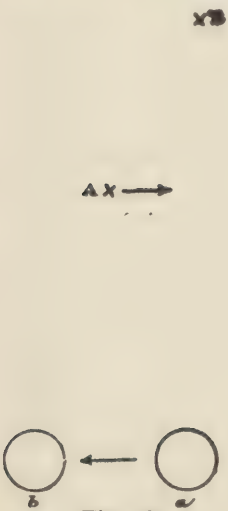


Fig. 187.

When we look with the two eyes, each eye receives a perspective image of the objects situated in front of us; as the two eyes are not at the same place, there result between the images differences which

are the more pronounced the smaller the distance of the object. If, on the contrary, we look at a plane image with both eyes, the retinal images are identical. This, therefore, is a sign by which the appearance of an object of three dimensions is distinguished from a plane image. It is only for near objects that this difference exists: if the objects are at a great distance, the retinal images are alike; thus a landscape presents almost the same appearance whether we close one eye or whether we open both.

Bibliography.—*Œuvres de Young,* edited by Tscherning, p. 244.

CHAPTER XXIII

BINOCULAR PERCEPTION OF DEPTH

130. Influence of Convergence.—The most important information on the distance of an object is furnished us by the degree of convergence which it is necessary to use to fix it binocularly. Just as for the judgment of the direction of the visual line in uniocular vision (see ch. XXI), it is the degree of innervation used which guides us, and not at all the sensation of the position of the eyes, which is always very vague. It is solely for *differences* of convergence that we have a very exact sensation; we can judge with very great exactness whether one object is nearer or farther away than another; the judgment of absolute distance is very uncertain.—When we fix a distant object, a near object appears in double crossed images. Although we may not often perceive these images, they give us, nevertheless, a vague idea of the distance of the object, for they suffice to give a pretty accurate impulse to convergence, since, guided by them, we converge for the object without much effort. But it is only after having accomplished convergence and having seen that the innervation given has attained its object, that we have an accurate idea of the distance. The difference between the two judgments is almost analogous to that which we find when we wish to measure the distance between two points. Suppose that we wish to measure this distance with a compass, provided with a scale graduated in millimeters, telling the distance between the two points. We can readily, at first sight, give to the compass approximately the aperture which is necessary, but we obtain a more exact and distinct idea of the distance when we make the measurement and see how much must be added to or taken away from the estimated distance.

131. The Stereoscope.—The advantage of binocular vision was made clear only by the invention of the stereoscope by *Wheat-*

stone (1833). With this instrument we obtain an impression of depth much superior to that which any other representation can give of it.

Each of the images of the stereoscopic representation is drawn in such a way as to form in the eye a retinal image like that which the object would form there. Distant objects are, therefore, represented by images which are identical, while the images of near objects are different.

Stereoscopic Parallax.—In order to account for the manner in which objects are represented on stereoscopic images, we may suppose two transparent plates (MM, fig. 188), placed in

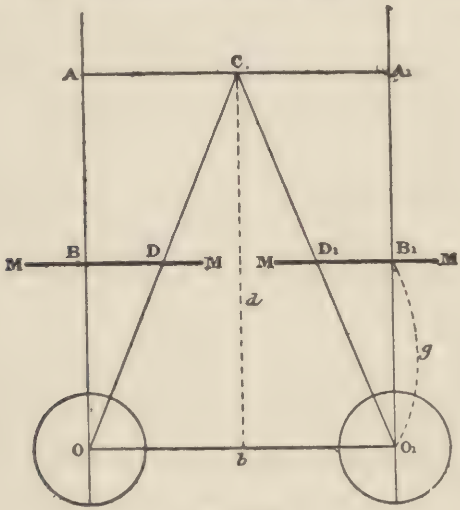


Fig. 188.

front of the eyes at the place which the stereoscopic image will occupy later. From all the exterior points we suppose straight lines directed towards the eyes. There start thus from each exterior point two of these lines, and the point at which each of these straight lines cuts the corresponding plate is the reproduction of the exterior point. If the latter is at infinity the two straight lines are parallel, and the distance BB_1, between the two points, is equal to the base line. If we place the two transparent stereoscopic figures one over the other, so that the

two reproductions of the same point situated at infinity overlap, we can make the reproductions of all the points situated at infinity coincide two by two.—If, on the contrary, the exterior point (C, fig. 188) is not at infinity, the distance between the two reproductions is less than that of the eyes. We designate the difference by the name *stereoscopic parallax.* The parallax of the point C is $BD+B_1D_1=E$. Designating the distance between the two eyes by *b*, that of the object from the eyes by $AO=d$, and the distance of the plate from the eyes by *g*, we have

$$\frac{b-E}{d-g} = \frac{b}{d} = \frac{E}{g} \text{ or } E = \frac{bg}{d}.$$

The parallax increases, therefore, with the distance between the two eyes, and it is the greater as the object is nearer the observer.

METHODS OF OBSERVING THE STEREOSCOPIC IMAGES.—*a.* Making the visual lines parallel, we can without further trouble blend the two images into one, which appears in relief. We then see three images, the middle one of which gives the relief; for each eye sees not only the image which is intended for it, and which is blended with that of the other eye, but also the image which is intended for the other eye; we can eliminate the two useless images by placing the hand as a partition between the eyes. It may be difficult to make the visual lines parallel while accommodating for a quite short distance, but if we succeed in doing so, the illusion is as perfect as with the stereoscope. Frequently we do not succeed with the ordinary stereoscopic images because, being intended for the stereoscope of *Brewster,* they are calculated for too long a base line, which obliges us to make the visual lines diverge in order to fuse them.

We can also look at the images by directing the right eye towards the image of the left, and *vice versa,* so that the visual lines intersect at a point situated in front of the image. It is then necessary to place on the left the image intended for the right eye, under penalty of seeing the relief reversed, if the supposed object lends itself to such an interpretation.—The fused

image appears diminished and situated in front of the plane of the drawing, at the point of intersection of the visual lines.

b. The stereoscope of *Wheatstone,* the first which was constructed, is composed of two plane mirrors (bd and bd_1), forming a right angle (fig. 189); the eye O_1 looks into the mirror on the right at the image of the drawing B_1 D_1, which it sees at ff_1; the eye O sees the image of BD at the same place; the two images are fused into a single one presenting relief. In order not to have the relief reversed or *pseudoscopic,* it is necessary to present to the left eye the image intended for the right eye, since the mirrors reverse the images.

c. The stereoscope most used is that of *Brewster:* each eye looks through a prism with convex surfaces, the apex of which

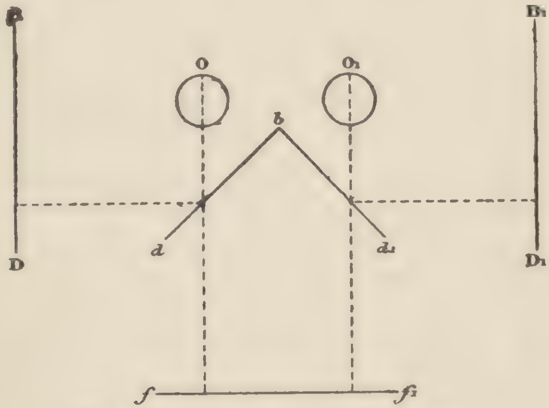


Fig. 189.—Stereoscope of Wheatstone.

is turned towards the nose. The glasses produce a certain magnification, and their prismatic effect renders it unnecessary to make the visual lines parallel.

We can replace the glasses of the stereoscope of *Brewster* by ordinary convex lenses, by decentering them; that is to say, by placing them so that the distance between the centers of the two glasses is greater than the distance between the eyes.

d. When the image represents an object which is symmetrical in relation to the median plane, the two drawings are symmetrical. We can, therefore, in this case obtain a stereoscopic effect by looking with one eye at an ordinary drawing, with the other at its image by reflection, since the reflection produces a symmetrical image of it. The most convenient way is to look through a prism with total reflection.

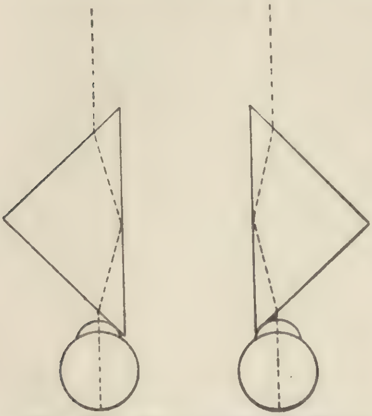


Fig. 190.—Pseudoscope of Wheatstone.

e. Placing a prism with total reflection in front of each eye, we obtain pseudoscopic relief when we look at any object, providing such an interpretation is possible. A cigar is thus presented as a hollow leaf of tobacco, etc. *Wheatstone* had constructed an instrument of this kind named *pseudoscope* (fig. 190).

f. The *telestereoscope* of *Helmholtz* is composed of four mirrors arranged as we see in figure 191. The rays *ab, a′b′*, coming from a landscape, are reflected by the large mirrors towards the small ones, and by the latter towards the eyes. We obtain the same effect as if the eyes A and B were in the position of their images (A_1 B_1) produced by the double reflection. We have seen that binocular relief is due to the distance which separates the two eyes. The greater this distance is the more pronounced is the relief. The instrument gives relief to objects which, under ordinary circumstances, are too distant to give this perception; at the same time it makes them appear nearer and smaller, almost as if we looked at a diminished model of them.

g. The iconoscope of *Javal* resembles somewhat an inverted telestereoscope, the eyes having taken the place of the object (a and a_1), and the object that of the eyes (in the direction of AB).

The instrument acts as if the eyes were very near each other, at c and c_1. Looking at objects through this instrument, the relief disappears: the object appears flat, as in a painting. On the contrary, if we observe an engraving through the instrument, it presents a more pronounced relief than under ordinary circumstances. For, the binocular vision then ceases to make us observe that the different parts of the image are in the same plane, which destroys the illusion. Looking through the iconoscope the relief is more marked than when simply closing one eye.

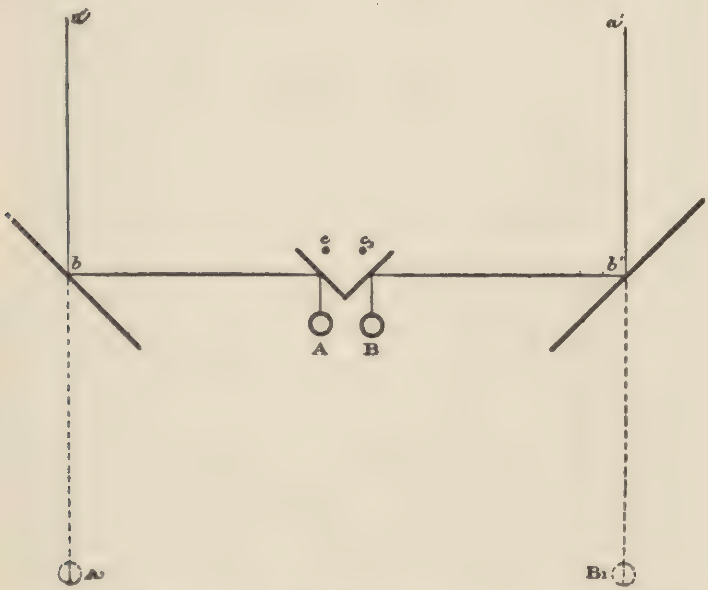


Fig. 191.—Telestereoscope of Helmholtz.

h. The binocular ophthalmoscope of *Giraud-Teulon* is analogous to the iconoscope. The mirrors are replaced by two glass rhombohedra, each of which covers half of the opening of the ophthalmoscope. As in the preceding case, the rays reach the eye after a double reflection on the small surfaces of the rhombohedron. The instrument acts as if the eyes were at cc_1 (fig. 192).

i. We draw the two figures, over each other, one with red lines, the other with blue lines. Looking through a red glass we do not see the red lines, and *vice versa.*—If we look at these *anaglyphs,* placing a red glass in front of one eye and a blue

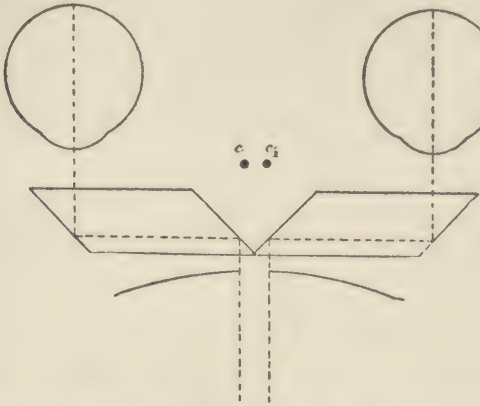


Fig. 192.—Binocular ophthalmoscope of Giraud-Teulon.

glass in front of the other, we obtain a stereoscopic effect. Changing the glasses the relief is reversed, if the nature of the object permits such an interpretation (*d' Alméida*).

132. The effect of the stereoscope is to give an idea of the third dimension, such as no other representation can give of it. Its use has become especially popular since stereoscopic photographs have been made, for though we can make stereoscopic *drawings* of stereometric figures, etc., it is impossible to make them of a landscape so that the reproduction may be exact. *Dove* used the stereoscope to see whether a bank note was false, by placing it in one of the fields and putting a genuine note in the other. If it was false he saw some of the letters leave the plane of the paper, for it is impossible to make an entirely exact counterfeit of an engraving, and the least difference in the distance of the letters produces relief.

STEREOSCOPIC LUSTRE.—Under ordinary circumstances there are usually formed only in one eye images of the same objects

as in the other; as long as we place in the stereoscope images of real objects only, we simply see the relief. I have already said that, in the case of the controlled reading of *Javal,* we see at the same place the stick and the letters which it should conceal. The observer gets over the difficulty by supposing the stick transparent. Another interpretation of the same kind is known as *stereoscopic lustre* (*Dove*). If we draw one of the stereoscopic figures with black lines on a white ground, the other with white lines on a black ground, we observe that the fused image presents a certain brightness, almost as if it was covered with a layer of plumbago. Replacing the black surfaces by colored surfaces, we sometimes obtain the metallic lustre.—Every bright body, in fact, sends back two kinds of light: regularly reflected white light and diffuse light which has the color of the body itself. When, in the stereoscope, we see at the same place white light and colored light, the contradiction is explained by supposing that the object we look at is bright.

ANTAGONISM OF THE VISUAL FIELDS.—When the images placed in the two fields are so different that they cannot be fused, as, for example, if we present to one eye horizontal lines and to the other vertical lines, we observe the phenomenon known as *antagonism of the visual fields:* it is sometimes one, sometimes the other field which predominates, and while one predominates the other is suppressed; we do not see it at all. It is not the field of the same eye which predominates everywhere; the common field is composed of parts belonging to either eye. When one of the fields has predominated at one place for some time, the appearance changes, the other field getting the upper hand. The change often takes place under an external influence; a winking of the eyelids or a change in the direction of the look sometimes suffices to bring it about. Furthermore, the phenomena vary much according to the objects.

If we present to each eye outline pictures which do not correspond to each other, drawn on a uniform ground, but different for both eyes, we observe that the ground of each field predominates near the picture which belongs to it. The following experiment demonstrates this fact in a quite striking manner.

We draw in one of the fields a large black vertical bar, in the other, another similar but horizontal bar: on blending the fields the bars form a cross (fig. 193), the middle of which, situated at the point where the two bars cross, is black; the parts next to the middle are whitish, because the outline picture makes the white ground predominate. The extremities of the arms appear, on the contrary, almost as black as the middle, in spite of the superimposing of the white on the other field.

In making this experiment, we experience a difficulty in fixing the images on each other: the vertical arm glides on the horizontal arm. This is due to the fact that there are no common vertical lines which can guide us for the degree of convergence. On account of their importance for convergence we designate the vertical lines as the *dominating outlines*. To prevent the two figures from gliding on each other, we place at the middle of each line a small white cross. The tendency to fuse these small crosses suffices to fix the vertical bar at the middle of the horizontal bar.

Fig. 193.—After *Helmholtz*.

When the two fields have not the same color, we generally observe antagonism of the visual fields. I have thus arranged the experiment with colored shadows (page 289) so as to have one of the shadows in each field of the stereoscope. On blending them it was sometimes one, sometimes the other color which predominated. I repeated the experiment with several of my pupils, none of whom succeeded in seeing the gray shadow.—There are authors, however, who claim to have obtained the color of the mixture; the phenomenon is then, perhaps, of the same order as stereoscopic lustre.

133. Identical Points of the Retinæ.—We say that one point of a retina is *corresponding* to, or *identical* with, a point of the other one, when the images of the same exterior point falling on these two retinal points are blended into a single image. If, in the second eye, the image is formed on any other point, it is not blended with that of the first eye: the point is seen double.

It is evident that the two *foveas* are corresponding points, since the object fixed is always single. To find the other identical points, *Johannes Müller* has given the following rule. We suppose the retina divided into quadrants by a horizontal meridian and a vertical meridian, both passing through the fovea. The position of each point is then determined, as on a terrestrial globe, by its longitude and latitude in relation to these two meridians. Two points having the same longitude and latitude are identical. The rule of *Müller* agrees with that which we have laid down in chapter XXI, according to which an object is seen single when the two eyes see it in the same direction in relation to the point fixed.

The researches of *Volkmann* have shown that the law of Müller is not wholly exact, and that it is necessary to replace the vertical meridians by *apparently vertical* meridians, which, for a person standing upright and looking towards the horizon, converge about two degrees in the downward direction, so as to almost meet at the ground (see page 356). We then suppose the retina divided by circles parallel to this meridian as well as to the horizontal meridian, and the law of Müller is applicable. —Placing in each field a really vertical line, these lines appear to converge upwards and must, consequently, cross if we try to blend them. In order that the experiment may succeed it is necessary, however, to arrange them so that one line may be white on a black ground, the other black on a white ground. Otherwise the lines are blended nevertheless.

Theories on the Nature of Identity.—The question of knowing why two points are corresponding while two others are not, has been much discussed. Most of the advocates of the *theory of identity* suppose that there exists an anatomical relation between the two corresponding points. They suppose that

the nerves conducting the impressions of two corresponding points unite, on their way to the chiasma, into one which conducts the impression to the brain. This idea was already expressed by *Galien,* and has been confirmed by *Newton, Wollaston* and others. The so-called *theory of projections* is expressed almost as we have described it in chapter XXI: a point on the left retina, situated 10 degrees to the left of the *fovea,* localizes its impression at 10 degrees to the right of the point of fixation; the point situated at 10 degrees to the left of the right *fovea* localizes its impression in the same direction; and as the two impressions are localized in the same direction, they are blended into one. The identity of the two *foveas* might be a result acquired by experience. This theory has been upheld by *Kepler, Porterfield* and, under an erroneous form, by *Giraud-Teulon.*

Immediately after the invention of the stereoscope and the studies of the production of relief to which this invention gave rise, there was an inclination to abandon the idea of corresponding points, for the stereoscopic experiments seem opposed to what we have said on these points. Indeed, let us look in the stereoscope at a representation of the two points A and B, both situated in the median plane, and fix the more distant A. The images of B are not formed on two corresponding points, since in one eye its image is to the right, in the other to the left of the *fovea.* Nevertheless, we see it single and in relief; that is to say, nearer than A.—On account of this apparent contradiction, *Wheatstone* inclined towards the theory of projections. In despair of a better explanation, the advocates of the theory of identity supposed that a point of one of the retinæ does not correspond to a point, but to a small surface of the other (*Panum*). An image falling on the point of the first retina could then become blended, either without relief, with an image formed at the middle of the small surface of the other, or with relief, with an image formed on a more peripheral point of the small surface. But, under this form, the theory of identity was not tenable; it would be necessary, indeed, to suppose that the same two points could be sometimes corresponding, sometimes not

corresponding, which is scarcely admissible. The question was cleared up only by the labors of *Javal.*

THEORY OF JAVAL ON THE PRODUCTION OF RELIEF.—This theory calls especially for two factors, the *neutralization* (partial suppression of one of the images) and the *influence* of the *ocular movements,* on which *Brücke* had already insisted. In chapter XXI reference was made to the suppression of one of the images, which takes place when different images are formed on two corresponding parts of the retinæ. We then see, sometimes the image of one eye, sometimes that of the other, and while we see the image of one eye, the corresponding part of the image of the other disappears absolutely. In normal persons the suppression especially manifests itself alternately for both eyes, under the form of *antagonism of the visual fields;* in strabismic patients, on the contrary, we often have occasion to observe the constant neutralization of a great part of the visual field of one eye.

Brücke was the first who insisted on the great importance of the ocular movements for the perception of relief. Anyhow, it is certain that without them we could have only a very vague notion of it. Looking into a stereoscope, especially if the images are difficult to fuse, it is only after I have permitted my look to wander for some time on the figures, fusing sometimes the images of the distant objects, sometimes those of the near objects, that relief appears to me. As long as the sensation of relief is not produced I see double, sometimes the near objects, sometimes the distant ones; but at the moment when relief appears, I see all of them single. Certain authors claim that they have observed relief by illuminating the stereoscopic images with an electric spark, the duration of which light is so short that all ocular motion is necessarily excluded. This would certainly be impossible in my case, for there always elapses a certain time before the real illusion, which does not prevent me from being able to form all at once a vague notion of relief.

According to *Javal,* it is necessary, indeed, to distinguish between the *idea of relief,* which is produced by the fact that we see near objects in double crossed images, and the *measurement*

of relief, which depends on the *sensation* of the degree of innervation necessary to converge towards the near object. To account for the manner in which we come to obtain the sensation of relief, it is preferable to use images which are quite difficult to blend, the stereoscopic parallax of the objects represented being quite strong. We immediately fuse the images of distant objects, and all the others appear in double images. We then allow the look to stray on the figure, which forces convergence more or less, according as the object is represented more or less distant. After having continued thus for some time, relief manifests itself almost in the same way as we can with closed eyes obtain a very distinct idea of the form of an object by feeling it with the fingers. At the same time that relief appears, the double images disappear; the image of one or other eye is suppressed. If one of the eyes plays the part of the *directing eye* (see page 371) it is usually the images of the other eye which are suppressed, unless the image of the preponderating eye is much more peripheral than that of the other. In cases in which this preponderance is not developed, the double images seem to appear following the law of *Javal:* we suppress that one of the images which occupies the smallest retinal surface. We can account for the manner in which we suppress the images by looking at a rule which is held obliquely before the eyes, so that it presents a greater surface to one eye than to the other. Whether it occupies the position AA (fig. 194), or the position BB, it seems to me, seen binocularly, to have the same appearance as when I close the left eye. Persons in whom the preponderance of one eye is not developed see the rule binocularly, as it is presented to the left eye, if it occupies the

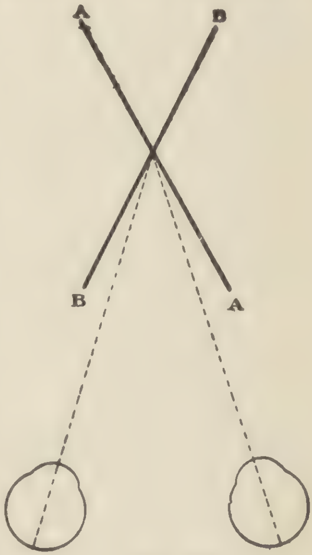


Fig. 194.

position AA. In the position BB they see it, on the contrary, as it presents itself to the right eye.

The discussion of the two theories of binocular vision, that of *identity* and that of *projections,* has not yet closed. The explanation of *Javal* is applicable in reality as well to one as to the other. We *can* imagine the projection learned by experience; and even the fact of always projecting the images of the two *foveas* at the same place, the foundation stone of binocular vision, may be something learned. It is, perhaps, the superiority of the fovea, as to visual acuity, which causes us to always bring the images of the object which interests us to form themselves on both foveas, and we may thus have been led to always localize the impression of the two foveas at the same place. On the other hand, the advocates of the theory of identity take their stand on the anatomical observations of the semi-decussation in the chiasma, and especially on comparative anatomy, which shows that in many animals—fish, for example—whose eyes are placed so as not to have a common visual field, the optic nerves cross completely. Clinical observations in hemianopsia, especially those of partial hemianopsia, are a further argument in favor of this theory. The study of the vision of strabismic patients, which is perhaps the best means of deciding the question finally, shows, as we shall see in the following chapter, that, in consequence of a false position of the eyes, there may be developed a kind of correspondence between two retinal points which, under ordinary circumstances, are not corresponding; but this relation never assumes the character of true binocular vision with fusion, and it sometimes suffices, in a person who has squinted since childhood, to place the eyes in an approximately correct position, in order that, in the course of a fortnight, correct projection may gain the upper hand.

Bibliography.—Wheatstone (C.). *Contributions to the Physiology of Vision. On some Remarkable and hitherto Unobserved Phenomena of Binocular Vision. Phil. trans.,* 1838, II, p. 371-394.—Wheatstone (C.). *Contribution to the Physiology of Vision,* II. *Phil. Mag.,* 4, III, p. 149-152 and p. 504-523.—Brewster (D.). *The stereoscope.* London, 1858.—Helmholtz (H.). *Das Telestereoskop. Pogg. Ann.,* CI, p. 494-CII, p. 167.—

Javal (E.). *Sur un instrument nommé Iconoscope, destiné à donner du relief aux images planes examinées avec les deux yeux.* Report, LXIII, 927. —Javal (E.). *De la neutralisation dans l'acte de la vision. Ann. d'oc.*, LIV, p. 5.—Müller (Johannes). *Beiträge zur vergleichende Physiologie des Gesitchtssinnes.* Leipzig, 1826, p. 191.—Volkmann (A. W.). *Physiologische Untersuchungen im Gebiete der Optik, II.* Leipzig, 1864.—Newton (J.). *Opticks*, 1717, p. 320.—Panum (P. L.). *Physiologische Untersuchung über das Sehen mit zwei Augen.* Kiel, 1858.—Brücke. *Ueber die stereoscopische Erscheinungen.* Müller's Archiv für Anat. u. Physiol., 1841, p. 459.—Nagel (A.). *Das Sehen mit zwei Augen und die Lehre von den identischen Netzhautstellen.* Leipzig, 1861.—Javal (E.). *Manuel du strabisme.* Paris, Masson, 1896.

CHAPTER XXIV

STRABISMUS

134. Different Forms of Strabismus.— We say that there is strabismus when the two visual lines do not intersect at the point fixed. The image of the point fixed is not, therefore, formed on the two *foveas,* and since the two *foveas* are always corresponding points, there is no binocular vision. One might, therefore, define strabismus as the condition in which binocular vision is wanting, at least at certain moments or for certain directions of the look. It must be observed, however, that we may meet with cases in which the visual lines have the proper direction, at least apparently, but in which binocular vision is, nevertheless, wanting; this case often presents itself in persons affected with strabismus, who have undergone a successful operation. It is also customary to speak of strabismus when one eye deviates, even if it is completely blind. The study of strabismic patients is very important for different questions of physiologic optics.

We distinguish two forms of strabismus: *paralytic strabismus,* due to a paralysis of one or more muscles, and *concomitant strabismus,* which, in the great majority of cases, is due to the defect of innervation (*Hansen-Grut*). The symptoms by which we make the differential diagnosis between these two forms of strabismus are well known. They have passed from the classic memoir of *Graefe* into all treatises of ophthalmology. In cases of paralytic strabismus the excursion of the eye is less on the side of the paralyzed muscle, and the secondary deviation is greater than the primary. Patients present diplopia, either spontaneously, or more especially if we examine them with a candle and a colored glass. The distance between the two images increases when the look is directed towards the side of the diseased muscle, and it is the image of the diseased eye which is farthest away in this direction.

When the patient closes the healthy eye and looks towards

an object situated on the side of the diseased muscle, the projection is false; for, as it is necessary, on account of the paresis, to use a stronger innervation to bring the eye to fix the object, the patient thinks that this object is situated more to one side than it really is, and when he wants to grasp it quickly he brings the hand too far to that side. I have already observed (page 367) the importance of this observation to demonstrate that we judge the direction of the look above all by the degree of innervation used to bring it into this direction.

Concomitant Strabismus.—When we speak of strabismus without other qualification it is generally this form that we mean.—In this strabismus the deviation is almost the same for all directions of the look, except that generally the convergence is more pronounced for the downward than for the upward look. The secondary deviation is equal to the primary deviation. The patient does not complain of diplopia, but we may always bring it about by the means which I shall describe forthwith. The distance between the two images is the same everywhere, to whichever side the patient looks. The simplest means of diagnosing strabismus is to make the patient fix an object, the finger of the observer, for example. If one of the eyes seems to deviate, we cover the other, and if the former then makes a movement to fix, it was deviated: strabismus is, therefore, proved. This examination must be repeated for a distant object. If we do not discover strabismus by this means, it may, nevertheless, happen that the patient has it, but in a very slight degree, or, in other words, that he has no binocular vision; we may, in this case, place a prism, apex inwards, in front of the eye. If there is binocular vision the eye makes a movement of convergence to neutralize the effect of the prism (*Graefe*).—If the strabismus is periodic we can sometimes discover it by making the patient fix a very small object, a word printed in very small type, for example; the patient is obliged to accommodate to distinguish the word, and the effort of accommodation may then cause strabismus.

Latent Strabismus.—In order to see whether there is latent strabismus, we make the patient fix the finger of the observer;

we cover one eye and examine, on uncovering it, whether the eye deviated under the hand and whether it straightened itself in order to fix. If the deviating eye does not straighten itself, the strabismus has become manifest; if it does straighten itself, it is latent.—According to *Graefe,* we make the patient observe a long vertical line which has at the middle a black spot, or, which is preferable, a candle, while we place in front of one of his eyes a prism, apex upwards. If there is latent strabismus, the patient sees two objects placed exactly one above the other (if the apex of the prism forms a horizontal line). If not, there is latent strabismus, and we can then measure the degree of it by placing the prism of *Crétès* before the other eye and finding the degree of this prism which makes one image appear above the other. We can also use the *Maddox test,* etc. *Javal* placed a ground glass lens before one of the eyes of the patient; this glass prevents the eye which it covers from distinguishing anything, while the observing eye sees the covered eye sufficiently well to judge of its position.

Making the examination in this way, we find, in many people, a slight degree of latent divergent strabismus for near vision. This condition is often designated as *insufficiency of the internal recti.* This expression is ill-chosen and should be discontinued. The internal recti are not weaker than in the normal eyes, as *Hansen-Grut* has shown, for, otherwise this weakness ought to manifest itself also for the associated movements. If the right internal rectus were really weaker than in the normal state, we should, when looking to the left, see the phenomena appear which characterize paresis of the right internal rectus, which is by no means the case. It is not in the muscles, it is in the innervation of convergence that we must search for the cause of this deviation. We might, therefore, speak of an insufficiency of convergence, but this also would be a bad expression, for many patients affected with this deficiency can converge as well as normal persons; it is only the stimulus of convergence that is wanting. (1)

(1) [In this country Stevens' nomenclature has been generally accepted. According to him this condition is called *exophoria.*]—*W.*

135. Measurement of Strabismus.— 1° We cover the good eye; the strabismic eye straightens itself, and we value, in millimeters, the extent of the displacement of the cornea.

2° *Javal* has proposed to measure the deviation in degrees by means of the perimeter. He places the patient so that the strabismic eye is in front of the point of fixation of the perimeter. The patient fixes this point with his good eye. The observer then moves a candle along the arc of the perimeter, sighting in the direction of this candle towards the strabismic angle. He finds the position in which the corneal image is at the middle of the pupil, which indicates approximately the direction of the visual line of the strabismic eye. In the keratoscopic arc of *de Wecker,* the candle is replaced by a white mire, and at the point of fixation is a small mirror in which is reflected a distant object which serves as the point of fixation.

3° We can use the distance of the two images as a measure of the strabismus if there is diplopia. We can measure this distance with the prism of *Crétès,* or by projecting the images on a wall provided with a graduation in degrees (*Hirschberg, Landolt*) or on a *Prentice* scale.

The deviation often varies much with the distance of the object fixed. It may also be useful to determine the deviation at different distances, at 4 meters and at 25 centimeters, for example, as *Schioetz* has proposed.

136. The etiology of concomitant strabismus is a quite complex question on which opinions are still divided. *Boehm* discovered the relation which exists between hypermetropia and convergent strabismus, and *Donders,* in a general way, announced the part that the anomalies of refraction play in the etiology of strabismus. This influence cannot be denied, and it is especially striking for convergent strabismus. In my extensive compilation of statistics of young conscripts (see page 101) there were 42 cases of convergent strabismus, of whom 31 were hypermetropes, 7 emmetropes and 4 myopes; that is to say, that about 70 per cent. of the persons squinting inwards were hypermetropes. But, on the other hand, there were in all 301 hyper-

metropes (of 2 dioptries or more); only a very small minority of the hypermetropes squint, therefore.

The manner in which *Donders* explained the relation between convergent strabismus and hypermetropia is well known. When an emmetrope fixes a near object, it is above all the necessity of seeing it single which regulates the position of his eyes. But, if we cover one of the eyes, this need no longer exists, and, nevertheless, the observed person generally continues to converge towards the point fixed; this is due to the relationship which exists between accommodation and convergence. Even if the observed person is sufficiently myopic to make it unnecessary for him to accommodate for the object, the covered eye converges, nevertheless, pretty exactly for the object. This is due to what *Hansen-Grut* termed *sensation of the distance;* knowing that the object is at a short distance away, the patient converges because he is accustomed to do so in the interest of binocular vision, even in a case in which this interest no longer exists.

These three factors regulate the degree of convergence to be used. Under ordinary circumstances, it is the first factor which is of most importance; but, in cases of hypermetropia, it may happen that, in order to sustain his accommodation, the patient converges more than is necessary for fusion. He then sacrifices his binocular vision to obtain distinct vision with one eye only, and this happens with special ease when the vision of the other eye is diminished for one reason or another (opacities of the cornea, astigmatism, etc.). In a certain number of cases we find vision greatly diminished without any perceptible reason. We cannot yet say whether this diminution is a consequence of strabismus (*amblyopia ex anopsia*), or whether it is not rather a cause of strabismus, due to a congenital anomaly.

If we thus explain why a hypermetrope may become strabismic, we cannot well understand why the great majority of hypermetropes do not squint. They often seem to have quite as much reason to squint as strabismic patients. *Javal* supposes that strabismus has developed under the influence of paresis of the accommodation which is cured later. The existence of such

paresis is certainly hypothetical, but it would very well explain the origin of strabismus; the parents of strabismic children are quite frequently affected with convulsions, intestinal worms, which might have produced nervous troubles, etc. According to *de Wecker,* a certain number of cases of convergent strabismus might be due to a paralysis of one of the external recti acquired during infancy. Paralytic strabismus would be transformed later into concomitant strabismus.

Myopia plays, in the production of divergent strabismus, a less important role than hypermetropia in the production of convergent strabismus. As the myope does not accommodate at all, or only slightly for near objects, one of the factors which sustains convergence is wanting. If the eyes are very unequal, there may readily follow a divergent strabismus relative to near objects. On the other hand, distant vision is so diffuse for the more imperfect eye that binocular vision is of little usefulness, and this eye then easily deviates outwards. Generally speaking, every eye, the vision of which is destroyed or greatly diminished, has a tendency to deviate outwards.—In very rare cases we meet in myopes a special form of convergent strabismus.

The ideas on the nature of strabisums are much divided. Most authors find the cause of strabismus in the muscles, for instance, *v. Graefe* ("excess of average contraction"), *Schweigger* ("excess of elasticity of the muscles"), etc. Others, *Alfred Graefe* and *Javal,* for instance, attribute periodic strabismus and the variable part of permanent strabismus to innervation, while they suppose that the permanent part is due to consecutive muscular alterations. The theories which attribute the vast majority of cases of strabismus to a defect of innervation are beginning to gain ground. They have been advocated by *Stellwag, Rahlmann, Hansen-Grut* and *Parinaud.* The theory of *Hansen-Grut* seems to me to adapt itself best to the phenomena.

According to this author, the whole muscular theory collapses before the following observation. Suppose a left convergent strabismus of 6 mm.: if this strabismus had a muscular origin, it would be necessary that the limit of excursion outwards of the left eye would be displaced inwards 6 mm. But we never

find anything of the kind. If the limit is sometimes displaced a little inwards, this is due to a lack of habit, since we never have occasion to make so great a motion with the strabismic eye.

Hansen-Grut distinguishes between the *position of anatomic equilibrium* and the *position of functional equilibrium* of the eyes. The former is the position which the eyes assume apart from all nervous influence. When the eyes are in this position (during sleep, after death, etc.), the visual lines diverge in nearly all patients. The position of *functional* equilibrium is the position which the eyes assume when we look at a distant object with one eye covered. In this position the visual lines are parallel in normal persons. The convergent strabismus is due to the fact that there is developed an unusual position of functional equilibrium; the divergent strabismus, on the contrary, is due to the fact that such a position is not developed at all, so that the eyes are placed in the position of anatomic equilibrium.

137. Vision of Strabismic Patients.— Except in cases of *convergent strabismus of myopes,* strabismic patients do not generally complain of diplopia; they suppress the image of the deviated eye, so that the strabismic eye serves only to slightly increase the visual field. We may, however, always cause diplopia by holding a red glass in front of the good eye, by keeping this eye closed for some days, etc.; but then we often meet with the singular phenomenon termed *paradoxical diplopia.* This diplopia was discovered by *v. Graefe.* Examining persons affected with convergent strabismus, in whom he had performed a tenotomy which *partly* corrected the defect, he found crossed diplopia, although the visual lines were still convergent, and the patients, according to the ordinary rule, should have indicated homonymous diplopia. *Javal* was the first to study this phenomenon on patients not operated on. The explanation of this fact is that there is developed what has been very improperly named a vicarious *fovea.* The patient has first cultivated the habit of suppressing the image of the strabismic eye; then there is gradually formed an idea of the false position of the strabismic eye; he has learned that an object which forms its image on the

fovea of the good eye, forms its image at a point (b) inwards from the *fovea* of the strabismic eye, and he has learned to localize this image at the place where the object to which it belongs is situated. If we place a prism, apex down, in front of the good eye, the patient sometimes says that he sees only the image of the strabismic eye; the patients localize it almost on the same vertical line as the image of the good eye, instead of indicating widely separate homonymous images. It is, therefore, as if there was developed a correspondence between the point b and the *fovea* of the good eye. But the localization of the image is always very uncertain; the patient sometimes says that he sees both images well, but that it is impossible to tell which is the image of the strabismic eye.

If we perform a tenotomy which does not completely correct the deviation, the image of the point fixed is no longer formed either on the true *fovea* or the vicarious *fovea,* but between the two. Patients first project the image according to the vicarious *fovea:* as it is formed on a part of the retina situated outside of the latter, the patient sees the object in crossed images. Later, especially if we make systematic exercises in order to reach it, the true *fovea* comes to exert its preponderating influence: the patient sees the object in homonymous images. Following the development of the change of vision of the patient, we sometimes succeed in finding a time when the patient projects the image of the strabismic eye according to both foveas at once: he sees with the strabismic eye, at the same time, one image to the right and another to the left of the object. This singular form of vision has been described by *Javal* under the name *binocular triplopia.* I have had occasion to study a case of this character.

138. Treatment of Strabismus.—If we confine ourselves to the treatment by operation, it is prudent not to completely correct convergent strabismus, for the strabismic eye has a tendency to put itself in divergence, a tendency which sometimes suffices by itself to finally cause the convergent strabismus to disappear. On the contrary, when it is our intention to reëstablish binocular

vision, we must try to make the position of the eyes as correct as possible. This reëstablishment is often a very long and difficult matter; the task is less arduous in cases in which there still exists binocular vision in a part of the field. In certain cases, such as the periodical divergent strabismus and the convergent strabismus of myopes, we succeed by means of some exercises, or even by the simple operative treatment. According to *Javal,* who especially devoted his attention to this question, the course of the treatment is as follows:

a. Reëstablishment of diplopia and, if possible, *of the vision of the strabismic eye.* We keep the good eye covered by means of a blind patch; if the vision of the other eye is very bad, in order to less annoy the patient, we allow him to wear the patch on the bad eye during several hours of the day, but it is necessary, during this period of treatment, never to allow the two eyes to be uncovered at the same time, under penalty of never seeing the neutralization disappear or of seeing the strabismus increase; for, as the diplopia annoys so much less as the images are more distant from each other, the patient tries to squint more strongly in order to separate the images.

b. Reëstablishment of the approximately correct position of the eyes by way of operation.

c. Stereoscopic exercises.—We begin by placing in each field, on each visual line, a round spot. If the patient fuses them, we move them farther or nearer, in order to develop in him the necessity of seeing single. The stereoscope of *Javal,* an imitation of that of *Wheatstone* (fig. 189), but with a variable angle between the mirrors, lends itself very well to this exercise. As soon as the patient sees double, we begin. When the patient has succeeded, we make him fuse letters by giving him smaller and smaller characters. For all these tests it is necessary to add to each figure numerous small marks, different ones for each eye, in order to make certain that the patient really fuses. He ought to see the figure with both series of marks; otherwise, he neutralizes one of the figures, instead of fusing both. When beginning these exercises, we often encounter the phenomenon which *v. Graefe* designated under the name of *antipathy to single*

vision. When we place the round spots in positions corresponding to the visual lines, the patient converges or diverges in order not to fuse them; if we try, in this new position of the eyes, he makes his convergence change again, and so forth. *Javal* invented a very ingenious card to surmount this difficulty, which is often very great.

d. Exercises without the stereoscope.—There often exists a part of the field in which the patient sees single; then we make him exercise in order to increase this part, for example, by placing a candle in the part of the field in which the patient fuses and bringing it towards the other part; when the patient sees double, we begin again.

e. If the patient stands these different tests, we begin to make him do *controlled reading.* We interpose a pencil between the eyes and the book; reading can then take place without interruptions only by using both eyes. This exercise must be continued for months. It is only a long while after the reëstablishment of binocular vision that the patient can see relief.

Bibliography.—Böhm. *Das Schielen.* Berlin, 1845.—v. Gräfe (A.). *Ueber Doppeltschen nach Schieloperationen und Incongruenz der Netzhaüte. Arch. f. Ophth.*, I, 1, p. 82.—v. Gräfe (A.). *Ueber eigenthümliche zur Zeit noch unerklärliche Anomalien in der Projection der Netzhautbilder. Arch. f. Ophth.*, II, 1, p. 284.—v. Gräfe (A.). *Symptomenlehre der Augenmuskellähmungen.* Berlin, 1867.—Donders (F. C.). *Anomalies of the Refraction and Accomodation of the Eye.* London, 1864. Hansen-Grut (E.). *Pathogeny of concomitant squinting* (Bowman lecture). *Transactions of the Ophthalmological Society of the United Kingdom,* Vol. X, 1890.—Javal (E.). *Manuel du strabisme.* Paris, Masson, 1896.

CHAPTER XXV

OPTIC ILLUSIONS

139.—We designate by the above name cases in which the visual impressions give rise to a false judgment on the nature of the object. Illustrations, paintings and, generally, all representations of an object have the effect of producing these illusions; and all optic instruments act in a like manner. In the former part of the book I have mentioned several times illusions of a more special character; I shall here describe briefly some others, the explanation of which, in most cases, is quite obscure.

a. A first series of illusions is based on the fact that a line or space seems larger when it is divided than when it is not. This is the reason why the two parts *ab* and *bc* of the line (fig.

Fig. 195.

195) have the same length, but that still the part *bc* appears longer, because it has divisions. The two illustrations of figure

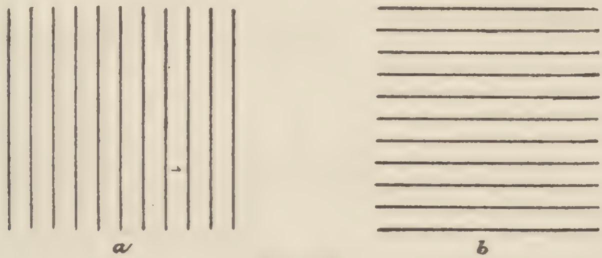


Fig. 196.

196 are square, but the illustration *a* seems wider and the illustration *b* higher, on account of the divisions. For the same reason, a space filled with furniture appears larger than when it is empty.

b. Very small angles are estimated to be larger than they are in reality. The following illusions may be considred as examples of this rule. The lines *ab* and *cd* of figure 197 are situated in the prolongation of each other, but *cd* seems displaced upwards. The illusion increases if we move the figure farther away. We may conceive that if we judge the acute angle to be too large, the line *cd* ought to seem to have undergone a rotation around the point *c*, the line *ab* around the point *b*, which would produce the illusion in question.

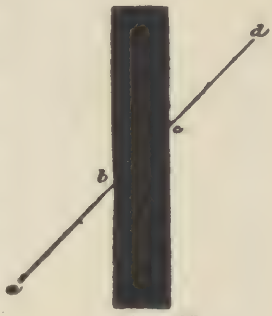


Fig. 197.

The same error of judgment seems to take place in the illusion produced by the designs of figure 198 (*Hering*) and figure 199 (*Zöllner*).

In figure 198 the long lines are straight and parallel, but seem curved; in the upper part of the figure they appear to have their concave sides turned towards each other; in the lower part the

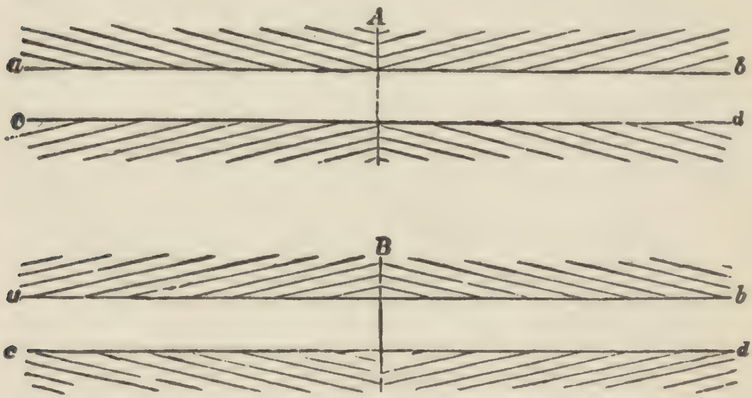


Fig. 198.

contrary takes place. In the figure of *Zöllner,* the long straight lines, which are parallel, seem to converge or diverge upwards, following the direction of the small oblique lines. We can conceive that these illusions would be produced if the judgment at-

tributes a too large size to the acute angles. According to *Helm-*

Fig. 199.

holtz, the movements of the look play a great part in the production of these illusions; they appear much more pronounced

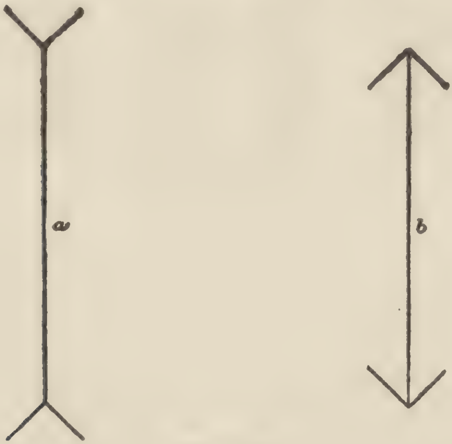


Fig. 200.

if we keep the look quiet. If we bring a point slowly from right to left in front of the figure of *Zöllner,* while fixing it with the

look, the lines seem to move; those which appear to incline their upper extremity to the right seem to ascend, while the others seem to descend, and the inclination seems at the same time more pronounced. If we bring the point from left to right, the lines affect reverse movement. The experiment is not very easy to perform, but we can obtain the same effect more easily by keeping the point which we fix motionless and moving the drawing.

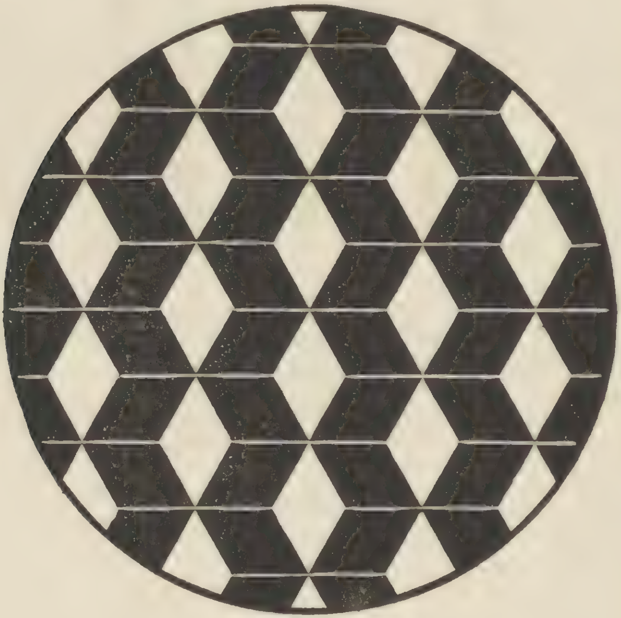


Fig. 201.

c. The two long straight lines of figure 200 have the same length, but *b* appears smaller than *a*.

d. We frequently estimate cylinders too large. If we place a large bottle on a sheet of paper, and trace its circumference, we can with difficulty conceive, after having taken away the bottle, that we are not deceived, so small is the circle. Another error of judgment is well known: we present a tall hat to some one, asking him to indicate on the wall its height, starting from

the ground. Generally the height pointed out is about half too large.

e. I have already mentioned the reverse of relief which we observe when we change the stereoscopic images sideways, and which is known under the name of *pseudoscopia*. We sometimes observe the same phenomenon under other circumstances. If, for example, we fix with one eye the posterior part of the upper border of a lamp chimney, we obtain quite easily the illusion that this part is in front, and the glass seems at the same time to lean towards the observer.—Observing with one eye the cast of a medal, it may be difficult to tell whether the figure is hollow or in relief.

Analogous phenomena often present themselves in cases in which a drawing may be interpreted in two different ways. Thus figure 201 seems composed of cubes, the illuminated side of which is turned sometimes to the right, sometimes to the left. When one interpretation has predominated for a certain time, the other suddenly presents itself. We can instigate the change by quickly imagining the contrary relief.

f. We mention, finally, the illusions of movements of exterior objects, which often present themselves in consequence of the false judgment of the movements which we ourselves make. One of the best-known examples is that of the apparent movements of objects when we are traveling by rail; the traveler does not take into account his own change of position and attributes the movement to the exterior objects. The reverse illusion often presents itself when one train stops alongside of another; if the latter is put in motion, we often attribute the movement to our own train. Waltzers see exterior objects rotate around them in a direction contrary to their own rotation. The movement seems to continue for some time after stopping, on account of the persistence of the jerking movements of the eyes (page 360).

Generally, exterior objects do not appear to be displaced during the movements of the look, but if we bring the look quickly from one of the limits of the field to the other, exterior objects seem to move in the contrary direction.

Aubert has described the following illusion, which is due to a like reason. In the shutter of a completely dark room we make a vertical slit, which is then the only object visible. Leaning the head towards one of the shoulders, the slit seems to undergo a rotation in the reverse direction; it no longer appears vertical. We judge the inclination of the head to be less than what it is, almost in the same manner as the movements which we cause the eyes to make while keeping the lids closed, always seem less than they really are. The experiment also succeeds outside of the dark room, especially if we place ourselves in such a way as not to see any other lines, the direction of which we know to be vertical.

Bibliography.—Zöllner. *Ueber eine neue Art von Pseudoscopie. Pogg. Ann.*, CX, p. 500.—Hering (E.). *Beiträge zur Physiologie.* Leipzig, 1861, I, p. 65.—Aubert (H.). *Physiologie der Netzhaut.* Breslau, 1865.

TREATISES TO CONSULT

Œuvres ophtalmologiques of THOMAS YOUNG, translated and annotated by M. TSCHERNING. Copenhagen, Hoest, 1894. The memoires of *Young* were published at the beginning of the century in the *Transactions of the Royal Society of London* and reprinted in his *Lectures* (London, 1807). A later reprint in *Peacock Works of Thomas Young*, London, 1855, is not to be recommended, the reproduction therein of the pretty illustrations of *Young* being quite defective. The works of *Young* are often of a very difficult reading, but many of the modern ideas on ocular dioptrics and on the vision of colors dated from him. On account of the great importance of the works of *Young*, I have published a French edition of them which I have tried to make of an easier reading by explanatory notes.

v. HELMHOLTZ (H.). *Handbuch der physiologischen Optik.* Leipzig, 1867. This monumental work is indispensable to all those who desire to make a profound study of physiologic optics, but it is not a very easy study. The book contains nearly all that was known on the subject of physiologic optics at the time of its appearance and a complete bibliography. In 1885, the author began a new edition of it (Leop. Voss, Hamburg), which was continued after his death by A. KŒNIG. The only difference between it and the former consists of a number of intercalations, which, however, are not of very great importance, if we except those of the second part which contain the results of the researches on the vision of colors of *Kœnig, Dieterici, Brodhun, Uhthoff*, etc. The latter portion of the work contains, from the hand of *Kœnig*, a complete bibliography, which will be very useful to the investigators of the future.—The work of HELMHOLTZ was translated into French by E. JAVAL and N. T. KLEIN (Masson, 1867), but this translated edition is exhausted.—The student of physiologic optics must not dispense with reading the original memoirs of this great scholar.

HERMANN (L.). *Handbuch der Physiologie der Sinnesorgane.* 2 vol. Leipzig, 1879. The part which has to do with vision has been treated by FICK (A.) (*Dioptrics*), KUEHNE (*Chemistry of the Retina*) and HERING (E.) (*Movement of the Eyes, Binocular Vision*).

Less important works and of an easier reading:

FICK (A.). *Lehrbuch der Anatomie und Physiologie der Sinnesorgane.* Lahr, 1864.

KAISER (H.). *Compendium der physiologischen Optik.* Wiesbaden 1872. Apart from some parts which the author has treated in an original manner, this work is an extract from v. HELMHOLTZ.

AUBERT (H.). *Physiologische Optik,* in *Handbuch der gesammten Augenheilkunde* von A. GRAEFE und TH. SAEMISCH. Leipzig, 1876. The most original part is an extract from:

AUBERT (H.). *Physiologie der Netzhaut.* Breslau, 1865, a book which contains a great number of very elaborate researches on the retinal functions.

LE CONTE (JOSEPH). *Sight.* London, 1881. In spite of some errors this work is very instructive on account of its originality.

From the time prior to v. HELMHOLTZ dates MACKENZIE (W.). *The Physiology of Vision.* London, 1841, being based especially on the works of YOUNG and WHEATSTONE.

What was known on the subject of physiologic optics in the last century is found in:

PORTERFIELD (WILLIAM). *A Treatise on the Eye.* 2 vol. Edinburgh, 1759, and in:

JURIN (JACQUES). *Essai sur la vision distincte et indistincte* in the great treatise on optics of ROBERT SMITH (*A Complet System of Opticks*). London, 1738. In French *Cours complet d'optique* of ROBERT SMITH, translated by PEZENAS. Paris, 1767.

The work of JURIN on indistinct vision is still the best on this somewhat neglected question.

Of the works on more or less important branches of physiologic optics we may cite:

DONDERS (F. C.). *On the Anomalies of Accommodation and Refraction of the Eye.* London, 1864. In German by O. BECKER. Wien, 1866. In French by E. JAVAL, in DE WECKER. *Traité des maladies des yeux.* Paris, 1866. On account of its remarkable clearness DONDERS is of a very easy reading, and may be recommended to every young medical student who desires to begin the study of this branch of opthalmology.

The same subject has been treated in:

NAGEL (A.). *Die Anomalien der Refraction und Accommodation des Auges* in *Gräfe* und *Sämisch. Handbuch der Augenheilkunde.* Leipzig, 1880.

LANDOLT (E.), in DE WECKER and LANDOLT. *Traité complet d'ophtalmologie,* 1884.

MAUTHNER (L.). *Vorlesungen über die optischen Fehler des Auges.* Wien, 1876.

MAUTHNER (L.). *Farbenlehre.* Second edition. Wiesbaden, 1894. The books of Mauthner are written in a very clear style and bear the impress of great learning.

Mémoires d'ophtalmométrie, annotated and preceded by an introduction by E. JAVAL. Paris, Masson, 1890. This work contains a great number of notes on ophthalmometry by different authors.

E. JAVAL. *Manuel de Strabisme.* Paris, Masson, 1896. This work is important for the study of binocular vision.

INDEX

A

Abduction, 363
Aberration, chromatic, 96, 120, 131, 133, 137
 produced by accommodation, 211
 spherical, 96, 114, 125
Aberroscope, the, 123
Aberroscopic phenomena, 172, 173, 206
Absorption of light, 2
Accommodation, 46
 amplitude of, 97, 192
 astigmatic, 155
 author's theory of, 200
 central and peripheral, 208
 Cramer's theory of, 197
 Helmholtz theory of, 198
 H. Muller's theory of, 199
 influence of, 377
 mechanism of, 195, 196, 198, 201
 paralysis of, 194
 relative amplitude of, 365
 skiascopic examination of, 209
 spasm of, 194
 Young's theory of, 201
Accommodation and convergence, relation between, 365
Achloropsia, 322
Acuity, visual, 335
 peripheral, 341
Adduction, 363
Aerial images, 42
 perspective, 378
After-images, 291
 positive, 291
 negative, 291
Akyanopsia, 323
Amblyopia exanopsia, 401
Ametropia, 9
Anaglyphs, 338
Anerythropsia, 322
Angle alpha, 45, 77
 critical, 11
 meter, 364
 of convergence, 12
 of deviation, 12
 of incidence, 2
Angle of refraction, 2
 of visibility, 336
Aniridia, 198
Antagonism of the visual fields, 389
Aperture of an optic system, 41
Aphakia, 96, 111
Asthenopia, accommodative, 109, 193
 of astigmatic patients, 158
 tarsal, 177
Astigmatic persons, examination of, 159
 surfaces, 75
Astigmatism, 138, 166
 against the rule, 150
 by incidence, 115, 143
 crystalline, 150
 corneal, 147, 149, 151
 irregular, 96, 164, 166
 latent, 155
 oblique, 147
 of the human eye, 145
 physiologic, 146
 post-operative, 156
 produced by the form of the surfaces, 138
 regular, 96, 138, 141
 ophthalmometric and subjective, 150
 supplementary, 151
 symptoms of, 158
 with spherical aberration, 169
 with the rule, 150, 158
Arteries, pulsation of, 240
Atropine, 255
Auto-ophthalmoscope, 241

B

Base line, 364
Binocular ophthalmoscope, 387
Binocular vision, 346
 projection in, 369
 theories of, 391, 393, 395
Black, sensation of, 287
 absolute, 287
Brightness, 284
Brushes of Haidinger, 187

C

Cardinal points, 23
    methods of finding, 25, 26
    of the crystalline lens, 29
    of the human eye, 39
Cataract, 203, 281
Cat's eye, amaurotic, 229, 230
Centering, defect of, 80
Characteristic part of a pencil of light, 167
Chess-board of Helmholtz, 260
Chromatic aberration, 96, 120, 131, 133, 137
    correction of, 137
Chromatoptometer of Chibret, 325, 326
Ciliary corona, 188
Ciliary muscle, discovery of, 203
    structure of, 205, 224, 225
Cocaine, 255
Color-blindness, 317
Color-box of Maxwell, 299, 305
Color curves of Maxwell, 306
    of a dichromatic, 321
Color phenomena of contrast, 287, 290
Colors, complementary, 287
    equation of, 298
    methods of mixing, 298
    results of mixtures of, 301
    sensations of, 285
    spectral, 298
    the principal, 328
    the standard, 305
Color sense, 282
    clinical examination of, 324
Color table of Helmholtz, 313
    of Maxwell, 304, 308, 319
    of Newton, 285, 303
Color vision, mechanism of, 327
    Ebbinghaus's theory, 331
    Helmholtz theory, 329
    Hering's theory, 330
    Koenig's theory, 331
    Parinaud's theory, 331
    Young's theory, 327
Concave spherical mirrors, 4
    aperture of, 4
    apex of, 4
    axis of, 4
    principal focus of, 4
    principal focal distance of, 4, 7
    reflection on, 5
Conjugate points, 2, 6
Conoid of Sturm, 138
Contact glasses, 174
Contact of corneal images, 59
Controlled reading, 405
Convergence, defect of, 363
    measurement of, 363
    negative, 362
Convex mirrors, 7
Co-ordinates, center of, 369
    polar, 369
Cornea, basilar part of, 67
    conical, 66
    examination of peripheral parts of, 67
    increase in curvature of, 195
    in keratonconus, 72, 73, 74
    optic part of, 67
    refracting power of, 38, 69
    results of measurements of, 66
    utilized part of, 65
Crystalline lens, 34
    accommodative layer of, 222
    advance of, 195
    astigmatic accommodation of, 153
Crystalline lens, catoptric images of, 196, 197
    change in thickness of, 219
    contents of, 222
    cortical portion of, 36
    deformity of, during accommodation, 216
    increase in curvature of, 195
    measuring aberration of, 128
    measuring surfaces of, 81, 82, 83, 84
    luxation of, 96
    nucleus of, 36, 222
    obliquity of, 153
    refracting power of, 38
    total index of, 37
Cylindrical glasses, 145, 158
Czermak, experiment of, 90

D

Daltonism, 317
    bilateral, 318
    monolateral, 318
Decentered eyes, 158
Deformity of internal surfaces in astigmatism, 151
Descartes, law of, 9, 25
Dichromasia, 377, 320
Dichromatopsia, 377
Diffraction in the eye, 188
Diffusion circles, 88, 117, 207
    size of, 88
    examination of, 117
Diplopia, physiologic binocular, 370
    paradoxical, 390
Disc keratoscopic, 74
    of Benham, 277

Disc of Helmholtz, 277
of Masson, 276
of Placido, 74
of Volkmann, 356
Dispersion, 131, 136
Distance, indirect judgment of, 377
sensation of, 401
Doubling, methods of in ophthalmometry, 59
Dove, experiment of, 289

E

Empiric theories, 261
Entoptic phenomena, 176
analysis of, 180
manner of observing, 176
parallax of, 181
Entoptic object, determination of position of, 182
examination of refraction of, 182
Entoptic observation of vessels of retina, 183
Entoptoscope, the, 180
Eye, an artificial, 262
aperture of the optic system of the, 41
color of fundus of the, 239
center and axes of rotation of, 346
directing, 371
emmetropic, 97
methods of illuminating fundus of the, 229
muscles of, 248
Eye, optic axis of the, 45
optic constants of the, 33
optic system of the, 33, 38
schematic, of Helmholtz, 34
the simplified, 32
Eyes, associated movements of the, 361
jerking movements of the, 360
relative movements of the two, 360
rotary movements of, 358
Erect image, examination by, 233, 237
Eserine, 255
Exophoria, 399

F

Far point, 97
Fixation, point of, 44
Fechner, law of, 270
explanation of the, 270
verification of the, 270, 271, 272, 273, 274
Focal distance, anterior, 23
of a convex mirror, 7
of a concave mirror, 8
posterior, 13, 23
principal, 4
Focal interval of Sturm, 195
lines, 138, 139, 171
Focus, anterior, 23
posterior, 23
principal, 4, 5
Form sense, the, 273
measure of the, 272, 336
Foucault, principle of, 119
Fovea, 44, 95, 238, 270, 279
Fraunhofer, experiments of, 134
lines of, 132, 283, 295

G

Gauss, theory of, 23, 41
Glabella, 373
Globe, elongation of, 195, 202

H

Hemeralopia, 280
Hess and Heine, observations of, 218, 226
Homatropine, 255
Hooke, experiments of, 234, 235, 236
Horopter, 374
Hue, of color, 284
changes of, 284
Hypermetropia, 96, 99, 109
absolute, 109
axial, 95
correction of, 99
latent, 109, 233
Hypoconchia, 104

I

Iconoscope of Javal, 386
Identical points of the retina, 391
Identity, theories on the nature of, 391
Image, 2
defects of the, 141
erect, examination by, 232, 237
inverted, examination by, 241
of mirrors, 3, 4, 5
of lenses, 18

Image of any optic system, 24
produced by a small aperture, 2
real, 2
useful, 46
virtual, 2
Images, displacement of in accommodation, 217, 218
manner of observing the, 50, 54
of Purkinje, 34, 35, 48, 50, 78, 79
of the eye, false, 47
of the second order, false, 54
suppression of double, 375
Innervation, judgment of, 367
Intensity, 284
Inter-focal distance, 138, 139
Internal surfaces, position of, 80
centers of, 83
deformity of, 151
Interval of an optic system, 27, 30
Inverted image, examination by, 241
Iris, 197
apparent, 41
Iridodonesis, 257
Isopters, 242

J

Jaeger, test-types of, 338
Javal, test chart of, 338
Judgment, unconscious, 377

K

Keratoconus, 96, 157, 213, 214
Keratoscope of de Wecker and Massilon, 213
Keratoscopic disc, 74
image, 73, 74, 75, 76

L

Lens, 17
achromatic, 133
aplanatic, 114
axis of, 17
concave, 19
crossed, 115
crown, 115
flint, 115
focal distance of a, 17, 20
infinitely thin, 17, 28
measuring focal distance of, 20
Lens, optic center of, 17
over-corrected, 114
phenomena dependent on spherical aberration of, 115
refracting power of, 22
Lenticonus, 96
false, 96
Leucoma, central, 281
Leucoscope, the, 325
Light, harmful, 47
lost, 47
monochromatic, 188, 282
quantity reflected, 10
rectilinear propagation of, 1
useful, 47
Light sense, the, 270
measurement of, 274
Lithium flame, 282
Listing, axes of, 353
law of, 261, 262, 348, 349, 352, 354, 357, 358
Luminous point, analysis of the, 171
figures of, 171
Luminous rays, 1
incident, 4
reflected, 4

M

Macula, 238, 281, 315
Maddox test, 399
Meissner, experiments of, 335
Menisci, 20
Meridian, apparently vertical, 335
Meter angle, 364
Meyer, H., experiment of, 287
Micrometer, 237
Microphthalmia, 67
Mile, experiment of, 90
Mires, 57
Mirrors, concave spherical, 4
plane, 3
portion of used, 8
Monochromasia, 324
Muscæ volitantes, 179, 186
Mydriatics and Myotics, 255
Myopia, 96, 101, 107, 196
atropine treatment of, 107
axial, 95
correction of, 98
dangerous, 102
treatment of, 107

N

Nativistic theories, 263
Near point, 98
  determination of, 192
Neutral point in the spectrum of color-blind, 317
Nicol prism, 188
Nodal points, 23, 39
Normal, of a surface, 18
Nyctalopia, 281

O

Oblique illumination, 256
Ocular movements, 346, 360
  muscles, action of, 248
Opaque bodies, 1
Ophthalmometer, 58, 59, 60
  of Brudzewski, 72
  of Helmholtz, 59, 68
  of Javal and Schioetz, 61, 68
Ophthalmometry, 57
Ophthalmodynamometer of Landolt, 363
Ophthalmophakometer, 53, 77, 211, 216
Ophthalmoscope, 229
  binocular, 387
  of Coccius, 8
  of Cramer, 196, 227
  of Helmholtz, 231
  principle of, 231
Ophthalmoscopic examination of refracting media, 245
  field, 236, 243
  magnification, by erect image, 233
  magnification by inverted image, 241
Ophthalmoscopy, 229
Optic axis, 45
  constants of the eye, 33
  illusions, 407
  properties of bodies, 1
Optic system of the cornea, 38
  of the crystalline lens, 38
  of the eye, 38
  aperture of the eye, 41
  obliquity of the eye, 171
Optogram, 270
Optometer, 100
  of Badal, 192
  of George Bull, 192
  of Mile, 90
  of Scheiner, 90
  of Weiland, 162
  of Young, 91, 172, 208

P

Papillary excavations, 237, 239
Papilla, 237, 238, 286
  scleral border of, 239
Paracentesis, 227
Paracentral shadow, 252
  theory of, 251
  explanation of, 252
Parallax, influence of the binocular, 382
Penumbra, 2
Perception of depth, monocular, 377
  binocular, 382
Periscopic glasses, 115, 162
Phosphene of Czermak, 186
Phosphorescence, 229
Photoptometer of Charpenteir, 275, 297
  of Foerster, 275
Placido, disc of, 74
Plates of Helmholtz, 246
Point of fixation, 44
Position of anatomic equilibrium, 403
  of cardinal points, 30
  of the centers, 81
  of the surfaces, 80
  of functional equilibrium of eye, 403
Presbyopia, 193
Primary direction of eye, 349
  position, 349
Principal focus, 4
  focal distance, 4
  meridians, 138
  planes, 24
  points, 23
Prism, achromatic, 132, 133
  a vision directe, 132, 133
  Nicol, 187
  refraction by a, 11
  with total reflection, 10
  Wollaston, 61
Projection in binocular vision, 369
Projections, center of, 370
  general laws of, 366
  theory of, 392
Pseudoscope, the, 386
Pseudoscopia, 411
Punctum proximum, 92, 93
  remotum, 92, 93
Pupil, 254
  apparent, 42, 254
  contraction and dilatation of, 254
  in accommodation, 256
  influence of light on, 255
  movements of, 255

Pupil, nerve control of, 254
of albinos, 230
of entrance, 43
of exit, 43
real, 42
variations of refraction in, 173
Purity of color, 284

R

Radii, direct determination of, 84
Radius vector, 368
Ragona Scina, experiment of, 288
Reflection, 2
images of the eye, 211, 212
regular, 2
total, 10
on a concave mirror, 4, 5
on a plane mirror, 3
Refracting surface, power of, 16
simple, 28
Refraction, 10
anomalies of, 95
by a parabolic surface, 214
by a prism, 12
by a spherical surface, 13, 14
by plane parallel plates, 11
by a surface of revolution of the second degree, 17
index of, 10
in the pupil, 173
ophthalmoscopic and subjective, 237
Relief, idea of, 393
measurement of, 393
theory of, 393
Retina, 264, 266
changes of, 266
detachment of, 280
functions of, 266
pigment of, 267
Retina of frog, section of, 268
Retina seen by the ophthalmometer, 240
Retina's own light, 272
Retinal horizon, 354
Retinal purple, 239, 266
discovery of, 267
Retinal oscillations, 293

S

Saturation of color, 284
Scheiner, experiment of, 91, 115, 300
Scopolamine, 255
Secondary direction, 349
Shade of color, 284
Shadows, 1
colored, 288
deformity of the, 118
experiments with, 290
Sight, lines of, 89
Skiascopic examination for astigmatism, 160, 248
field, 24, 25
examination of optic anomalies, 252
Skiascopy, 246
application of, 246
with concave mirror, 248
with plane mirror, 246
Snellen, charts of, 337
Sodium flame, 282
Spectacles, choice of, 104, 193
Spectroscope, 282
Spectrum, 282
colors of, 284
of diffraction, 283
of refraction, 284
Spot of Mariotte, 76, 286, 342
Spherical aberration, 96, 114, 125
Spherometer, 21
Staphyloma, 237
Stenopaic opening, 92
Stereoscope, 382
effect of, 388
of Helmholtz, 386
of Wheatstone, 386
Stereoscopic exercises, 405
images, methods of observing, 384
lustre, 388
parallax, 383
photographs, 388
Strabismic patients, vision of, 403
Strabismus, 397
cause of, 402
concomitant, 397, 398, 400
convergent, of myopes, 403
latent, 398
measurement of, 400
nature of, 402
paralytic, 397
relation between convergent and hypermetropia, 400
relation between divergent and myopia, 402
treatment of, 404
Strontium flame, 285
Synchisis scintillans, 246
Syringe of Pravaz, 228, 257

T

Tapetum, 229
Telescopic system, 27

Telestereoscope of Helmholtz, 387
Thallium flame, 282
Threshold, the, 274
    determination of, 279
Tint, 284
Tore, 142, 163
Translucent bodies, 1
Transparent bodies, 1
Trichromasia, abnormal, 315
Triplopia, binocular, 404
Troxler, phenomenon of, 292, 343

V

Veins, pulsation of, 239
Vision, "recurrent," 292
    single, antipathy to, 405
Visual acuity, 335
    central, 334
Visual, measurement of, 335, 336
    peripheral, 341
Visual acuity and illumination, relation between, 340
Visual field, projection of the, 366
Visual fields, antagonism of the, 276, 389
Visual impressions, projection of, 366
Visual line, 44
Volkmann, disc of, 356
    experiments of, 120

W

White, normal, of Koenig, 287
Wollaston, experiment of, 134
    prism of, 61

LIST OF AUTHORS

Abbe, 32, 34, 43
Agabobon, 293
Airy, 145
Alméida (d'), 388
Argyll Robertson, 256
Arlt, 104, 113, 196, 206, 226, 257, 265
Aubert, 68, 87, 314, 412, 413

Babbage, 232
Badal, 100, 101, 192, 243
Becker, 56, 414
Beer, 229
Bellarminoff, 231, 253
Benham, 278
v. Bezold, 137
Bidwell, 292
Bitzos, 252
Bjerrum, 243, 253, 278, 281, 342, 245
Blix, 56
Bœhm, 110, 113, 400, 406
Boll, 267, 269
Bouguer, 271, 274, 281
Bourgeois, 66
Bouty, 32
Bowman, 203, 228
Brewster, 181, 182, 191, 384, 385, 395
Brodhun, 294, 333
Brown-Séquard, 254
Brudzewski, 72, 87, 126, 128, 130
Bruecke, 132, 203, 228, 229, 232, 253, 393, 396
Bull (George), 155, 159, 163, 177, 178, 192, 193, 265
Burkhardt, 338
Burow, 184

Charpentier, 275, 281, 293, 297
Chibret, 326, 333
Coccius, 56, 60, 100, 200, 226, 189, 253
Cohn, 102, 256, 325
Coronat, 197
Cramer, 196, 197, 198, 216, 218, 219, 224, 227
Crétés, 363, 400
Crzellitzer, 190, 224, 228
Cuignet, 246, 253
Cumming, 229, 232, 253
Czermak, 90, 187, 200

Daae, 325
Dalton, 317, 323, 332
Darier, 178, 191
Darwin, 264
Davis, 292
Demicheri, 35, 52, 96, 113, 173, 209, 214, 222, 245, 253
Descartes, 9, 26, 195
Dieterici, 285, 312, 316, 321, 333
Dimmer, 112, 113
Dobrowolsky, 155
Dojer, 346
Dollond, 133
Doncan, 181, 182, 191
Donders, 61, 66, 100, 103, 106, 108, 109, 110, 113, 146, 150, 163, 181, 182, 189, 193, 203, 237, 264, 316, 318, 346, 349, 352, 358, 359, 365, 400, 406, 414
Dove, 289, 388, 389
Druault, 188, 189, 191
Dubois (Raphaël), 59
Dubois-Reymond, 256, 265, 269

Ebbinghaus, 331, 333
Eissen, 150
Eriksen, 68, 70, 72, 87, 155
Euler, 133

Fechner, 270, 271, 272, 273, 274, 275, 277, 279, 280, 293, 294
Fick, 354, 359, 413
Foerster, 205, 228, 275, 280
Fontana, 205
Fraunhofer, 132, 135, 137, 284, 297
Fukala, 108

Galien, 392
Gariel, 32
Gauss, 23, 32, 33, 41
v. Genderen Stort, 268, 269
Giraud-Teulon, 387, 388, 392
Goulier, 146, 163
v. Graefe, 100, 129, 228, 241, 397, 402, 403, 405, 406
Graefe (Alfred), 198, 228, 402, 406
Green, 338
Groenouw, 342, 345
Guillery, 338, 342, 345

Haidinger, 187, 191
Hamer, 66
Hansen Grut, 280, 397, 399, 401, 402, 403, 406
Hay, 352
Heath, 32
Heine, 218, 226, 228
v. Helmholtz, 6, 32, 34, 37, 57, 59, 61, 66, 68, 95, 131, 135, 137, 146, 178, 198, 199, 200, 203, 205, 206, 219, 220, 222, 226, 228, 229, 231, 232, 233, 246, 253, 257, 260, 261, 262, 263, 264, 277, 298, 301, 302, 313, 322, 329, 330, 331, 332, 334, 335, 337, 346, 349, 354, 359, 378, 386, 387, 390, 395, 409, 413, 414
Hencke, 199
Henle, 44
Hensen, 199, 225
Hering, 264, 324, 330, 331, 332, 350, 358, 361, 372, 376, 408, 412, 413
Hermann, 153, 352, 413
Herschel, 32
Hess, 218, 226, 228
Heuse, 56
v. Hippel, 318, 319, 329
Hirschberg, 100, 400
Hocquard, 222
Holmgren, 322, 324
Holth, 72, 74, 222, 343, 344, 345
Home, 195
Hooke, 334, 335, 336, 345
Hueck, 198, 219, 228, 256, 358, 359
Huyghens, 332

Iwanoff, 226

Jackson, 125, 130, 210, 253
Jaeger, 241, 338
Jamin, 32
Javal, 44, 48, 60, 61, 62, 63, 66, 68, 73, 75, 87, 100, 107, 137, 146, 147, 150, 151, 153, 158, 162, 165, 224, 278, 289, 338, 339, 349, 356, 357, 359, 364, 365, 371, 375, 376, 386, 389, 393, 395, 400, 401, 403, 404, 405, 406, 413, 414
Johnsson, 91
Jurin, 94, 414

Kagenaar, 60
Kaiser, 371, 376, 413
Kepler, 46, 195, 392
Klein, 281, 413
Knapp (H), 146, 150, 163
Knapp, Jr., 372
Kœnig, 285, 287, 294, 299, 312, 316, 317, 321, 322, 326, 331, 333
Koster, 207, 209, 212, 220, 332, 333
Krause, 221, 228
Krenchel, 280, 281, 325, 333
v. Kries, 331, 333
Kuehne, 267, 269, 413

Laiblin, 187
Lambert, 271, 281, 287, 301, 332
Lamare, 360, 365
Landolt, 113, 358, 363, 400, 414
Langenbeck, 196, 228
Leber, 322
LeConte, 414
Leonardo da Vinci, 46
Leroy, 237, 250, 251, 253
Listing, 37, 46, 180, 191, 262, 348, 349, 351, 352, 353, 354, 355, 357, 358, 359, 374
Lorenz, 32

Mace dé Lépinay, 294, 332
Mackenzie, 414
Maddox, 399
Mannhardt, 204, 227
Mariotte, 286, 342, 343, 344, 354
Martin, 150, 155
Mascart, 95, 132
Masselon, 147, 213
Masson, 276, 277, 278, 280, 290, 300, 313
Matthiessen, 34, 37, 46, 68
Mauthner, 61, 113, 200, 323, 414
Maxwell, 298, 300, 302, 304, 305, 306, 308, 309, 310, 312, 313, 315, 316, 319, 320, 321, 328, 329, 332
Meissner, 355, 356, 359
Meyer (H.), 130, 287
Mile, 90, 94
Müller (H.), 184, 185, 191, 199, 203, 204, 205, 228, 266, 332
Müller (Joannes), 374, 375, 391, 396

Nagel, 364, 365, 396, 414
Newton, 6, 98, 285, 287, 301, 302, 303, 307, 312, 332, 392, 396
Nicati, 294, 332
Nordenson, 150, 163

Ostwalt, 112, 113, 154

Panum, 392, 396
Parent, 246, 250, 253
Parinaud, 279, 296, 297, 331, 333, 402
Petit (Jean Louis), 221, 228
Pfalz, 150
Pflüger, 112, 287, 325
Placido, 74, 147
Porta, 46
Porterfield, 392, 414
Pouillet-Müller, 32
Pravaz, 228, 257
Preyer, 323
Prentice, 365, 400
Purkinje, 48, 49, 50, 54, 56, 78, 97, 183, 186, 190, 196, 200, 227, 240, 257, 292, 293, 313, 332

Raehlmann, 402
Ragona Scina, 288
Ramsden, 195
Rayleigh, 310, 316, 332, 379
Rée, 166, 167, 168, 169, 170, 171, 175
Risley, 105
Rochon Duvignaud, 228
Rose, 326
Ruete, 241, 253, 358, 359

Salomansohn, 188, 191
Scheiner, 46, 91, 92, 94, 115, 120, 195, 300
Schioetz, 48, 60, 61, 62, 63, 68, 147, 150, 163, 165, 188, 189, 191, 224, 349, 400
Schlemm, 204
Schmidt-Rimpler, 245
Schweigger, 163, 402
Seebeck, 322, 325, 332
Smith (Robert), 345, 414
Snellen, 101, 337, 338, 339
Snellius, 9
Sous, 100
Stadfeldt, 41, 46, 112, 128, 129, 130, 224, 228, 257
Steiger, 66
Stellwag, 110, 113, 338, 345, 402
Stilling, 104, 325
Stokes, 162
Sturm, 138, 158, 163, 195
Sulzer, 67, 68, 70, 72, 87, 154, 155, 174.

Troxler, 292, 343, 344, 360
Tscherning, 46, 56, 61, 87, 94, 113, 130, 137, 163, 175, 191, 209, 228, 265, 332, 345, 359
Turk, 240

Uhthoff, 332

Vacher, 155
Verdet, 32
Vierordt, 187
Vœlkers, 199, 225
Volkmann, 120, 121, 130, 348, 356, 357, 358, 359, 391, 392, 396

Wecker (de), 106, 147, 213, 246, 400, 402
Werlein, 91
Weyde (v.d), 321
Wheatstone, 382, 385, 386, 392, 395, 405, 414
Wollaston, 60, 61, 134, 137, 162, 392
Wüllner, 32

Young, 37, 46, 56, 91, 92, 121, 122, 123, 124, 134, 135, 136, 145, 165, 172, 173, 187, 188, 192, 193, 201, 202, 203, 208, 209, 237, 265, 289, 308, 327, 328, 329, 331, 359, 377, 379, 381, 413, 414

Zeiss, 133
Zinn, 223
Zoellner, 408, 409, 412
Zumft, 332, 333

Nany